

Secondaryk-Kernel Symmetric Intuitionistic Fuzzy Matrices

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Abstract

The secondary k- kernel symmetric Intuitionistic fuzzy matrices are described in this study as well as examples. Discussion is held regarding the relationships among s- k- kernel symmetric Intuitionistic fuzzy matrix, s- kernel symmetric Intuitionistic fuzzy matrix, k- kernel symmetric intuitionistic fuzzy matrix and kernel symmetric intuitionistic fuzzy matrix. For a matrix to be an Intuitionistic fuzzy matrix with an s-k kernel symmetric, necessary and sufficient conditions are established.

Keywords: Intuitionistic fuzzy matrix, kernel symmetric

Intuitionistic fuzzy matrix, s-k- kernel symmetric Intuitionistic fuzzy matrix.

1. Introduction

We deal with Intuitionistic fuzzy matrices, concluded a fuzzy algebra F with finding $[0, 1]$ under maximum and minimum operations, are the only matrix taken into consideration in this study. If $R(A) = R(A^T)$, then a Intuitionistic fuzzy matrix A is range symmetric Intuitionistic fuzzy matrix, and if $N(A) = N(A^T)$, then it is kernel symmetric Intuitionistic fuzzy matrix. It is commonly known that the concepts of range and kernel symmetry apply to complex matrices. For an Intuitionistic fuzzy matrix, this fails. This inspired us to research s-k kernel symmetric Intuitionistic fuzzy matrices. The study of secondary symmetric Intuitionistic fuzzy matrices whose elements are symmetric about the secondary diagonal, was started by Lee [1]. Communication theory applications of persymmetric fuzzy matrices, or matrices that are symmetric about both the diagonals, have been examined by

Cantoni and Paul [2]. As a generalization of s-real and s-hermitian matrices, Hill and Waters [3] created a theory of k-real and k-hermitian matrices. Meenakshi and Jayashree [5] build k- kernel symmetric fuzzy matrices in a manner similar to how k-real and k-hermitian of a complex matrix [3] are developed.

As a generalization of secondary hermitian and hermitian matrices, Meenakshi and Krishnamoorthy[6] introduced the idea of s-k hermitian matrices. In this study, as a special instance of the findings on complex matrices discovered in [7][8], we extend [9] the notion of s-k kernel symmetric intuitionistic fuzzy matrix and Equivalent conditions for various g-inverses of secondary k- kernel Intuitionistic fuzzy matrices to be secondary K-Kernel symmetric are determined.

2. Preliminaries

Let K be the related permutation matrix, and let 'k' be a fixed product of disjoint transpositions in $S_n = \{1, 2, \dots, n\}$. Throughout, let v be the permutation intuitionistic fuzzy matrix with units in its secondary diagonal.

Let the function is defined $\kappa(x) = (x_{k[1]}, x_{k[2]}, x_{k[3]}, \dots, x_{k[n]}) \in F_{n \times 1}$ for $x = x_1, x_2, \dots, x_n \in F_{[1 \times n]}$, where K is involuntary, the following are satisfied the associated permutation matrix where K is a permutation matrix.

$$(P.2.1) \quad KK^T \sqsubseteq K^T K \sqsubseteq I_n, K \sqsubseteq K^T, K^2 \sqsubseteq I \quad \text{and} \quad (\kappa) \sqsubseteq Kx$$

By the definition of V ,

$$(P.2.2) \quad V \sqsubseteq V^T, VV^T \sqsubseteq V^T V \sqsubseteq I_n \text{ and } V^2 \sqsubseteq I$$

$$(P.2.3) \quad N(A) \sqsubseteq N(AV), N(A) \sqsubseteq N(AK)$$

$$(P.2.4) \quad (AV)^T \sqsubseteq VA^T, (VA)^T \sqsubseteq A^T V$$

If A^+ exists, then

$$(P.2.5) \quad (AV)^\square \sqsubseteq VA^\square, (VA)^\square \sqsubseteq A^\square V$$

Definition 2.1. Intuitionistic fuzzy matrix A is kernel symmetric intuitionistic fuzzy matrix iff $N(A) \sqsubseteq N(A^T)$.

Lemma 2.1. For Intuitionistic fuzzy matrix A belongs to F_n and a permutation Intuitionistic

fuzzymatrix P , null space of A equal to null space of B iff $N(PAP^T) \sqsubseteq N(PBP^T)$.

Lemma 2.2. Intuitionistic fuzzymatrix $A = KA^T$ $K \sqsubseteq KA = (KA)(KA)^T(KA)$, Intuitionistic fuzzymatrix $\sqsubseteq \sqsubseteq AK = (AK)(AK)^T(AK)$ Intuitionistic fuzzymatrix.

3. Secondary k -kernel symmetric Intuitionistic fuzzy matrices

Definition 3.1. For Intuitionistic fuzzymatrix A belongs to F_n iss-symmetric Intuitionistic fuzzymatrix iff $A \sqsubseteq VA^TV$

Definition 3.2. For Intuitionistic fuzzymatrix A belongs to F_n iss-kernel symmetric Intuitionistic fuzzymatrix if $N(A) \sqsubseteq N(VA^TV)$.

Definition 3.3. For Intuitionistic fuzzymatrix A belongs to F_n iss- k -kernel symmetric Intuitionistic fuzzymatrix if $N(A) \sqsubseteq N(KVA^TVK)$.

Lemma 3.1. A Intuitionistic fuzzymatrix A belongs to F_n iss-kernel symmetric Intuitionistic fuzzymatrix $\sqsubseteq \sqsubseteq V$ is kernel symmetric Intuitionistic fuzzymatrix $\sqsubseteq VA$ is kernel symmetric Intuitionistic fuzzymatrix.

Proof.

A iss-kernel symmetric Intuitionistic fuzzymatrix [By Definition 3.2][By P. 2.2]
 $\sqsubseteq N(A) \sqsubseteq N(VA^TV)$

$$\sqsubseteq N(AV) \sqsubseteq N((AV)^T)$$

$$\sqsubseteq N(VAVV^T) \sqsubseteq N(VVA^TV)$$

$$\sqsubseteq N(VA) \sqsubseteq N((VA)^T)$$

$$\sqsubseteq VA \text{ is kernel symmetric.}$$

Remark 3.1. To be more specific, when $\kappa(i) = i$ for $i = 1, 2, \dots, n$ then the corresponding Intuitionistic fuzzy permutation matrix K reduces to the identity Intuitionistic fuzzy matrix and Definition (3.3) reduces to $N(A) = N(VA^TV)$ implying that the matrix is s -kernel symmetric.

Remark 3.2. The equivalent permutation intuitionistic fuzzy matrix K simplifies to V for $k(i) = n - i + 1$. $N(A) = N(A^T)$ in definition (3.3) implies that A is a kernel symmetric intuitionistic fuzzy matrix.

Remark3.3. We observe that s-k-symmetric Intuitionistic fuzzy matrix is s-k-kernel symmetric Intuitionistic fuzzy matrix because if A is s-k-symmetric then $A = KVA^T VK$, which means that A is s-k-kernel symmetric Intuitionistic fuzzy matrix, then $N(A) = N(KVA^T VK)$.

The reverse, however, is not always true. This is demonstrated in the example that follows.

Example3.1. For $k=(1,2)$, $A = \begin{bmatrix} \langle 1,0 \rangle & \langle 0.2,0.3 \rangle \\ \langle 0.2,0.3 \rangle & \langle 0.4,0.3 \rangle \end{bmatrix}$,

$$KVA^T VK = \begin{bmatrix} \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 1,0 \rangle & \langle 0.2,0.3 \rangle \\ \langle 0.2,0.3 \rangle & \langle 0.4,0.3 \rangle \end{bmatrix} \\ = \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle \end{bmatrix}, \\ KVA^T VK = \begin{bmatrix} \langle 0.4,0 \rangle & \langle 0.2,0 \rangle \\ \langle 0.2,0 \rangle & \langle 1,0.2 \rangle \end{bmatrix} \neq A$$

Here $A \neq KA^T K$

Therefore A is symmetric IFM, κ -symmetric IFM, s- κ -kernel symmetric IFM but not κ -symmetric IFM.

Example3.2. For $k=(1,2)$ $V = \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle \end{bmatrix}$, $A = \begin{bmatrix} \langle 0.5,0.2 \rangle & \langle 0.4,0.2 \rangle \\ \langle 0.4,0.2 \rangle & \langle 0.5,0.2 \rangle \end{bmatrix}$

$$KVA^T VK = \begin{bmatrix} \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0.5,0.2 \rangle & \langle 0.4,0.2 \rangle \\ \langle 0.4,0.2 \rangle & \langle 0.5,0.2 \rangle \end{bmatrix} \\ = \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle \end{bmatrix} \\ KVA^T VK = \begin{bmatrix} \langle 0.5,0.2 \rangle & \langle 0.4,0.2 \rangle \\ \langle 0.4,0.2 \rangle & \langle 0.5,0.2 \rangle \end{bmatrix} = A$$

A is symmetric, s- κ -symmetric and hence therefore s-k-kernel symmetric.

Example3.3. For $k=(1,2)(3)$

$$K = \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \end{bmatrix}, V = \begin{bmatrix} \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \end{bmatrix}$$

$$K \neq V, K \neq I \text{ and } KV \neq VK$$

$$A = \begin{bmatrix} \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 0.2,0.3 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 0.2,0.3 \rangle & \langle 0.4,0.5 \rangle & \langle 0,0 \rangle \end{bmatrix}$$

$$KV = \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \end{bmatrix}$$

$$KV = \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \end{bmatrix}$$

$$VK = \begin{bmatrix} \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \end{bmatrix}$$

$$VK = \begin{bmatrix} \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \end{bmatrix}$$

$$KVA^T VK = \begin{bmatrix} \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0,0 \rangle & \langle 0.2,0.3 \rangle & \langle 0.2,0.3 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0.4,0.5 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 1,0 \rangle & \langle 0,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \end{bmatrix}$$

$$KVA^T VK = \begin{bmatrix} \langle 1,0 \rangle & \langle 0.4,0 \rangle & \langle 0,0 \rangle \\ \langle 0,0 \rangle & \langle 0,0 \rangle & \langle 1,0 \rangle \\ \langle 0.2,0 \rangle & \langle 1,0 \rangle & \langle 0,0 \rangle \end{bmatrix} \neq A$$

$$A \neq KVA^T VK$$

Hence A is not s-k-symmetric. But s-k- kernel symmetric.

$$i.e) N(A) \sqcap N(KVA^T VK) = \langle 0,0 \rangle$$

Theorem 3.1. For Intuitionistic fuzzy matrix A belongs to F_n the following are equivalent

$$(1) N(A) = N(KVA^T VK)$$

$$(2) N(KVA) = N((KVA)^T)$$

$$(3) N(AKV) = N((AKV)^T)$$

$$(4) N(AVK) = N((AVK)^T)$$

$$(5) N(VKA) = N((VKA)^T)$$

$$(6) N(VA) = N(K(VA)^T K)$$

$$(7) N(AV) = N(K(AV)^T K)$$

$$(8) N(AK) = N(V(AK)^T V)$$

$$(9) N(KA) = N(V(KA)^T V)$$

$$(10) N(A^T) = N(KVA)$$

$$(11) N(A) = N(KVA^T)$$

Proof:

$$N(A) = N(KVA^T VK)$$

$$\square N(A) = N(KVA^T) \quad (\text{Definition 3.2})$$

$$\square N(AVK) = N((AVK)^T) \quad (\text{by P}_{2.3})$$

$$\square \square V \square \square \square \text{ is kernel symmetric}$$

$$\square \square VK)(AVK)(VK^T) \text{ (kernel symmetric Lemma 2.1)}$$

$$\square VKA \text{ is kernelsymmetric}$$

$$\square KA \quad (\text{s- kernel symmetric})$$

Therefore , (1) iff (4) iff (5) iff (9) holds

(2) iff (6)

$$\square N(KVA) = N((KVA)^T)$$

$$\square N(K^T KVA) = N((K^T KVA)^T)$$

$$\square N(VA) = N(VA)^T$$

$$\square N(VA) = N(K(VA)^T K)$$

Therefore , VA is K kernel symmetric

(2) iff (6) holds

(2) iff (10)

$$\square N(KVA) = N((KVA)^T)$$

$$\square N(KVA) = N(A^T) \quad (\text{By P. 2.3})$$

Thus (2) iff (10) hold.

(4) iff (11)

$$\square N(AVK) = N((AVK)^T)$$

$$\square N(A) = N(KVA^T) \quad (\text{By P. 2.3})$$

(4) iff (11) hold

(1) iff (4) iff (7)

$$\square N(A) = N(KVA^T VK)$$

$$\square N(A) = N((AVK)^T)$$

$$\square N(AVK) = N((AVK)^T) \quad (AVK \text{ is kernel symmetric})$$

$$\square N(AVKK^T) = N((AVKK^T)^T)$$

$$\square N(AV) = N((AV)^T)$$

$$\square N(AV) = N(K(AV)^T K) \quad (AV \text{ is k- kernel symmetric})$$

(1) iff (4) iff (7) hold

(3) iff (8)

$$\square N(AKV) = N((AKV)^T)$$

$$\square N(AK) = N(V(AK)^T V)$$

$\square \square \square \square \square \square \square \square \square \square \square \square \square \square \square \square$ iff (8) holds

Hence the Theorem

Corollary 3.1. For Intuitionistic fuzzy matrix A belongs to F_n the following are equivalent

$$(1) N(A) = N(VA^T V)$$

$$(2) N(VA) = N((VA)^T)$$

$$(3) N(AV) = N((AV)^T)$$

$$(4) N(A^T) = N(VA)$$

$$(5) N(A) = N(VA^T)$$

Proof: (1) implies (2)

$$\square N(A) = N(VA^T V)$$

$$\square N(VA) = N(VVA^T V)$$

$$\square N(VA) = N(V^2 A^T V)$$

$$\square N(VA) = N((VA)^T)$$

(1) implies (2) hold

(1) implies (3)

$$\square N(A) = N(VA^T V)$$

$$\square \square \square \square \square \square \square N(AV) = N(VA^T VV)$$

$$\square \square \square \square \square \square \square N(AV) = N(VA^T V^2)$$

$$\square \square \square \square \square \square \square N(AV) = N(AV^T)$$

(1) implies (3) holds

(1) implies (4)

$$N(A^T) = N((VA^T V)^T)$$

$$N(A^T) = N((VA^T V)^T)$$

$$N(A^T) = N(VA)$$

(1) implies (4) holds

(1) implies (5) easily verified

(5) implies (1) easily

verified

Hence the Theorem

Lemma 3.2. For Intuitionistic fuzzymatrix A belongs to F_n , if $(VKA)^+$ exists $\square$ $(KA)^+$ exists $\square$ A^+ exists.

Proof: $(VKA)^+$ exists iff $(VKA)^T \in (VKA)\{1\}$

$$\text{iff } VKA = VKA(VKA)^T VKA$$

$$\text{iff } VKA = VKA(KA)^T VV(KA)$$

$$\text{iff } KA = KA(KA)^T (KA)$$

$$\text{iff } (KA)^+ \text{ exists iff } A^+ \text{ exists} \quad [\text{Lemma 3.4 in [8]}]$$

Lemma 3.3 For Intuitionistic fuzzymatrix A belongs to F_n if $(KVA)^+$ exists $\square$ A^+ exists.

Proof: $(A)^+$ exists iff $(VA)^+$ exist

$$\text{iff } VA = (VA)(VA)^+(VA)$$

$$KVA = K(VA)(VA)^+(VA)$$

$$KVA = K(VA)(VA)^+ KK(VA)$$

$$KVA = (KVA)(KVA)^+(KVA)$$

$$(KVA)^T \in (KVA)\{1\}$$

Therefore, $(KVA)^+$ exist.

Theorem 3.2. For Intuitionistic fuzzymatrix A belongs to F_n . Then any two of the following conditions imply the other one.

$$(1) \quad N(A) = N(KA^T K)$$

$$(2) \quad N(A) \sqsubseteq N(KVA^T VK).$$

$$(3) \quad N(A^T) = N((KAV)^T)$$

Proof: 

$$(1) \& (2) \sqsubseteq (3)$$

By Theorem 3.1

$$N(A) \sqsubseteq N(KVA^T VK) \text{ implies } N(A) = N((AVK)^T)$$

$$N(KAK) \sqsubseteq N(VA^T K) \quad (\text{Lemma 2.1})$$

$$N(A) \sqsubseteq N(KA^T K)$$

$$N(KAK) \sqsubseteq N(A^T) \quad (\text{Lemma 2.1})$$

$$N(A^T) = N((KAV)^T)$$

Therefore, (3) hold

$$(1) \& (3) \Rightarrow (2)$$

$$N(A) = N(KA^T K) \text{ implies } N(KAK) = N(A^T)$$

$$\text{Hence (1) and (3) } N(KAK) = N((KAV)^T)$$

$$N(A) = N(KVA^T) \quad (\text{Lemma 2.1})$$

$$A \text{ is s-k kernel symmetric} \quad [\text{Theorem 3.1}]$$

Therefore (2) holds.

$$(2) \& (3) \Rightarrow (1)$$

A is s-k kernel symmetric

$$N(A) = N(KVA^T)$$

$$N(KAK) = N((VA^T K)^T)$$

Hence (2) and (3)

$$N(KAK) = N(A^T)$$

$$N(A) = N(KA^T K)$$

therefore (1) hold.

Hence the theorem.

4. s- k-Kernel Symmetric IFM Regular IFM

This section has established the existence of several generalized inverses of IFM in F_n . It is determined additional comparable conditions for different g-inverses of an s-k-kernel symmetric IFM to be s-k kernel symmetric IFM. Generalized inverses belonging to the sets $A \{1, 2\}$, $A \{1, 2, 3\}$ and $A \{1, 2, 4\}$ of s-k-kernel symmetric IFM A are characterized.

Definition:4.1 The IFM $A \in F_{m \times n}$ is said to be regular (or g-inverse) if there exists another IFM, $X \in F_{n \times m}$ such that $AXA = A$. The generalized inverse of an IFM is not unique that is an IFM has many generalized inverses exists .

Definition:4.2 For IFM $A \in F_{m \times n}$ and another IFM, $X \in F_{n \times m}$ satisfies the given equation $AXA = A$ and $(AX)^T = AX$, then X is called least square generalized inverse of A which is called it as $A\{1,3\}$ inverses.

Definition:4.3 For IFM A and another IFM, $X \in F_{n \times m}$ satisfies the given equation $AXA = A$ and $(XA)^T = XA$, then X is called minimum norm generalized inverse of A which is called it as $A\{1,4\}$ inverses.

Definition:4.4 For IFM $(XA) \in F_{m \times n}$ and another IFM, $(XA)^T \in F_{n \times m}$ is said to be Moore Penrose inverse of XA if $XA(XA)^T XA = XA$, $(XA)^T XA(XA)^T = (XA)^T$, $[XA(XA)^T]^T = XA(XA)^T$ and $[(XA)^T XA]^T = (XA)^T XA$.

The Moore Penrose inverse of XA is denoted by $(XA)^+$

Theorem 4.1: Let A belongs to F_n , X belongs to $A\{1,2\}$ and AX, XA , are s- κ -Kernelsymmetric IFM. Then A is s- κ -kernel symmetric IFM $\Leftrightarrow X$ is s- κ -kernel symmetric IFM.

Proof: $N(KVA) = N(KVAXA) \subseteq N(XA)$ [since $A = AXA$]

$$= N(XVVA) = N(XVKKVA) \subseteq N(KVA)$$

$$\text{Hence, } N(KVA) = N(XA)$$

$$= N(KV(XA)^T VK) [XA \text{ is s- } \kappa\text{-kernel symmetric IFM}]$$

$$= N(A^T X^T VK)$$

$$= N(X^T VK)$$

$$= N((KVX)^T)$$

$$N((KVA)^T) = N(A^T VK)$$

$$= N(X^T A^T VK)$$

$$= N((KVAX)^T)$$

$$= N(KVAX) [\text{is s- } \kappa\text{-kernel symmetric}]$$

$$= N(KVX)$$

$$KVX \text{ is kernel symmetric} \Leftrightarrow N(KVA) = N((KVA)^T)$$

$$\Leftrightarrow N((KVX)^T) = N(KVX)$$

$$\Leftrightarrow KVX \text{ is kernel symmetric}$$

$$\Leftrightarrow X \text{ is s- } \kappa\text{-kernel symmetric.}$$

Theorem 4.2: Let A belongs to F_n , $X \in A \{1,2,3\}$, $N(KVA) = N((K VX)^T)$. Then A is s - κ -Kernel symmetric IFM $\Leftrightarrow X$ is s - κ -kernel symmetric IFM.

Proof: Since X belongs to $A \{1,2,3\}$ we have $AXA = A$, $XAX = X$, $(AX)^T = AX$

$$N((KVA)^T) = N(X^T A^T VK) \quad [\text{By using } A = AXA]$$

$$= N(KV(AX)^T)$$

$$= N((AX)^T) \quad [\text{By P.2.3}]$$

$$= N(AX) \quad [(AX)^T = AX]$$

$$= N(X) \quad [\text{By using } X = XAX]$$

$$= N(KVX) \quad [\text{By P.2.3}]$$

$$KVA \text{ is kernel symmetric IFM} \Leftrightarrow N(KVA) = N((KVA)^T)$$

$$\Leftrightarrow N((K VX)^T) = N(KVX)$$

$$\Leftrightarrow KVX \text{ is kernel symmetric}$$

$$\Leftrightarrow X \text{ is } s\text{-}\kappa\text{-kernel symmetric.}$$

Theorem 4.3: Let A belongs to F_n , $A \{1,2,4\}$ $N((KVA)^T) = N(KVX)$. Then A is s - κ -Kernel symmetric IFM $\Leftrightarrow X$ is s - κ -kernel symmetric IFM.

Proof: Since $X \in A \{1, 2, 4\}$, we have $AXA = A$, $XAX = X$, $(XA)^T = XA$

$$N(KVA) = N(A) \quad [\text{By P. 2.3}]$$

$$= N(XA) \quad [XAX = A, AXA = A] = N((XA)^T) \quad [(XA)^T = XA]$$

$$= N(A^T X^T)$$

$$= N(X^T)$$

$$= N((K VX)^T). \quad [P.2.3]$$

$$KVA \text{ is kernel symmetric IFM} \Leftrightarrow N(KVA) = N((KVA)^T)$$

$$\Leftrightarrow N((K VX)^T) = N(KVX)$$

$$\Leftrightarrow KVX \text{ is kernel symmetric IFM}$$

$$\Leftrightarrow X \text{ is } s\text{-}\kappa\text{-kernel symmetric IFM.}$$

In particular for $K = I$, the above Theorems reduces to equivalent conditions for various g -inverses of a s -kernel symmetric IFM to be secondary kernel symmetric IFM.

Corollary 4.1: Let A belongs to F_n , X belongs $A \{1, 2\}$ and AX , XA are s -kernel symmetric IFM. Then A is s -kernel symmetric IFM $\Leftrightarrow X$ is s -kernel symmetric IFM.

Corollary 4.2: Let A belongs to F_n , X belongs to $A \{1, 2, 3\}$, $N(KVA) = N((VX)^T)$. Then A is s -kernel symmetric IFM $\Leftrightarrow X$ is s -kernel symmetric IFM.

Corollary 4.3: Let A belongs to F_n , X belongs to $A \{1, 2, 4\}$, $N((VA)^T) = N(VX)$. Then A is s -kernel symmetric IFM $\Leftrightarrow X$ is s -kernel symmetric IFM.

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