

Site Selection of Solar Plant based on Normal Wiggly Hesitant Bipolar-Valued Fuzzy Set

R. Rajalakshmi¹, Dr. K. Julia Rose Mary²

¹ Research Scholar, Department of mathematics, Nirmala college for women, India.

² Associate Professor, Department of mathematics, Nirmala college for women, India.

Corresponding author: rajimat2020@gmail.com

Article Info

Page Number: 4558 - 4583

Publication Issue:

Vol 71 No. 4 (2022)

Abstract

Hesitant fuzzy set is a popular and powerful tool to express multiple values in decision making. If they are unable to provide a resolution, it is important to prioritize and prioritize different values. In this study, we explore the potential/deep uncertainties of decision makers(DMs) when using HBFSs to disclose their rating information in the decision-making process. Here, we propose the Normal wiggly Hesitant Bipolar Fuzzy set (NWHBFS) as an extension of the normal wiggly hesitant fuzzy set. The NWHBFS can express deep opinions about positive and negative membership information. NWHFSs can hold hesitant fuzzy information and deeply uncertain information into hesitant fuzzy information. After that, we propose the normal wiggly hesitant bipolar fuzzy information scoring function to differentiate NWHBFS. Furthermore, in order to understand and apply NWHBFS, we explore some of the characteristics of NWHBFs and propose NWHBFS preliminary aggregation operators. Finally, we use a numerical example.

Article History

Article Received: 25 March 2022

Revised: 30 April 2022

Accepted: 15 June 2022

Publication: 19 August 2022

Keywords: Hesitant bipolar ,Normal wiggly hesitant bipolar fuzzy set , Solar site selection, MCDM (multi criteria decision making) and COCOSO (Combined Compromise Solution) method.

1 Introduction

In real life there are many things that many characters decide, and more and more people face decision problems in all aspects of their business life and personal life, such as what is the best location to open a new restaurant, How does a farmer decide to test a new crop and so on. Faced

with these results, it is important to help decision makers express their evaluation information based on some naturally conflicting criteria and to summarize all the information for the final ranking decisions of the alternatives. Thus a series of MADM(multi-attribute decision making)models have been proposed and extensively used in practice. Since Zadeh (1965) first proposed the concept of fuzzy sets to describe the ambiguity, many efficient representation models have beenuniversallystudied by scholars, such as interval-valued intuitionistic fuzzy sets (1989), hesitant fuzzy sets(HFs) (2010), interval type-2 FSs(2000), pythagorean FSs (2019) and so on. Goal of the aforementioned different types of fuzzy sets is to more effectively communicate the decision makers unknown and complex evaluation information. In recent decades, these different types of FSs have been widely organized in academic research and have achieved great research achievements, according to different types of application environment. The embedding of HFs in them is defined by Torra (2010), which is free to deal with uncertainties, allowing DMs to describe their evaluation information with a set of values belonging to $[0,1]$. However, the main problem for DMs in a complex real MCDM is how to determine the smoothness value based on a given criterion to express their uncertainty and ambiguity. For example, if the DMs cannot decide which specific number should be given to the alternative under a certain attribute. He/She may give multiple numbers instead of a specific number to represent his/her assessment information. Therefore, compared to other extended forms of FSs. HFs have a wide range of applications and of more practical importance.

In addition to the aforementioned extensions of fuzzy set theory. Zhang (1994) introduced the concept of bipolar fuzzy sets (BFs) and proposed the use of two membership functions representing membership in a set and membership in a complementary set. Although there is an advantage that can be achieved by using bipolar fuzzy logic, they are significantly less used for solving MCDM problems compared to other fuzzy logic extensions. The following examples may be mentioned as some of the more rare applications of BFs for solving MCDM problems. Alghamdi et al. (2018) and Akram and Arshad (2018) proposed bipolar fuzzy extensions of TOPSIS and ELECTRE I methods. Han et al (2018) present a comprehensive mathematical approach based on the TOPSIS method to improve the accuracy of bipolar disorder clinical diagnosis.

However, in the context of many real-life decision making applications, decision makers need to shows their opinions in more complex forms than some specific numerical values. In other words,

DMs cannot represent all uncertain evaluation points from a few precise values. In other words, the existing representative model cannot contain all the hesitant information given to the decision makers in a centralized way, that is, the DMs cannot provide complete assessment information through the actual models. Therefore, to deal with this kind of complex MCDM problems more successfully and obtain the possible information from the original estimation in HFs format, Ren et al (2018) first proposed the notion of Normal wiggly hesitant fuzzy sets. This new extended form of HFs shows that it is a more appropriate and logical approach to obtain reliable hesitant fuzzy information in the decision making process. It is worth emphasizing that the NWHFS is based on the assumption that population uncertainty can be considered as a normal wiggly range for possible values. The NWHBFS is expressed by the real preference degree and normal wiggly range, which cannot only retain the decision makers original preferences. but more beneficially mines deep uncertain information. In addition, the great advantage of NWHBFS is that it exposes potential uncertainty information and enables decision makers to obtain more fair and accurate discretionary information.

Decision making methods play an important role in dealing with decision making problems and are widely used in many forms of practical applications in various fields such as VIKOR method, MULTIMOORA method, ELECTRE method, TOPSIS method and PROMETHEE method. The relatively new COCOSO method developed by Yazdani et al(2019a) is based on the integration of simple additive weighting and exponential weighting product modeling. The essence of this method lies in the comparison of compromise perspectives, which ultimately reconciles the evaluation criteria, which are often contradictory. The COCOSO method provides an overview of possible compromise solutions available to the decision maker. There are many people who save various problems with COCOSO methodlike Bagal et al(2021), Deveci et al (2021), Ecer (2021), Peng et al(2021), Peng & Luo (2021), Torkayesh et al. (2021a; b), Ulutaş et al (2021), Stanujkic et al(2020) and Wen et al(2019). The aforementioned studies make good use of the COCOSO method to mine the psychological behavior of DMs to overcome MADM problems. However, until now research on COCOSO methods is very limited. Also, NWHFS can explore the possible and uncertain information behind DMs feelings without someone giving them directly.

The demand for electricity in the modern world is very high. Among the renewable energy resources, solar energy is attracting considerable interest due to its availability and economic aspects. Nevertheless, the solar plant imposes some strict rules, for example, solar thermal power

plants are best suited for places with a minimum of 2000 kW h/m²/year of solar radiation, low humidity and dust and other agents that prevent the absorption of solar radiation. In Site selection for renewable energies has high importance (Noorollahi et al 2015). Hence, MCDM approach has been widely used in solar technology site selection (Sánchez-Lozano et al 2013).

According to previous reviews, this research paper gives deeper into the concept of motivation and decision-making. A Normal wiggly is used to clearly convey their reluctance. Incorporating normal wiggly into the MCDM technique helps to explore the hesitancy of various assumptions and provide the correct decision. We propose an extended normal wiggly method to assist decision makers in selecting desirable products by maintaining the advantages of decision makers psychological behaviour and traditional COCOSO method and NWHBF information.

The Contribution of the research work is given below,

- We proposed the normal wiggly hesitant bipolar fuzzy sets to enrich the theories of NWHBFS.
- We present a normal wiggly hesitant bipolar fuzzy COCOSO method to deal with MADM problems with normal wiggly hesitant bipolar fuzzy information.
- Make use of the extended NWHBF-COCOSO method to rank the alternatives and provide the best choice.
- Compare with the existing approach to verify the rationality and validity of our proposed approach.

2 Preliminaries

Definition 2.1

Let U be a fixed set, the hesitant bipolar fuzzy set on U is defined as follows,

$$B^* = \{\langle u, h_B^*(u) \rangle | u \in U\} \quad (1)$$

In above equation $h_B^*(u)$ is represent the hesitant bipolar fuzzy set. The hesitant bipolar fuzzy set $h_B^*(u)$ is contain the membership degree. The positive membership degree is $\alpha_{B^*}^P(u)$ and the negative membership degree is $\alpha_{B^*}^N(u)$. Each element of positive membership degree is $\alpha_{B^*}^P(u): U \rightarrow [0,1]$ and each element of negative membership degree is $\alpha_{B^*}^N(u): U \rightarrow [-1,0]$, respectively for every $u \in U$, which satisfies the following condition

$$0 \leq \alpha_{B^*(u)}^P \leq 1, -1 \leq \alpha_{B^*(u)}^N \leq 0$$

The pair $\hat{h}(x) = \{\langle \alpha^P(u), \alpha^N(u) \rangle\}$ is defined as hesitant bipolar fuzzy number is called by $\hat{h} = (\alpha^P, \alpha^N)$. where, $0 \leq \alpha_1^P \leq 1, -1 \leq \alpha_1^N \leq 0, (\alpha_1^P, \alpha_1^N) \in (\alpha^P, \alpha^N)$

Definition 2.2

The bipolar fuzzy numbers for triangle is defined by

$$\hat{B}^* = \{\langle x, \mu_B^P(x), \nu_B^N(x) \rangle\} \quad (2)$$

where $\mu_B^P(x), \nu_B^N(x)$ are positive and negative membership function respectively. The positive and negative membership function is given as

$$\mu^P(x) = \begin{cases} \frac{x-t_L}{t_P-t_L} & \text{if } t_L \leq x < t_P, \\ \frac{x-t_R}{t_P-t_R} & \text{if } t_P \leq x < t_R, \\ 0 & \text{Otherwise} \end{cases}$$

$$\mu^N(x) = \begin{cases} \frac{-(x-t_L)}{t_N-t_L} & \text{if } t_L \leq x < t_N, \\ \frac{-(x-t_R)}{t_N-t_R} & \text{if } t_N \leq x < t_R, \\ 0 & \text{Otherwise} \end{cases}$$

Definition 2.3

Let X be a fixed set, a hesitant triangular fuzzy set on X is in terms of a function that when applied to each x in X and returns a subset of values in $[0,1]$.

To be easily understood, we express the HTFS by a mathematical symbol

$$E = \{\langle x, \hat{h}_E(x) \rangle / x \in X\} \quad (3)$$

where $\hat{h}_E(x)$ is a set of some possible triangular fuzzy values in $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to the set E .

For convenience, we call $\hat{h}_E(x) = \hat{h} = (\gamma^L, \gamma^M, \gamma^R)$ a hesitant triangular fuzzy elements and \hat{H} the set of all HTFEs.

Given three HTFEs $\hat{h} = (\gamma^L, \gamma^M, \gamma^R)$, $\hat{h}_1 = (\gamma_1^L, \gamma_1^M, \gamma_1^R)$ and $\hat{h}_2 = (\gamma_2^L, \gamma_2^M, \gamma_2^R)$ and $\lambda > 0$ we define their operations as follows.

1. $\hat{h}^\lambda = \bigcup_{\hat{\gamma} \in \hat{h}} \{(\gamma^L)^\lambda, (\gamma^M)^\lambda, (\gamma^R)^\lambda\}$
2. $\lambda \hat{h} = \bigcup_{\hat{\gamma} \in \hat{h}} \{1 - (1 - \gamma^L)^\lambda, 1 - (1 - \gamma^M)^\lambda, 1 - (1 - \gamma^R)^\lambda\}$
3. $\hat{h}_1 \oplus \hat{h}_2 = \bigcup_{\hat{\gamma}_1 \in \hat{h}_1, \hat{\gamma}_2 \in \hat{h}_2} \{\gamma_1^L + \gamma_2^L - \gamma_1^L \gamma_2^L, \gamma_1^M + \gamma_2^M - \gamma_1^M \gamma_2^M, \gamma_1^R + \gamma_2^R - \gamma_1^R \gamma_2^R\}$
4. $\hat{h}_1 \otimes \hat{h}_2 = \bigcup_{\hat{\gamma}_1 \in \hat{h}_1, \hat{\gamma}_2 \in \hat{h}_2} \{\gamma_1^L \cdot \gamma_2^L, \gamma_1^M \cdot \gamma_2^M, \gamma_1^R \cdot \gamma_2^R\}$

Definition 2.4

Let $p = \{\gamma_1, \gamma_2, \dots, \gamma_{\#p}\}$ and $q = \{\gamma'_1, \gamma'_2, \dots, \gamma'_{\#q}\}$ be the hesitant bipolar fuzzy element (HBFE) then, the hesitant bipolar fuzzy element of mean value is defined as

$$\bar{p} = \frac{1}{\#p} \sum_{i=1}^{\#p} \gamma_i \quad (4)$$

$$\bar{q} = \frac{1}{\#q} \sum_{i=1}^{\#q} \gamma'_i \quad (5)$$

Definition 2.5

Let $p = \{\gamma_1, \gamma_2, \dots, \gamma_{\#p}\}$ and $q = \{\gamma'_1, \gamma'_2, \dots, \gamma'_{\#q}\}$ be the HBFE. By the average value definition, we find the SD (standard deviation) of positive and negative member values in p and q as

$$\sigma_p = \sqrt{\frac{1}{\#p} \sum_{i=1}^{\#p} (\gamma_i - \bar{p})^2} \quad (6)$$

$$\sigma'_q = \sqrt{\frac{1}{\#q} \sum_{i=1}^{\#q} (\gamma'_i - \bar{q})^2} \quad (7)$$

The function $\tilde{f}: p \rightarrow [0, \sigma_p]$ and $\tilde{f}: q \rightarrow [\sigma'_q, 0]$ satisfy

$$\tilde{f}(\gamma_i) = \sigma_p \exp - \frac{(\gamma_i - \bar{p})^2}{2\sigma_p^2} \quad (8)$$

$$\tilde{f}(\gamma'_i) = \sigma_q \exp - \frac{(\gamma'_i - \bar{q})^2}{2\sigma_q^2} \quad (9)$$

Definition 2.6

The real preference degrees (rpd) of \tilde{p} and \tilde{q} are given by (Ren et al., 2018):

$$rpd(\tilde{p}) = \begin{cases} \sum_{i=1}^{\#\tilde{p}} \tilde{\gamma}_i(\frac{\#\tilde{p}-i}{\#\tilde{p}-1}) & \text{if } \tilde{p} < 0.5 \\ 1 - \sum_{i=1}^{\#\tilde{p}} \tilde{\gamma}_i(\frac{\#\tilde{p}-i}{\#\tilde{p}-1}) & \text{if } \tilde{p} > 0.5 \\ 0.5 & \text{if } \tilde{p} = 0.5 \end{cases} \quad (10)$$

$$rpd(\tilde{q}) = \begin{cases} \sum_{i=1}^{\#\tilde{q}} \tilde{\gamma}'_i(\frac{\#\tilde{q}-i}{\#\tilde{q}-1}) & \text{if } \tilde{q} < -0.5 \\ 1 - \sum_{i=1}^{\#\tilde{q}} \tilde{\gamma}'_i(\frac{\#\tilde{q}-i}{\#\tilde{q}-1}) & \text{if } \tilde{q} > -0.5 \\ -0.5 & \text{if } \tilde{q} = -0.5 \end{cases} \quad (11)$$

These degrees are used to measure certain inherent preferences of related individuals. It provides a complete priority estimation and detects inaccurate performance. It is also used for evaluation of the intrinsic preferences of decision makers. The rpd s of \tilde{p} and \tilde{q} depend on the measure.

3 The New Extension of the Hesitant bipolar Fuzzy set-Normal wiggly hesitant bipolar fuzzy set

After preparing the preliminaries, we propose a new MCDM method with the aid of NWHBF guidance. Furthermore, we show an illustrative example of site selection for a solar plant to evaluate the effectiveness of the proposed approach.

Definition 3.1

Let $E = (x, p(x)) / x \in X$ be a HFS on the reference set X . Then, the corresponding normal wiggly hesitant bipolar fuzzy set (NWHBFS) on X can be denoted as:

$$NWHBF = \langle X, p(x), q(x), \chi(p(x)), \chi(q(x)) \rangle, x \in X \quad (12)$$

where $p(x)$ and $q(x)$ are the hesitant fuzzy elements in E . Also, $p(x)$, $q(x)$ denote the positive and negative degrees of membership of elements belonging to E to $X (x \in X)$, respectively.

$$\begin{aligned} \chi(p(x)) &= \{\bar{\eta}_1, \bar{\eta}_2, \dots, \bar{\eta}_{\#p(x)}\}, \bar{\eta}_1 = \{\eta_i^L, \eta_i^M, \eta_i^U\} \\ &= \{\max(\eta_i - \bar{f}(\eta_i), 0), (2rpd(\bar{p}(x)) - 1)\bar{f}(\eta_i) + \eta_i, \min(\eta_i + \bar{f}(\eta_i), 1)\} \end{aligned}$$

η_i is one of the values of $p(x)$. Similarly,

$$\begin{aligned} \chi(q(x)) &= \{\bar{\zeta}_1, \bar{\zeta}_2, \dots, \bar{\zeta}_{\#q(x)}\}, \bar{\zeta}_1 = \{\zeta_i^L, \zeta_i^M, \zeta_i^U\} \\ &= \{\max(\zeta_i - \bar{f}(\zeta_i), -1), (2rpd(\bar{q}(x)) - 1)\bar{f}(\zeta_i) + \zeta_i, \min(\zeta_i + \bar{f}(\zeta_i), 0)\} \end{aligned}$$

where ζ_i is one of the values of $q(x)$.

Also $\bar{f}(\eta_i)$ and $\bar{f}(\zeta_i)$ are wiggly parameters of η_i, ζ_i and $rpd(\bar{p}(x))$ and $rpd(\bar{q}(x))$ is the real

preference degree of $p(x), q(x)$

We call $\chi(p(x))$ and $\chi(q(x))$ as normal wiggly elements. The wiggly range of each value in the HBFE is part of the triangle formed by the triangle numbers in the Normal wiggly element (NWE). A pairwise $\langle p(x), \chi(p(x)) \rangle$ and $\langle q(x), \chi(q(x)) \rangle$ is called NWHBFE (Normal Wiggly Hesitant Bipolar Fuzzy element), is represented as $\langle p, q, \chi(p), \chi(q) \rangle$.

Definition 3.2

Let $\langle p, q, \chi(p), \chi(q) \rangle$ denote the NWHBFE, and mean value of all positive and negative membership values as \bar{p}, \bar{q} and standard deviation of positive and negative membership values as σ_p, σ_q .

Then, the score function of $\langle p, \chi(p) \rangle$ and $\langle q, \chi(q) \rangle$ is

$$S_{NWHBF}(\langle p, \chi(p), q, \chi(q) \rangle) = [\gamma(\bar{p} - \sigma_p) + (1 - \gamma)(\frac{1}{\#p} \sum_{i=1}^{\#p} \tilde{\eta}_i - \sigma_{\tilde{\eta}_i}), \gamma'(\bar{q} - \sigma_q) + (1 - \gamma')(\frac{1}{\#q} \sum_{i=1}^{\#q} \tilde{\zeta}_i - \sigma_{\tilde{\zeta}_i})]$$

where, $\tilde{\eta}_i = \frac{\eta_i^L + \eta_i^M + \eta_i^U}{3}$, $\tilde{\zeta}_i = \frac{\zeta_i^L + \zeta_i^M + \zeta_i^U}{3}$ and

$$\sigma_{\tilde{\eta}_i} = \sqrt{(\eta_i^L)^2 + (\eta_i^M)^2 + (\eta_i^U)^2 - (\eta_i^L \eta_i^M) - (\eta_i^L \eta_i^U) - (\eta_i^M \eta_i^U)}$$

$$\sigma_{\tilde{\zeta}_i} = \sqrt{(\zeta_i^L)^2 + (\zeta_i^M)^2 + (\zeta_i^U)^2 - (\zeta_i^L \zeta_i^M) - (\zeta_i^L \zeta_i^U) - (\zeta_i^M \zeta_i^U)}$$

Now, $\gamma, \gamma' \in (0, 1)$.

Definition 3.3

Let $\langle p_1, q_1, \chi(p_1), \chi(q_1) \rangle$ and $\langle p_2, q_2, \chi(p_2), \chi(q_2) \rangle$ be two different NWHBFEs and their score values are $S_{NWHB}(\langle p_1, q_1, \chi(p_1), \chi(q_1) \rangle)$ and $S_{NWHB}(\langle p_2, q_2, \chi(p_2), \chi(q_2) \rangle)$.

• if $S_{NWHB}(\langle p_1, q_1, \chi(p_1), \chi(q_1) \rangle) > S_{NWHB}(\langle p_2, q_2, \chi(p_2), \chi(q_2) \rangle)$, then $\langle p_1, q_1, \chi(p_1), \chi(q_1) \rangle$ is superior to $\langle p_2, q_2, \chi(p_2), \chi(q_2) \rangle$ and we denote,

$$\langle p_1, q_1, \chi(p_1), \chi(q_1) \rangle > \langle p_2, q_2, \chi(p_2), \chi(q_2) \rangle \quad (13)$$

• if $S_{NWHB}(\langle p_1, q_1, \chi(p_1), \chi(q_1) \rangle) = S_{NWHB}(\langle p_2, q_2, \chi(p_2), \chi(q_2) \rangle)$, then $\langle p_1, q_1, \chi(p_1), \chi(q_1) \rangle$ and $\langle p_2, q_2, \chi(p_2), \chi(q_2) \rangle$ are indistinguishable and we denote,

$$\langle p_1, q_1, \chi(p_1), \chi(q_1) \rangle = \langle p_2, q_2, \chi(p_2), \chi(q_2) \rangle \quad (14)$$

Definition 3.4

For $\langle p_1, q_1, \chi(p_1), \chi(q_1) \rangle$ and $\langle p_2, q_2, \chi(p_2), \chi(q_2) \rangle$, we define some operations as:

1. $\langle q_1, p_1, \chi(q_1), \chi(p_1) \rangle \oplus \langle q_2, p_2, \chi(q_2), \chi(p_2) \rangle = \langle \cup_{\eta_1 \in q_1, \eta_2 \in q_2} \eta_1 + \eta_2 - \eta_1 \eta_2, \cup_{\zeta_1 \in p_1, \zeta_2 \in p_2} \zeta_1 + \zeta_2 - \zeta_1 \zeta_2, \cup_{\tilde{\eta}_1 \in \chi(q_1), \tilde{\eta}_2 \in \chi(q_2)} \tilde{\eta}_1 \oplus \tilde{\eta}_2, \cup_{\tilde{\zeta}_1 \in \chi(p_1), \tilde{\zeta}_2 \in \chi(p_2)} \tilde{\zeta}_1 \oplus \tilde{\zeta}_2 \rangle$
2. $\langle q_1, p_1, \chi(q_1), \chi(p_1) \rangle \otimes \langle q_2, p_2, \chi(q_2), \chi(p_2) \rangle = \langle \cup_{\eta_1 \in q_1, \eta_2 \in q_2} \eta_1 \eta_2, \cup_{\zeta_1 \in p_1, \zeta_2 \in p_2} \zeta_1 + \zeta_2 - \zeta_1 \zeta_2, \cup_{\tilde{\eta}_1 \in \chi(q_1), \tilde{\eta}_2 \in \chi(q_2)} \tilde{\eta}_1 \otimes \tilde{\eta}_2, \cup_{\tilde{\zeta}_1 \in \chi(p_1), \tilde{\zeta}_2 \in \chi(p_2)} \tilde{\zeta}_1 \otimes \tilde{\zeta}_2 \rangle$
3. $\lambda(p_1, \chi(p_1)) = \{\cup_{\eta_1 \in p_1} 1 - (1 - \eta_1)^\lambda, \cup_{\tilde{\eta}_1 \in \chi(p_1)} \lambda \tilde{\eta}_1, \lambda > 0\}$
4. $((p_1, \chi(p_1)))^\lambda = \{\cup_{\eta_1 \in p_1} \eta_1^\lambda, \cup_{\tilde{\eta}_1 \in \chi(p_1)} \tilde{\eta}_1^\lambda, 1 - (1 - \zeta_1)^\lambda, \lambda > 0\}$

4.PROBLEM FORMULATION

According to the NWHBF definition, the NWHBFs matrix is shown in table 1.

$\langle p_{ij}, q_{ij}, \chi(p_{ij}), \chi(q_{ij}) \rangle$ is a NWHBFE that contains p_{ij} and q_{ij} . It contains deeply uncertain information, the proposed method NWHBF keeps the original data and also provides deeper information about the selected problem. The process of the MCDM method is as shown in figure 1.

Table 1: Normal wiggly Hesitant Bipolar Fuzzy Decision matrix

	C_1	C_2	...	C_n
A_1	$\langle p_{11}, q_{11}, \chi(p_{11}), \chi(q_{11}) \rangle$	$\langle p_{12}, q_{12}, \chi(p_{12}), \chi(q_{12}) \rangle$...	$\langle p_{1n}, q_{1n}, \chi(p_{1n}), \chi(q_{1n}) \rangle$
A_2	$\langle p_{21}, q_{21}, \chi(p_{21}), \chi(q_{21}) \rangle$	$\langle p_{22}, q_{22}, \chi(p_{22}), \chi(q_{22}) \rangle$...	$\langle p_{2n}, q_{2n}, \chi(p_{2n}), \chi(q_{2n}) \rangle$
\vdots	\vdots	\vdots		\vdots
A_m	$\langle p_{m1}, q_{m1}, \chi(p_{m1}), \chi(q_{m1}) \rangle$	$\langle p_{m2}, q_{m2}, \chi(p_{m2}), \chi(q_{m2}) \rangle$...	$\langle p_{mn}, q_{mn}, \chi(p_{mn}), \chi(q_{mn}) \rangle$
	$>$	$>$		$>$

4.1 Proposed methodology

To solve Multi criteria decision problem, we have now proposed a normal wiggly hesitant bipolar COCOSO method with the aid of entropy weight criteria.

Let $A_i (i = 1, 2, \dots, m)$ be a set of all alternatives and $C_j (j = 1, 2, \dots, n)$ be a set of criteria. Hesitant bipolar fuzzy MADM problems can be explained by a hesitant bipolar result expressed by the matrix $H = (h_{ij})_{m \times n}$ where h_{ij} is the evaluation of the alternative $A_i (i = 1, 2, \dots, m)$ according to the $C_j (j = 1, 2, \dots, n)$ criteria provided by the decision maker. This is considered a hesitant bipolar fuzzy number.

This paper expands the decision of the hesitant fuzzy set matrix by mining the basic/deep

uncertainty message when hesitant bipolar fuzzy set is used by decision makers to reveal evaluation information in the decision making process. In order to eliminate the influence of various physical dimensions and types, we normalize the hesitant bipolar fuzzy decision matrix $H = (h_{ij})_{m \times n}$

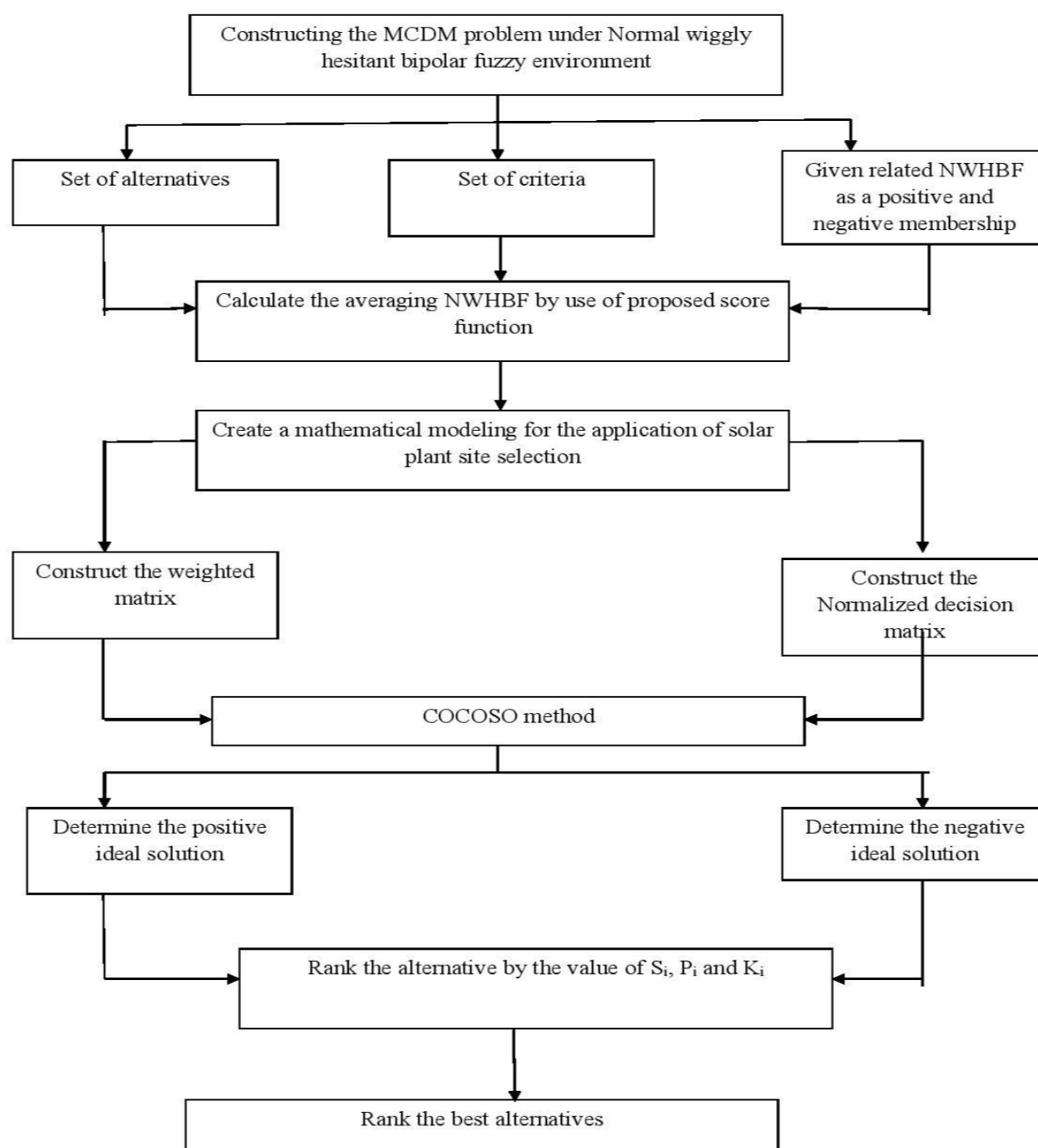


Figure 1: Flow diagram of Normal wiggly hesitant bipolar fuzzy-COCOSO method

Step 1:

Create the NWHBFD are following in the table.2

$$\begin{aligned}\hat{H}_{ij} &= \{\tilde{h}_{ij} \in X\} \\ &= \{[\chi(P_{ij}, \chi(q_{ij})/\chi(P_{ij}), \chi(q_{ij})) \in X]\end{aligned}\quad (15)$$

Table 2: Decision matrix

	C_1	C_2	...	C_n
A_1	$\langle p_{11}, q_{11} \rangle$	$\langle p_{12}, q_{12} \rangle$...	$\langle p_{1n}, q_{1n} \rangle$
A_2	$\langle p_{21}, q_{21} \rangle$	$\langle p_{22}, q_{22} \rangle$...	$\langle p_{2n}, q_{2n} \rangle$
\vdots	\vdots	\vdots	\vdots	\vdots
A_m	$\langle p_{m1}, q_{m1} \rangle$	$\langle p_{m2}, q_{m2} \rangle$...	$\langle p_{mn}, q_{mn} \rangle$

Step 2:

Determine the scoring function for NWHBFE preference

$$\begin{aligned}S_{NWBF}(\langle p, \chi(p), q, \chi(q) \rangle) &= \{\gamma(\bar{p} - \sigma_p) + (1 - \gamma) \\ &\quad (\frac{1}{\#p} \sum_{i=1}^{\#p} \tilde{\eta}_i - \sigma_{\tilde{\eta}_i}), \gamma'(\bar{q} - \sigma_q) + (1 - \gamma')(\frac{1}{\#q} \sum_{i=1}^{\#q} \tilde{\zeta}_i - \sigma_{\tilde{\zeta}_i})\}\end{aligned}\quad (16)$$

Step 3

Calculate criterion weights using weighted entropy

$$\tilde{E}(A) = 1 - \sum_{i=1}^m \sum_{j=1}^n \frac{[\chi(p_{ij}) - (-\chi(q_{ij}))]}{2} = 0 \quad (17)$$

We calculate the entropy matrix \tilde{E} associated with the decision matrix \tilde{H} ,

$$\tilde{E} = \begin{pmatrix} E_{1,1} & E_{1,2} & \cdots & E_{1,n} \\ E_{2,1} & E_{2,2} & \cdots & E_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ E_{m,1} & E_{m,2} & \cdots & E_{m,n} \end{pmatrix}$$

Establish the normalized entropy value \tilde{E} using the following expressions

$$\tilde{E}_{ij} = \frac{\tilde{E}_{ij}}{\max\{\tilde{E}_{i1}, \tilde{E}_{i2}, \dots, \tilde{E}_{in}\}} \quad (18)$$

For $1 \leq i \leq m$ and $1 \leq j \leq n$. The criterion weights of W_j are estimated by the following equation.

$$W_j = \frac{1 - \sum_{j=1}^n E_j}{n - \sum_{i=1}^n \sum_{j=1}^n E_{ij}}, (1 \leq j \leq n) \quad (19)$$

where n is the number of indicators, and $0 \leq W_j \leq 1$, $\sum_{j=1}^n W_j = 1$

Step 4

The NWHBF-Positive Ideal Solution (NWHBF-PIS) \tilde{h}_+ and NWHBF-Negative Ideal Solution (NWHBF-NIS) \tilde{h}_- are represented by,

$$\hat{\Lambda}_+ = \{([max\chi(p_{ij}), min\chi(q_{ij})]/\chi(p_{ij})\epsilon\tilde{H}_{ij}, \chi(q_{ij})\epsilon\tilde{H}_{ij}) \\ ([min\chi(p_{ij}), max\chi(q_{ij})]/\chi(p_{ij})\epsilon\tilde{H}_{ij}, \chi(q_{ij})\epsilon\tilde{H}_{ij})\} \quad (20)$$

$$\hat{\Lambda}_- = \{([min\chi(p_{ij}), max\chi(q_{ij})]/\chi(p_{ij})\epsilon\tilde{H}_{ij}, \chi(q_{ij})\epsilon\tilde{H}_{ij}) \\ ([max\chi(p_{ij}), min\chi(q_{ij})]/\chi(p_{ij})\epsilon\tilde{H}_{ij}, \chi(q_{ij})\epsilon\tilde{H}_{ij})\} \quad (21)$$

Step 5

The following is an application of the COCOSO method based on the integration of SAW and MEP approaches

$$\tilde{S}_i = \sum_{j=1}^n r_{ij} w_j \quad (22)$$

$$\tilde{P}_i = \sum_{j=1}^n r_{ij} w_j^w \quad (23)$$

where S_i and P_i denote the sum of weighted comparable sequences and the weight multiplied comparable sequences of alternative i , respectively and w_j denote the criterion weights j .

Step 6

Ranking of alternatives considered. For ranking purposes, the COCOSO method uses the relative performance score K_i . Calculated on the basis of three total estimated results K_{ia} , K_{ib} and K_{ic} .

$$\tilde{K}_i = \frac{1}{3}(k_{ia} + k_{ib} + k_{ic}) + (k_{ia} k_{ib} k_{ic})^{\frac{1}{3}} \quad (24)$$

$$\tilde{k}_{ia} = \frac{P_i + S_i}{\sum_{i=1}^m (P_i + S_i)} \quad (25)$$

$$\tilde{k}_{ib} = \frac{S_i}{\min S_i} + \frac{P_i}{\min P_i} \quad (26)$$

$$\tilde{k}_{ic} = \frac{\mu S_i + (1-\mu)P_i}{\mu \max S_i + (1-\mu) \max P_i}; 0 \leq \mu \leq 1. \quad (27)$$

Equation (24) outputs a balanced compromise of WSM and WPM sample scores. Equation (25) expresses the arithmetic mean of the sums of WSM and WPM scores, while equation (26) expresses the balance of WSM and WPM sample scores. In equation (27) $\mu = 0.5$ is chosen by the decision maker. However, the flexibility and sustainability of the proposed COCOSO relies on other values. The final ranking of the alternatives is determined based on the k_i

Step 7

We create a ranking of the alternatives sorted in descending order by the values of each S_i and P_i . Here, \tilde{k}_i is the compromise solution for choosing alternative i , which subjects to the following rules,

C₁: Acceptable advantage

$\phi(A_2) - \phi(A_1) \geq \frac{1}{n-1}$ where $\frac{1}{n-1} = DQ$ and ranking list $\phi(A_i)$, the alternative $\phi(A_2)$ is in the second place.

C₂: Acceptable stability

when the rank S and P rank with $\mu = 0.5$, the alternative is constant. If the above conditions are not met, the comfort solution is given below.

1. Condition C_2 is not satisfied if $A_1[1]$ and $A_1[2]$ alternate.
2. If the condition C_1 is not satisfied, then A_1, \dots, A_n becomes the alternative. Here, A_n is evaluated by the equation $Q(A_n) - Q(A_1) \leq \frac{1}{n-1}$ for at most n .

In the list of ranked alternatives, the most preferred alternatives should be closed with the positive best solution and negative best solution for the maximization and minimization criteria, respectively.

5 NUMERICAL EXAMPLE

The proposed method is described for application to NWHBF solar site selection. The problem of solar plant site selection is important and sensitive MCDM problem. In Site selection for renewable

energies has high importance (Noorollahi et al 2015). Hence, MCDM methods has been widely used in solar technology site selection (Sánchez-Lozano et al 2013).

We focus on five criteria: Technique of operation, Economic, Environmental, Social and risk factor and three alternatives A_1 , A_2 and A_3 . Alternative sites used in this study are suggested by individual opinion.

Description of criterion

(i) Technique of operation(C_1)

The technical criteria described the technical factors, parameters and characteristics related to solar PV power plant design, construction and operation phases. we have to consider solar radiation, temperature, sunshine hours, distance to network connection and distance to residential areas.

(ii) Economic(C_2)

This is the type of cost associated with the initial phase of the investment, such as solar panels, construction, electrical and civil works associated with the project. capital costs are an item of initial investment costs.

(iii) Environmental(C_3)

Renewable energy projects are designed as sustainable types of energy resources and project related activities are designed to cause minimal damage to the environment and humans.

(iv) Social(C_4)

Apart from technical, economic and environmental aspects. it is important to analyze the social and political aspects of investments

(v) Risk(C_5)

Surveys, interviews and meetings are an important part of project preparation to understand and measure public acceptance and support. very strong opposition from local citizens puts the entire investment at risk. On the other hand massive support from local citizens can help the project and contribute positively to local development.

After the preliminary section, we numerically evaluate the site selection for the solar plant. the result of the Hesitant bipolar values is given in matrix table.

$A(J,I)$ denotes the value matrix of vary alternatives and their related criteria. J is the index for different alternatives, I is the index for different criteria. The calculation of each alternative based on different criteria shows that $A(J,I)$.

Here, the decision maker gives the evaluation in the type of hesitant bipolar values. If the computed values of any alternative provided by the decision makers are inconsistent, such values are included only in the non-hesitant fuzzy elements. Assume that the HBF decision matrix \hat{H} is given below.

Table 3: Normal wiggly hesitant bipolar Decision matrix

	A_1	A_2	A_3
C_1	(0.5, 0.6, 0.7), (-0.1, -0.1, -0.2)	(0.2, 0.7, 0.8), (-0.3, -0.4, -0.4)	(0.4, 0.7, 0.8), (-0.1, -0.2, -0.2)
C_2	(0.6, 0.7, 0.9), (-0.1, -0.1, -0.2)	(0.7, 0.8, 0.9), (-0.3, -0.3, -0.2)	(0.5, 0.5, 0.7), (-0.2, -0.2, -0.3)
C_3	(0.3, 0.5, 0.6), (-0.3, -0.4, -0.4)	(0.4, 0.7, 0.8), (-0.3, -0.4, -0.3)	(0.4, 0.4, 0.5), (-0.1, -0.2, -0.3)
C_4	(0.2, 0.5, 0.5), (-0.4, -0.5, -0.5)	(0.7, 0.7, 0.8), (-0.4, -0.2, -0.3)	(0.4, 0.5, 0.7), (-0.1, -0.1, -0.2)
C_5	(0.3, 0.5, 0.6), (-0.3, -0.4, -0.4)	(0.3, 0.5, 0.6), (-0.3, -0.3, -0.4)	(0.4, 0.5, 0.6), (-0.2, -0.3, -0.4)

Constructing the NWHBF decision matrix.

Calculate the scoring function for the choice of NWHBF using equation (16), The normal wiggly hesitant bipolar fuzzy set is discussed here as mentioned above.

First, we determine the score matrix C_{ij} placed on the positive and negative membership values expressed in figure (2) and (3) in equation (16)

Scorematrix:

$$\begin{bmatrix} (0.5145, -0.1867) & (0.5866, -0.1867) & (0.3286, -0.4234) & (0.2786, -0.524) & (0.3841, -0.4234) \\ (0.292, -0.4234) & (0.714, -0.3271) & (0.385, -0.4085) & (0.681, -0.3996) & (0.328, -0.3885) \\ (0.497, -0.2203) & (0.465, -0.2866) & (0.382, -0.2887) & (0.402, -0.1867) & (0.413, -0.3912) \end{bmatrix}$$

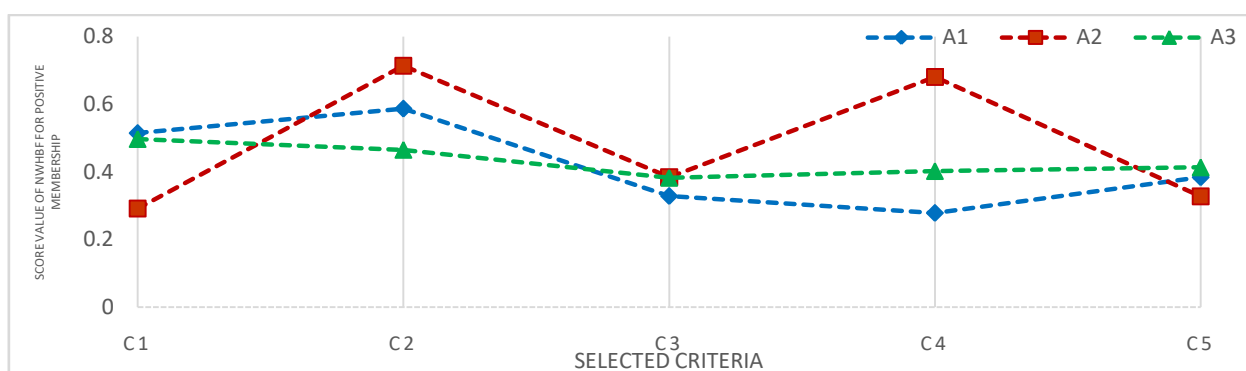


Fig.2 Score value of NWHBF for positive membership

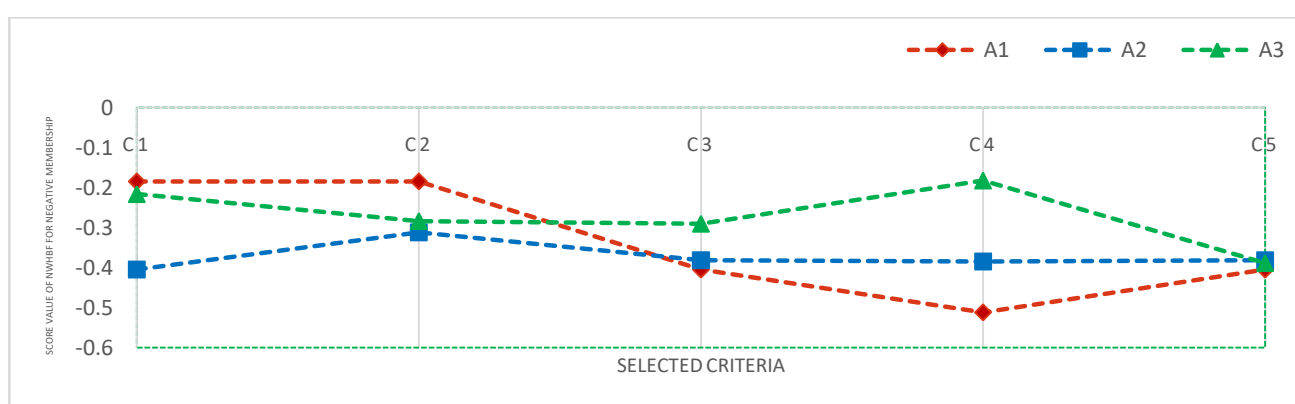


Fig.3 Score value of NWHBF for positive membership

The scoring function for positive and negative membership values for each alternate is given in tables (4)-(9).

Table 4: Positive membership of NWHBFDM for alternate A_1

	\bar{h}	σ_h	θ_i^L	θ_i^M	θ_i^U	$\sigma_{\theta_i}^L$	$\sigma_{\theta_i}^M$	$\sigma_{\theta_i}^U$	S_{NWHB}
$A_1 - C_1$	0.6	0.082	0.501	0.603	0.703	0.066	0.142	0.066	0.5145
$A_1 - C_2$	0.73	0.1247	0.603	0.655	0.902	0.125	0.248	0.085	0.5866
$A_1 - C_3$	0.46	0.1248	0.296	0.492	0.595	0.096	0.205	0.116	0.3286
$A_1 - C_4$	0.4	0.1414	0.195	0.491	0.491	0.091	0.095	0.095	0.2786
$A_1 - C_5$	0.46	0.1248	0.296	0.499	0.596	0.08	0.003	0.009	0.3841

Table 5: Positive membership of NWHBFDm for alternate A_2

	\bar{h}	σ_h	θ_i^L	θ_i^M	θ_i^U	$\sigma_{\theta_i}^L$	$\sigma_{\theta_i}^M$	$\sigma_{\theta_i}^U$	S_{NWHB}
$A_2 - C_1$	0.56	0.2625	0.212	0.727	0.82	0.182	0.408	0.305	0.292
$A_2 - C_2$	0.8	0.0816	0.701	0.802	0.902	0.067	0.1422	0.067	0.714
$A_2 - C_3$	0.56	0.17	0.408	0.512	0.805	0.1904	0.2787	0.110	0.385
$A_2 - C_4$	0.73	0.047	0.706	0.706	0.8	0.071	0.071	0.035	0.681
$A_2 - C_5$	0.46	0.1248	0.296	0.492	0.595	0.096	0.205	0.115	0.328

Table 6: Positive membership of NWHBFDm for alternate A_3

	\bar{h}	σ_h	θ_i^L	θ_i^M	θ_i^U	$\sigma_{\theta_i}^L$	$\sigma_{\theta_i}^M$	$\sigma_{\theta_i}^U$	S_{NWHB}
$A_3 - C_1$	0.63	0.17	0.405	0.7	0.807	0.118	0.007	0.1797	0.497
$A_3 - C_2$	0.56	0.094	0.503	0.503	0.703	0.132	0.132	0.0546	0.465
$A_3 - C_3$	0.43	0.047	0.399	0.399	0.499	0.065	0.065	0.026	0.382
$A_3 - C_4$	0.53	0.1247	0.404	0.507	0.703	0.125	0.212	0.085	0.402
$A_3 - C_5$	0.5	0.0816	0.401	0.501	0.599	0.068	0.1403	0.067	0.413

Table 7: Negative membership of NWHBFDm for alternate A_1

	\bar{h}	σ_h	θ_i^L	θ_i^M	θ_i^U	$\sigma_{\theta_i}^L$	$\sigma_{\theta_i}^M$	$\sigma_{\theta_i}^U$	S_{NWHB}
$A_1 - C_1$	-0.13	0.047	-0.097	-0.097	-0.198	0.064	0.064	0.03	-0.1867
$A_1 - C_2$	-0.13	0.047	-0.097	-0.097	-0.198	0.064	0.064	0.03	-0.1867
$A_1 - C_3$	-0.36	0.047	-0.299	-0.398	-0.398	0.029	0.064	0.064	-0.4234
$A_1 - C_4$	-0.46	0.047	-0.399	-0.499	-0.499	0.03	0.064	0.064	-0.524
$A_1 - C_5$	-0.36	0.047	-0.299	-0.398	-0.398	0.029	0.064	0.064	-0.4234

Table 8: Negative membership of NWHBFDm for alternate A_2

	\bar{h}	σ_h	θ_i^L	θ_i^M	θ_i^U	$\sigma_{\theta_i}^L$	$\sigma_{\theta_i}^M$	$\sigma_{\theta_i}^U$	S_{NWHB}
$A_2 - C_1$	-0.36	0.047	-0.299	-0.398	-0.398	0.029	0.063	0.063	-0.4234
$A_2 - C_2$	-0.26	0.047	-0.301	-0.301	-0.201	0.063	0.063	0.03	-0.3271
$A_2 - C_3$	-0.3	0.057	-0.29	-0.399	-0.3	0.091	0.03	0.063	-0.4085

$A_2 - C_4$	-0.3	0.082	-0.401	-0.201	-0.303	0.067	0.066	0.142	-0.3996
$A_2 - C_5$	-0.33	0.047	-0.298	-0.298	-0.399	0.064	0.064	0.03	-0.3885

Table 9: Negative membership of NWHBFD for alternate A_3

	\bar{h}	σ_h	θ_i^L	θ_i^M	θ_i^U	$\sigma_{\theta_i}^L$	$\sigma_{\theta_i}^M$	$\sigma_{\theta_i}^U$	S_{NWHB}
$A_2 - C_1$	-0.166	0.047	-0.098	-0.197	-0.197	0.029	0.063	0.063	-0.2203
$A_2 - C_2$	-0.233	0.047	-0.198	-0.198	-0.299	0.063	0.063	0.029	-0.2866
$A_2 - C_3$	-0.2	0.082	-0.095	-0.191	-0.295	0.067	0.144	0.067	-0.2887
$A_2 - C_4$	-0.13	0.047	-0.097	-0.097	-0.198	0.064	0.064	0.029	-0.1867
$A_2 - C_5$	-0.3	0.082	-0.197	-0.294	-0.397	0.067	0.143	0.067	-0.3912

Next, we calculate the normalized score matrix C_{ij} using equation (16). the positive and negative membership values as shown in figure (4) and (5).

Normalized score matrix:

$$\begin{bmatrix} (0.3947, -0.2248) & (0.3322, -0.2333) & (0.2999, -0.3778) & (0.2046, -0.4719) & (0.3414, -0.3519) \\ (0.2241, -0.5098) & (0.4044, -0.4088) & (0.3514, -0.3645) & (0.5001, -0.3599) & (0.291, -0.3229) \\ (0.3813, -0.2653) & (0.2634, -0.3583) & (0.3486, -0.2576) & (0.2952, -0.1681) & (0.367, -0.3252) \end{bmatrix}$$



Fig.4 normalized value of NWHBF for positive membership



Fig.5 normalized value of NWHBF for negative membership

The Normal wiggly hesitant bipolar fuzzy normalized entropy matrix using (17). The normalized entropy matrix values are,

$$\tilde{E} = \begin{pmatrix} 1 & 1 & 0.948 & 0.860 & 0.938 \\ 0.917 & 0.827 & 0.921 & 0.741 & 1 \\ 0.981 & 0.961 & 1 & 1 & 0.952 \end{pmatrix}$$

The each criterion weight is calculated using (19). The hesitant fuzzy entropy weight measures shown in figure. 6. The weight values are, $W_1 = 0.209$, $W_2 = 0.197$, $W_3 = 0.206$, $W_4 = 0.177$, $W_5 = 0.208$.

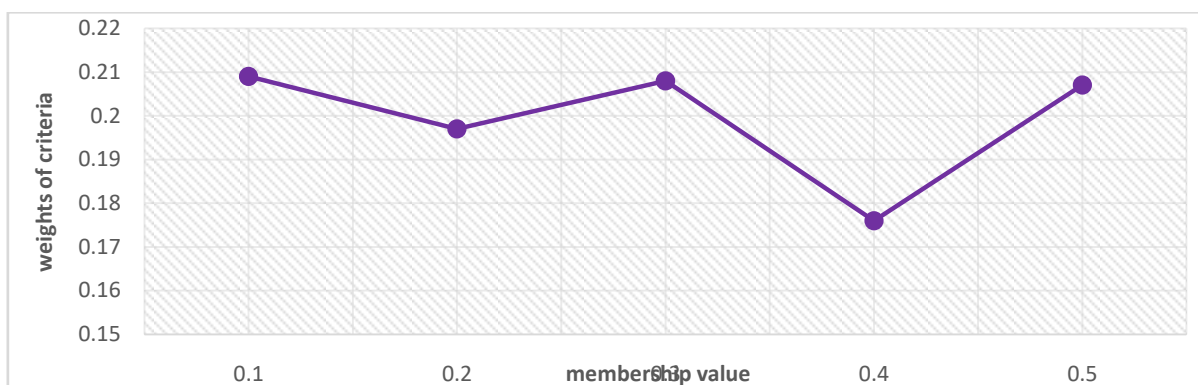


Figure .6: weighted criteria for NWHBFs

Utilize equation (20) and (21) to compute the PIS of (NWHBF) its symbolized as $\hat{\Lambda}_+$ and the NIS of (NWHBF) its symbolized as $\hat{\Lambda}_-$ are as follows,

$$\hat{\Lambda}_+ = 0.69, 0.71, 0.69, 0.76, 0.69 \quad (28)$$

$$\hat{\Lambda}_- = 0.63, 0.59, 0.64, 0.57, 0.65 \quad (29)$$

Calculate the sum of the weighted comparison order for each alternative as \tilde{S}_i using Eq.(22) as,

$$\tilde{S}_1 = 0.676, \tilde{S}_2 = 0.628, \tilde{S}_3 = 0.696 \quad (30)$$

Compute the total power weight of the comparable rows for each alternative as \tilde{P}_i by using Eq (23) as,

$$\tilde{P}_1 = 4.623, \tilde{P}_2 = 4.717, \tilde{P}_3 = 4.651 \quad (31)$$

Three evaluation scoring techniques are used to generate the relative weights of the other options obtained using the equation (25-27) and are shown as follows,

$$\tilde{K}_{1a} = 0.331, \tilde{K}_{2a} = 0.335, \tilde{K}_{3a} = 0.334 \quad (32)$$

$$\tilde{K}_{1b} = 2.07, \tilde{K}_{2b} = 2.02, \tilde{K}_{3b} = 2.11 \quad (33)$$

$$\tilde{K}_{1c} = 0.978, \tilde{K}_{2c} = 0.987, \tilde{K}_{3c} = 0.988 \quad (34)$$

Calculate the assessment value by the following equation (24) as,

$$\tilde{K}_1 = 2.00, \tilde{K}_{2a} = 1.987, \tilde{K}_{3a} = 2.03 \quad (35)$$

If we sort the alternatives in descending order by value K_i , we get, $A_3 > A_1 > A_2$ Hence, A_3 is the best alternative for solar plant site selection. The ranking values are given in the table below,

Table 4: The assessment value of each alternative

	S_i	P_i	K_i	Ranking order
A_1	0.676	4.623	2.00	2

A_2	0.628	4.717	1.987	3
A_3	0.696	4.651	2.03	1

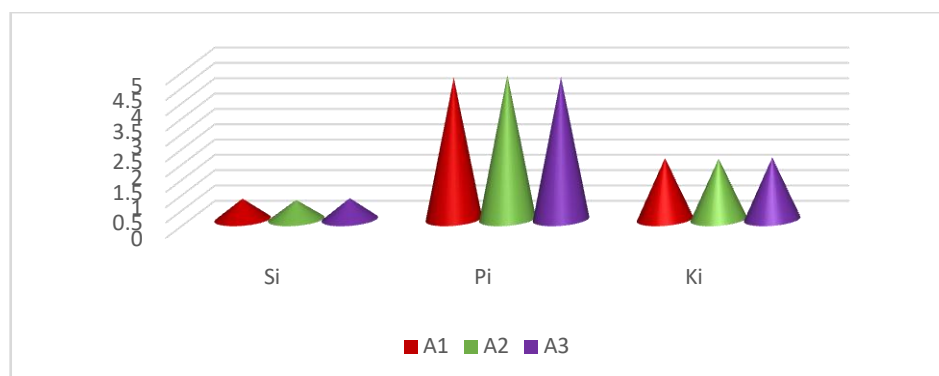


Figure .7: Graphical representation of COCOSO method

6 Comparative Analysis

In this section, we compare our proposed method with a triangular hesitant bipolar fuzzy set.

• A comparative analysis between Triangular hesitant bipolar fuzzy (THBF) and proposed method:

The NWHBF can be observed as a special case of evaluating the positive and negative membership degrees of THBFs. But in THBFs, the choice maker clearly assigns their values without any hesitancy. In NWHBFs the choice maker presents their reluctance. Therefore, an in-depth investigation of the hesitancy of the selected problem can be provided. Thus the chosen alternative is subject to various analysis and there is no obvious solution. The THBFs values are given in table 5. Now we compute the detailed evaluation value using the score function and calculate the normalized entropy matrix. We then use the COCOSO method to rank the alternatives. For that, we need to calculate the total power weight of the comparison sequences for each alternative and evaluation value and the sum of the weighted comparison sequence for each alternative. COCOSO method is one of the best ranking method in MCDM. Ranking results are shown in the table 6.

The ranking results are related to those of the proposed method. But NWHB is more workable than THBF. This is because they assume a situation which information about choice makers is deep and unearthing. In NWHBF decision makers prefer to use a number of hesitant possible values to express positive and negative membership. Figure .8 shows the result.

Table 5: Triangular hesitant bipolar Decision matrix

	A_1	A_2	A_3
C_1	(0.5, 0.6, 0.7), (-0.1, -0.1, -0.2)	(0.2, 0.7, 0.8), (-0.3, -0.4, -0.4)	(0.4, 0.7, 0.8), (-0.1, -0.2, -0.2)
C_2	(0.6, 0.7, 0.9), (-0.1, -0.1, -0.2)	(0.7, 0.8, 0.9), (-0.3, -0.3, -0.2)	(0.5, 0.5, 0.7), (-0.2, -0.2, -0.3)
C_3	(0.3, 0.5, 0.6), (-0.3, -0.4, -0.4)	(0.4, 0.7, 0.8), (-0.3, -0.4, -0.3)	(0.4, 0.4, 0.5), (-0.1, -0.2, -0.3)
C_4	(0.2, 0.5, 0.5), (-0.4, -0.5, -0.5)	(0.7, 0.7, 0.8), (-0.4, -0.2, -0.3)	(0.4, 0.5, 0.7), (-0.1, -0.1, -0.2)
C_5	(0.3, 0.5, 0.6), (-0.3, -0.4, -0.4)	(0.3, 0.5, 0.6), (-0.3, -0.3, -0.4)	(0.4, 0.5, 0.6), (-0.2, -0.3, -0.4)

Table 6: The evaluation value of each alternative

	S_i	P_i	K_i	Rank
A_1	0.591	4.502	2.04	2
A_2	0.526	4.394	1.94	3
A_3	0.633	4.563	2.12	1

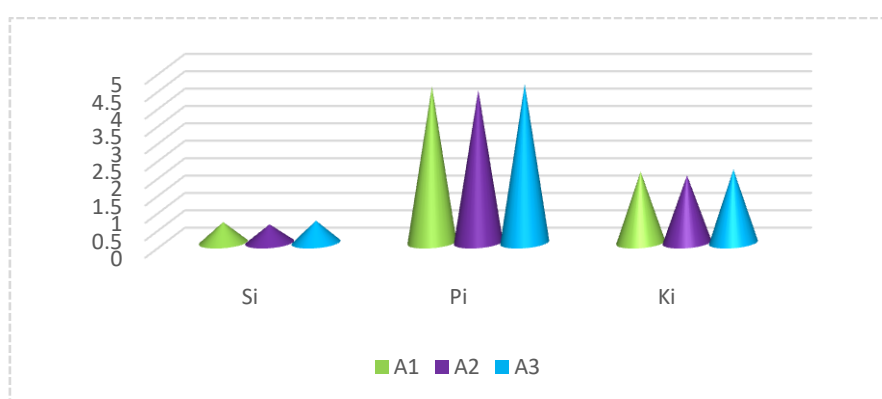


Figure.8: Ranking result of THBF-COCOSO method

7 Conclusion

In this paper, NWHBF-COCOSO under NWHBF context is proposed to extract the basic and uncertain information of decision makers by combining the traditional COCOSO approach. This

new extended NWHBF is a more appropriate and reasonable approach to obtain reliable hesitancy information, considering the potential uncertainty and ambiguity of DMs hidden behind their original estimates in the decision making process. On the basis, a case is presented to demonstrate the validity and rationality of the proposed method.

NWHBF is an extended work of NWHFs, which beneficially replaces the original evaluation of information by DMs in the decision making process. The NWHBF we developed mainly focusing on the unresolved thoughts of DMs. The wiggly range is also established the assumption that humans express their opinions and ideas. NWHBF's superior concept can be reflected in three aspects of 1.NWHBF maintains real non-reluctance fuzzy information and normal wiggly hesitation fuzzy information. 2. NWHBF can inherently detect deep uncertainty information based on the growth law of objects and intrinsic properties of the data set. 3.Mathematically NWHBFs can have all the properties of HBF, THBF and IVHBFs. This paper has presented a effective approach to deal with MCDM problems in a complicated world, which can not only represent uncertainty but also depict decision makers knowledge and cognitive behaviour of deep information. Also, considering the illustrative example of the solar plant site selection problem, we need to explore a new decision making method based on NWHBF.

References

- [1] Alghamdi, M.A., Alshehri, N.O., Akram, M. Multi-criteria decision-making methods in bipolar fuzzy environment. *International Journal of Fuzzy Systems*, 20(6) (2018), 2057–2064.
- [2] Akram, M., Arshad, M, A novel trapezoidal bipolar fuzzy TOPSIS method for group decision-making. *Group Decision and Negotiation*, 1–20, (2018).
- [3] A. Memari, A. Dargi, M.R.A. Jokar, et al., Sustainable supplier selection: A multi-criteria intuitionistic fuzzy TOPSIS method, *Journal of Manufacturing Systems* 50 (2019), 9–24.
- [4] A. Trivedi, S.K. Jha, S. Choudhary and R. Shandley, Fuzzy TOPSIS Multi-criteria Decision Making for Selection of Electric Molding Machine. *Innovations in Computer Science and Engineering*. In book: *Innovations in Computer Science and Engineering*, Springer, Singapore (2019), 325–332.
- [5] Bagal, D. K., Giri, A., Pattanaik, A. K., Jeet, S., Barua, A., Panda, S. N. (2021). MCDM

Optimization of Characteristics in Resistance Spot Welding for Dissimilar Materials Utilizing Advanced Hybrid Taguchi Method-Coupled CoCoSo, EDAS and WASPAS Method. In *Next Generation Materials and Processing Technologies* (pp. 475-490). Springer, Singapore.

- [6] Deveci, M., Pamucar, D., & Gokasar, I. (2021). Fuzzy Power Heronian function based CoCoSo method for the advantage prioritization of autonomous vehicles in real-time traffic management. *Sustainable Cities and Society*, 69, 102846.
- [7] Ecer, F. (2021). A consolidated MCDM framework for performance assessment of battery electric vehicles based on ranking strategies. *Renewable and Sustainable Energy Reviews*, 143, 110916.
- [8] G. Wei, J. Wang, J. Lu, et al., VIKOR method for multiple criteria group decision making under 2-tuple linguistic neutrosophic environment, *Economic Research-Ekonomska Istraživanja* (2019), 1–24.
- [9] H.Peng, K.Shen, S.He et al, Investment risk evaluation for new energy resources: An integrated decision support model based on regret theory and ELECTRE-III. *Energy conversion & management*. 183(2019), 332-348.
- [10] H. Liao, et al., Score-HeDLiSF: A score function of hesitant fuzzy linguistic term set based on hesitant degrees and linguistic scale functions: An application to unbalanced hesitant fuzzy linguistic MULTIMOORA, *Information Fusion* 48 (2019), 39–54.
- [11] Han, Y., Lu, Z., Du, Z., Luo, Q., Chen, S, A YinYang bipolar fuzzy cognitive TOPSIS method to bipolar disorder diagnosis. *Computer Methods and Programs in Biomedicine*, 158, 1–10, (2018).
- [12] J.H. Dahooie, E.K. Zavadskas, H.R. Firoozfar, et al., An improved fuzzy MULTIMOORA approach for multicriteria decision making based on objective weighting method (CCSD) and its application to technological forecasting method selection, *Engineering Applications of Artificial Intelligence* 79 (2019), 114–128.
- [13] J. Zhao, H. Zhu and H. Li, 2-Dimension linguistic PROMETHEE methods for multiple attribute decision making, *Expert Systems with Applications* 127 (2019), 97–108.
- [14] K. Atanassov, G. Gargov, Interval valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems* 31(3) (1989), 343-349.

- [15] L.A. Zadeh, Fuzzy sets, *Information & Control* 8(3) (1965) 338-353.
- [16] M.Y. Li and P.P. Cao, Extended TODIM method for multiattribute risk decision making problems in emergency response, *Computers & Industrial Engineering* 135 (2019), 1286–1293.
- [17] Noorollahi, Y., Ghasempour, R. and Jalilinasrabady, S. (2015). A GIS Based Integration Method for Geothermal Resources Exploration and Site Selection. *Energy Exploration & Exploitation* 33 (2): 243-257.
- [18] N. Chen and Z. Xu, Hesitant fuzzy ELECTRE II approach: a new way to handle multi-criteria decision making problems, *Information Sciences* 292 (2015), 175–197.
- [19] P. Liu and X. Qin, An extended VIKOR method for decision making problem with interval-valued linguistic intuitionistic fuzzy numbers based on entropy, *Informatica* 28(4) (2017), 665–685.
- [20] Peng, X., & Luo, Z. (2021). Decision-making model for China's stock market bubble warning: the CoCoSo with picture fuzzy information. *Artificial Intelligence Review*, 1-23.
- [21] Peng, X., Krishankumar, R., & Ravichandran, K. S. (2021). A novel interval-valued fuzzy soft decision-making method based on CoCoSo and CRITIC for intelligent healthcare management evaluation. *Soft Computing*, 25(6), 4213-4241.
- [22] Q.Liang and J.M.Mendel, Interval type-2 fuzzy logic systems:theory and design. *IEEE trans fuzzy syst* 8(2000), 535-550.
- [23] Sánchez-Lozano, J.M., García-Cascales, M.S. and Lamata M.T.(2015). Evaluation of suitable locations for the installation of solar thermoelectric power plant. *Computers & Industrial Engineering* 87, 343–355.
- [24] Stanujkic, D., Popovic, G., Zavadskas, E. K., Karabasevic, D., & Binkyte-Veliene, A. (2020). Assessment of Progress towards Achieving Sustainable Development Goals of the “Agenda 2030” by Using the CoCoSo and the Shannon Entropy Methods: The Case of the EU Countries. *Sustainability*, 12(14), 5717.
- [25] Torkayesh, A. E., Pamucar, D., Ecer, F., & Chatterjee, P. (2021a). An integrated BWM-LBWA CoCoSo framework for evaluation of healthcare sectors in Eastern Europe. *Socio-Economic Planning Sciences*, 101052.
- [26] Ulutaş, A., Popovic, G., Radanov, P., Stanujkic, D., & Karabasevic, D. (2021). A new hybrid

- fuzzy PSI-PIPRECIA-CoCoSo MCDM based approach to solving the transportation company selection problem. *Technological and Economic Development of Economy*, 27(5), 1227-1249.
- [27] V. Torra, Hesitant fuzzy sets, *International Journal of Intelligent Systems* 25(6) (2010) 529-539.
- [28] Wen, Z., Liao, H., Kazimieras Zavadskas, E., & Al-Barakati, A. (2019). Selection third-party logistics service providers in supply chain finance by a hesitant fuzzy linguistic combined compromise solution method. *Economic research-Ekonomska istraživanja*, 32(1), 4033-4058.
- [29] W. Zhang, Y. Zhu, D. Wang, et al., A Multi-attribute Decision Making Method Based on Interval Pythagorean Fuzzy Language and the PROMETHEE Method. *The International Conference on Natural Computation, Fuzzy Systems and Knowledge Discovery*. Springer, Cham, (2019), 818–826.
- [30] Yazdani, M., Zarate, P., Zavadskas, E. K., & Turskis, Z. (2019a). A Combined Compromise Solution (CoCoSo) method for multi-criteria decision-making problems. *Management Decision*, 57(9), 2501-2519.
- [31] Zhang, W.R. Bipolar fuzzy sets and relations: A computational framework for cognitive modeling and multiagent decision analysis. In *Proceedings of the IEEE Conference Fuzzy Information Processing Society Biannual Conference*, San Antonio, TX, USA, 18–21 December 1994; pp. 305–309.