Comparison of Least Square Method with MLE Method to Approximate as Determine the Parameters of Lomax Distribution by Simulation

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Abstract

In this paper, we mention the concepts of one of the most important statistical distributions Lomax, Lomax distribution has vast results in may area. Simulation studies are conducted to evaluate the performances of the proposed method. Here the estimation methods used are MLE LSE and these methods compare with the MSE and TD methods.

Keywords: Lomax distribution, MLE, LSE, shape parameter, scale parameter, simulation experiments.

Introduction

The Lomax (1954) has developed in the Lomax distribution and it is conditionally also called as Pareto type-II distribution. Pareto type-II is a main come of probability distribution. It is essentially a Pareto distribution that support begins at zero.

It has many uses in solving the problems on statistical into which are mainly used for the lifetime data, medical, Business, Biological Scenarios etc. for finding out the originality and justifying the issues faced.

Lomax distribution consists of two parameters which are denoted by Lomax(α, β), α is a shape parameter and β is the scale parameter.

The probability density function (p.d.f.)

$$f\left(x;\alpha,\beta\right)=\frac{\alpha}{\beta}\left[1+\frac{x}{\beta}\right]^{-(\alpha+1)};x>0,\alpha,\beta>0 \quad (1)$$

Mathematical model of Lomax distribution

The Lomax distribution, the properties are presenting here. Cumulative distribution function (c.d.f):

$$F\left(x;\alpha,\beta\right)=1-\left[1+\frac{x}{\beta}\right]^{-\alpha};x>0,\alpha,\beta>0 \quad (2)$$

The survival function or reliability function:

$$S(x) = \left[1+\frac{x}{\beta}\right]^{-\alpha} \quad (3)$$

The hazard function:

$$H(x) = \frac{\alpha\beta}{1+\alpha x} \quad (4)$$

Mean and variance of Lomax distribution:

$$E\left(X\right)=\frac{\beta}{\alpha-1} \;\; ; \alpha > 1 \quad (5)$$

$$Var\left(X\right)=\frac{\beta^2\alpha}{\left(\alpha-1\right)^2\left(\alpha-2\right)} - \frac{\beta^2}{\left(\alpha-1\right)^2} \;\; ; \;\; \alpha > 2 \quad (6)$$

Inverse Transformation method to generate a sample data for given parameters'

$$x=\beta\left(\left[1-u\right]^{-\frac{1}{\alpha}}-1\right); \;\; \text{u} : \;\; \text{U}\left[0,1\right] \quad (7)$$

where u ~ uniform(0,1), and the parameters α, β are known.

Parameter estimation methods:

Maximum Likelihood Estimation Method

$$L\left(x;\alpha,\beta\right)=\prod_{i=1}^{n} f\left(x;\alpha,\beta\right)$$

$$=\frac{\alpha^n}{\beta^n}\prod_{i=1}^{n}\left[1+\frac{x_i}{\beta}\right]^{-(\alpha+1)}$$

$$\ln\left[L\left(x;\alpha,\beta\right)\right]=n\ln\left(\alpha\right)-n\ln\left(\beta\right)-\left(\alpha+1\right)\sum_{i=1}^{n}\ln\left[1+\frac{x_i}{\beta}\right]$$

(8)

Estimation for the parameters

$$\hat{\alpha}_{MLE} = \frac{n}{\sum_{i=1}^{n} \ln\left[1+\frac{x_i}{\beta}\right]} \tag{9}$$

$$\hat{\beta}_{MLE} = \frac{1+\alpha}{n\beta} \sum_{i=1}^{n} \left[\frac{x_i}{\beta + x_i}\right] - \frac{1}{\beta} \tag{10}$$

Least Square Estimation Method:

The Estimation method used for this technique is a least square method. The least square method is the statistical process to find the best possible result for a set of values. It is mostly useful in solving and identifying the issues in DRDO, Medical, Engineering relate problem etc. In this we take the linear relation between scale and shape. We use the Least square method for the estimation of Lomax parameters.

Good and Kao had developed the graphical procedure used for Lomax distribution. The perseverance of parameters of the Lomax distribution is achieving well-defined or analytically.

$$F(x;\alpha,\beta) = F = 1 - \left[1+\frac{x}{\beta}\right]^{-\alpha}$$

$$1 - F = \left[1+\frac{x}{\beta}\right]^{-\alpha}$$

$$\frac{1}{1-F} = \left[1+\frac{x}{\beta}\right]^{\alpha}$$

$$Log\left(\frac{1}{1-F}\right) = \alpha \log\left[1+\frac{x}{\beta}\right]$$

$$= \alpha \log\left[\frac{\beta + x}{\beta}\right]$$

$$= \alpha \log[\beta + x] - \alpha \log[\beta] \tag{11}$$

$$y = Ax + B$$

$$S\left(\lambda_0,\lambda_1\right)=\left(A+Bx_1-y_1\right)^2+\left(A+Bx_2-y_2\right)^2+\cdots+\left(A+Bx_n-y_n\right)^2=\sum_{i=1}^{n}\left(A+Bx_i-y_i\right)^2$$

The essential and enough conditions for the function $S\left(A,B\right)$ to get the utmost result are given by:

$$\begin{aligned}&\frac{\partial S}{\partial A}=2\sum_{i=1}^{n}\left(A+Bx_i-y_i\right)=0\\&\frac{\partial S}{\partial B}=2\sum_{i=1}^{n}x_i\left(A+Bx_i-y_i\right)=0\end{aligned}\tag{12}$$

from where a system of linear algebraic equations for determination of coefficients A and B is gained:

$$\begin{aligned}&A\cdot n+B\cdot\sum_{i=1}^{n}x_i=\sum_{i=1}^{n}y_i\\&A\cdot\sum_{i=1}^{n}x_i+B\cdot\sum_{i=1}^{n}x_i^{\,2}=\sum_{i=1}^{n}x_iy_i\end{aligned}\tag{13}$$

By determination of determinants of equation systems above equations:

$$D=\begin{vmatrix}n & \sum_{i=1}^{n}x_i\\ \sum_{i=1}^{n}x_i & \sum_{i=1}^{n}x_i^{\,2}\end{vmatrix},D_0=\begin{vmatrix}\sum_{i=1}^{n}y_i & \sum_{i=1}^{n}x_i\\ \sum_{i=1}^{n}x_iy_i & \sum_{i=1}^{n}x_i^{\,2}\end{vmatrix},D_1=\begin{vmatrix}n & \sum_{i=1}^{n}y_i\\ \sum_{i=1}^{n}x_i & \sum_{i=1}^{n}x_iy_i\end{vmatrix}\tag{14}$$

values of required coefficients are gained:

$$A=\frac{D_0}{D}=\frac{\sum_{i=1}^{n}x_i^{\,2}\cdot\sum_{i=1}^{n}y_i-\sum_{i=1}^{n}x_i\cdot\sum_{i=1}^{n}x_iy_i}{n\cdot\sum_{i=1}^{n}x_i^{\,2}-\left(\sum_{i=1}^{n}x_i\right)^2}\tag{15}$$

$$B=\frac{D_0}{D}=\frac{n\sum_{i=1}^{n}x_iy_i-\sum_{i=1}^{n}x_i\cdot\sum_{i=1}^{n}y_i}{n\cdot\sum_{i=1}^{n}x_i^{\,2}-\left(\sum_{i=1}^{n}x_i\right)^2}\tag{16}$$

Depending on the association with the distribution parameters (*α, β*) and the coefficients of the straight line equation, is giving below:

$$Y = Log\left(\frac{1}{1-F}\right);\ A = \alpha;\ X = \log\left[\beta + x\right];$$

$$B = -\alpha \log\left[\beta\right] \rightarrow -\frac{B}{\alpha} = \log\left[\beta\right] \rightarrow \beta = \exp\left(-\frac{B}{\alpha}\right) \quad (17)$$

Computational Results:

The open minded result of our work is to compare the LSE method, and MLE method. We had taken the occasional samples by the known parameters. For every sample, we have taken the change of size from 5 to 100. To calculate the variations and find the total deviations (TD) for each method is in order: - α and β are parameters, $\hat{\alpha}$ and $\hat{\beta}$ are the estimated parameters

To compare, we calculated the total deviation (TD) for each method as follows:

$$TD = \left|\frac{\hat{\alpha} - \alpha}{\alpha}\right| + \left|\frac{\hat{\beta} - \beta}{\beta}\right|$$

In order to understand the accuracy of the Lomax estimation method, most of the studies address the question as a statistical error and calculate the mean square error (MSE) based on the measured data.

$$MSE = \frac{\sum_{i=1}^{n}\left(\hat{F}\left(x_i\right) - F\left(x_i\right)\right)^2}{n}$$

where $\hat{F}\left(x_i\right)$ is the measured value and $F\left(x_i\right)$ is the Lomax parameters based calculated value.

Simulation Study:

Two know the behavior of the shape and scale parameter. We find the values try using the Lomax distribution. In this paper, we had discussed about the results of the simulations study which has been done in different methods. The main course of the paper is to compare the values or results in the two different methods used for the estimation of shape. (α) = 0.5 (0.5) 3 and scale (β) = 0.5 (0.5) 3 parameters of two parameter Lomax distribution and used to generated 10,000 samples of sizes n= 5,10,20,30,50,75,100,200,250,300,400,500, and 1,000. The estimates are comparing using the values of MSE and TD. Here we used the R programming with version 4.0.5 for doing this process in finding out the statistical values.

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Table:

n	ALPHA	BETA	LSE				MLE				Results
			ALPHA	BETA	TD	MSE	ALPHA	BETA	TD	MSE	
5	0.5	0.5	-0.0148	0.3466	1.3364	0.0521	0.0093	0.5	0.9954	0.0025	MLE
10	0.5	0.5	-0.0257	0.3466	1.3583	0.0704	0.0038	0.5001	0.9926	0.8993	LSE
15	0.5	0.5	-0.0289	0.3466	1.3646	0.0756	0.0049	0.5001	0.9905	0.8794	LSE
20	0.5	0.5	-0.0278	0.3466	1.3625	0.0726	0.0058	0.5002	0.9888	0.8655	LSE
25	0.5	0.5	-0.0378	0.3466	1.3825	0.0895	0.0087	0.5	0.9965	0.0024	MLE
30	0.5	0.5	-0.037	0.3466	1.3808	0.0872	0.0053	0.5	0.9965	0.0025	MLE
50	0.5	0.5	-0.0489	0.3466	1.4046	0.1033	0.0085	0.5001	0.9968	0.0025	MLE
75	0.5	0.5	-0.0582	0.3466	1.4233	0.1143	0.0037	0.5	0.9964	0.0026	MLE
100	0.5	0.5	-0.0605	0.3466	1.4279	0.1161	0.0157	0.5001	0.9688	0.0027	MLE
5	1	0.5	-0.0403	0.6931	1.3875	0.0027	0.0077	0.5001	0.9963	0.0025	MLE
5	1.5	0.5	-0.0353	1.0397	1.3775	0.021	0.0063	0.5001	0.9969	0.0025	MLE
10	1	0.5	-0.112	0.6931	1.5309	0.0294	0.0064	0.5003	0.9941	0.7629	LSE
15	1	0.5	-0.1507	0.6931	1.6083	0.0548	0.0041	0.5	0.9959	0.0026	LSE
20	1	0.5	-0.1788	0.6931	1.6645	0.0783	0.0083	0.5005	0.9928	0.7474	LSE
25	1	0.5	-0.1857	0.6931	1.6782	0.0851	0.0091	0.5007	0.9922	0.737	MLE
30	1	0.5	-0.2407	0.6931	1.7883	0.1536	0.0037	0.5	0.9975	0.0025	MLE
50	1	0.5	-0.2867	0.6931	1.8803	0.236	0.0037	0.5	0.9963	0.0026	MLE
10	1.5	0.5	-0.1405	1.0397	1.5879	0.00060	0.0064	0.5003	0.9962	0.6274	LSE
15	1.5	0.5	-0.2358	1.0397	1.7784	0.0435	0.0037	0.5	0.9975	0.0025	LSE
20	1.5	0.5	-0.3134	1.0397	1.9337	0.1905	0.0071	0.5006	0.9966	0.6244	LSE
5	2.5	0.5	-0.0142	1.7329	1.3353	0.0752	0.0075	0.5001	0.9965	0.0025	MLE
10	2.5	0.5	-0.0445	1.7329	1.3958	0.0714	0.0087	0.5	0.9965	0.0024	MLE
15	2.5	0.5	-0.1034	1.7329	1.5137	0.0542	0.0059	0.5001	0.9978	0.0025	LSE
20	2.5	0.5	-0.1185	1.7329	1.5439	0.0476	0.0066	0.5	0.9974	0.0025	LSE
25	2.5	0.5	-0.1835	1.7329	1.6739	0.0151	0.0091	0.4999	0.9965	0.0024	MLE
30	2.5	0.5	-0.2351	1.7329	1.777	0.00010	0.005	0.5	0.9967	0.0025	MLE

Results and Discussion:

The outcome of this simulations is indicated in the above table. In the table, it has been noticed that, when the size increases, MSE decrease and respectively TD increases. Considerably, the tiny size, the execution of the models will vary from simultaneously.

In the above examination, of the content in the content in the table, MLE is far better than the method of LSE. It hasn't been any consistency in the executions of estimates by the LSE method. MLE method is much more better than the LSE method, MLE method is very much available to use by the practitioners.

Conclusion:

Here we had done an observation and recommended from LSE method to Lomax distribution. Most of the simulations have been modified and tested to the different verities.

MLE method is mostly unique for the estimation of the model parameters. This method is very helpful and it is most attractive method in different areas of lifetime data, execution of machine, engineering area, etc.,

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