Estimation of the Survival Function Using the New Mixed Distribution (Power Function-Truncated Burr III) With Application

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Abstract

In this research, a new mixture of distributions (Power Function-Truncated Burr III) (MPF-TBIII) was proposed. The proposed model with finding some mathematical and statistical properties of the distribution. Then the parameters and survival function of the proposed distribution (MPF-TBIII) were estimated using different methods of estimation, which are the greatest possibility method (MLE) and the fractional estimation method (PS).Fractional estimators (PS) were the best in estimation at large sample sizes, and then the fractional estimators (PS) method was used. Estimating the survival function of corona patients.

1 Introduction:

The life expectancy of humans, animals, and devices in engineering and medical sciences can be studied by means of survival distributions life Probability distributions are important in life for describing and predicting real-world phenomena. Distributions have been used to model data in many fields of medicine, environment, economics, engineering and others. But in some cases the probability distributions are not the classic is suitable for the phenomenon studied, and to improve the suitability of the data for distribution, new families of distributions have been created, namely the distributionsmixed. So the mixed distributions will be obtained by adding one or more parameters to the distribution so as to make these the distributions are more flexible for practical application to real data of one or more parameters to the survivability distribution of the mixed function so as to make this distributions are more flexible for practical application to real data.

2 Research problem:

With the development of data types, many problems began to appear in the modeling of those data, and probability distributions were used because the probability distributions are not sufficient to explain some scientific phenomena, the need has arisen to find probability distributions a new one that is more flexible in dealing with and modeling data, which is the mixed probability distributions.

3 Research aims:

The research aims to build a new probabilistic model using mixing distributions in addition to deriving the statistical properties of the distribution and then obtaining the best estimate of the distribution parameters by comparing the two methods of greatest possibility and fractional estimations based on an average indicatorIMSE squares, and then finding the best estimate of survival function for Corona patients by comparing the distribution of Power Function and Truncated Burr II and the new mixed distribution using the best method of estimation.

4 Power Function Distribution:[10]

The power function distribution is one of the important and frequently used distributions in failure data modeling, and the power function distribution is a special case of the Pareto distribution.

If x is a continuous random variable, the probability density function is as follows:

$$f(x) = \frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} \qquad 0 \leq x \leq \beta \qquad \alpha, \beta > 0 \qquad \ldots.(4-1)$$

Where α is a shape parameter, β Scale parameter.

The cumulative distribution function is as follows:

$$F(x) = \frac{x^{\alpha}}{\beta^{\alpha}} \qquad \ldots.(4-2)$$

5 Burr III distribution:[2],[5]

The Burr distribution was first introduced by researcher Burr (1942) who developed twelve types of cumulative function that were used in the analysis and modeling of lifetime data.

The Burr III distribution is a flexible distribution used for data modeling that was introduced by Burr. The cumulative function of the distribution is given as follows:

$$F(x) = (1 + x^{-\lambda})^{-\theta} \qquad \ldots (5-1)$$

As for the probability density function, it is given as follows:

$$f(x) = \lambda\theta x^{-\lambda-1}(1 + x^{-\lambda})^{-\theta-1} \quad ,0 < x < \infty \quad \lambda, \theta > 0 \qquad \ldots (5-2)$$

Where both parameters (λ, θ) are shape parameter.

6 Distribution of the truncated Burr III:[8]

Truncated probability distributions have many applications in various fields and are taken when the full range of a random variable is unobservable or when a set of random variable values are ignored.

The distribution will be subtracted from the right by applying the following formula:

$$f^*(x) = \frac{f(x)}{F(b)} \qquad 0 < x < b \qquad \ldots (6-1)$$

After substitution we get the truncated Burr III distribution:

$$f^*(x) = \frac{\lambda\theta x^{-\lambda-1}(1 + x^{-\lambda})^{-\theta-1}}{(1 + \beta^{-\lambda})^{-\theta}} \qquad 0 < x < \beta \qquad (6-2)$$

Where both parameters (λ, θ) are shape parameter, β is truncated parameter.

7 Mixed Power Function-Truncated Burr III:[3]

Mixed distributions are a combination of two or more distributions that arise when the populations are heterogeneous. Mixed distributions are of constant weight.

The two distributions will be mixed using the following formula:

$$g(x) = w_1 f_1(x) + w_2 f^*{}_2(x)$$

$$w_1 + w_2 = 1 w_2 = 1 - w_1$$

$$g(x) = w_1 f_1(x) + (1 - w_1) f^*{}_2(x)$$

$f_1(x)$: represents Power Function Distribution

$f^*{}_2(x)$: represents Truncated Burr III Distribution

Assuming that $w_1 = \gamma$ so:

$$g(x) = \gamma f_1(x) + (1-\gamma) f_2(x) \qquad \dots(7-1)$$

$$g(x) = \gamma \frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} + (1-\gamma) \frac{\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}} \quad ,0 < x < \beta \qquad \dots(7-2)$$

Where the parameters $(\alpha, \lambda, \theta)$ are shape parameters, and β is Scale parameter,

γ is Mixed parameter.

In order to prove the distribution function mentioned above that it is a probability density function, we must prove:

$$\int_0^{\beta} g(x, \alpha, \beta, \lambda, \theta, \gamma) dx = 1$$

$$\int_0^{\beta} \gamma \frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} + (1-\gamma) \frac{\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}} \, dx$$

$$= \frac{\gamma\alpha}{\beta^{\alpha}} \left[\frac{x^{\alpha}}{\alpha}\right]_0^{\beta} + \frac{(1-\gamma)}{\left(1+\beta^{-\lambda}\right)^{-\theta}} * \left(1+\beta^{-\lambda}\right)^{-\theta}$$

$$= \gamma + 1 - \gamma$$

$$= 1$$

8 Cumulative function of (Power Function-Truncated Burr III):[6]

$$G(x) = \int_0^{x} g(u) du$$

$$G(x) = \int_0^{x} \gamma \frac{\alpha u^{\alpha-1}}{\beta^{\alpha}} + (1-\gamma) \frac{\lambda\theta u^{-\lambda-1}\left(1+u^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}} du$$

$$G(x) = \frac{\gamma x^{\alpha}}{\beta^{\alpha}} + \frac{(1-\gamma)}{\left(1+\beta^{-\lambda}\right)^{-\theta}} * \left(1+x^{-\lambda}\right)^{-\theta} \qquad \dots(8-1)$$

9 Survival Function of (Power Function-Truncated Burr III):[4]

$$S(x) = 1 - G(x)$$

$$= \frac{\beta^{\alpha}\left(1+\beta^{-\lambda}\right)^{-\theta} - \gamma x^{\alpha}\left(1+\beta^{-\lambda}\right)^{-\theta} - (1-\gamma)\beta^{\alpha}\left(1+x^{-\lambda}\right)^{-\theta}}{\beta^{\alpha}\left(1+\beta^{-\lambda}\right)^{-\theta}} \quad \ldots (9-1)$$

10 Hazard Function of (Power Function-Truncated Burr III):[4]

$$h(x) = \frac{g(x)}{S(x)}$$

$$= \frac{\gamma\alpha x^{\alpha-1}\left(1+\beta^{-\lambda}\right)^{-\theta} + (1-\gamma)\lambda\theta\beta^{\alpha}x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\beta^{\alpha}\left(1+\beta^{-\lambda}\right)^{-\theta} - \gamma x^{\alpha}\left(1+\beta^{-\lambda}\right)^{-\theta} - (1-\gamma)\beta^{\alpha}(1+x^{-\lambda})^{-\theta}} \quad \ldots (10-1)$$

11 Structural Characteristics of the Power Function-Truncated Burr III:

1- Non-central moments:[9]

$$\mu_r^* = E(x^r) = \int_0^{\beta} x^r g(x)dx$$

$$\mu_r^* = E(x^r) = \frac{\gamma\alpha\beta^r}{\alpha+r} + \frac{(1-\gamma)\theta}{\left(1+\beta^{-\lambda}\right)^{-\theta}}(-1)^{-\frac{r}{\lambda}}\beta\left(\beta, 1-\frac{r}{\lambda}, -\theta\right) \quad \ldots (11-1)$$

If r=1, then it represents the arithmetic mean of the distribution.

To find the variance, we use the following formula:

$$v(x) = E(x^2) - \left(E(x)\right)^2 \quad \ldots (11-2)$$

2- Central moments:[7]

$$\mu_k = E(x - E(x))^k$$

$$\mu_k = E\left[\sum_{j=0}^{k} C_j^k (-1)^j x^{k-j} \mu_1^{*j}\right] \quad \ldots (11-3)$$

12 Estimation Method:

1- Maximum likelihood method:[1]

It is one of the most important methods of estimation due to the accuracy of its capabilities. This method is characterized by efficiency, adequacy, stability and consistency, in addition to unbiased.

$$l\,g(x,\alpha,\beta,\lambda,\theta,\gamma) = \prod_{i=1}^{n} g(x,\alpha,\beta,\lambda,\theta,\gamma)$$

$$\ln l\,g = \sum_{i=1}^{n} \ln\left[\gamma\frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} + (1-\gamma)\frac{\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}}\right]$$

$$\frac{d\ln lg}{d\alpha} = \left\{\sum_{i=1}^{n}\frac{1}{\left\{\gamma\frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} + (1-\gamma)\frac{\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}}\right\}} * \left[\frac{\gamma\alpha x^{\alpha-1}\ln(x) - \gamma\alpha x^{\alpha-1}\ln(\beta)}{\beta^{\alpha}}\right]\right\} = 0 \qquad \ldots(12-1)$$

$$\frac{d\ln lg}{d\beta} = \left\{\sum_{i=1}^{n}\frac{1}{\left\{\gamma\frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} + (1-\gamma)\frac{\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}}\right\}} * \left[\frac{-\gamma\alpha^{2}x^{\alpha-1}}{\beta^{\alpha+1}} - \frac{(1-\gamma)\lambda^{2}\theta^{2}\beta^{-\lambda-1}x^{-\lambda-1}(1+x^{-\lambda})^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta+1}}\right]\right\} = 0 \qquad \ldots(12-2)$$

$$\frac{d\ln lg}{d\lambda} = \left\{\sum_{i=1}^{n}\frac{1}{\left\{\gamma\frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} + (1-\gamma)\frac{\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}}\right\}} * \left[\frac{(1-\gamma)\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}} - \frac{(1-\gamma)\lambda\theta x^{-\lambda-1}\ln(x)\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}} - \frac{(1-\gamma)\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}(-\theta-1)x^{-\lambda}\ln(x)}{(1+x^{-\lambda})\left(1+\beta^{-\lambda}\right)^{-\theta}} - \frac{(1-\gamma)\lambda\theta^{2}x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}\beta^{-\lambda}\ln(\beta)}{\left(1+\beta^{-\lambda}\right)^{-\theta}(1+\beta^{-\lambda})}\right]\right\} = 0 \qquad \ldots(12-3)$$

$$\frac{d\ln lg}{d\theta}=\left\{\sum_{i=1}^{n}\frac{1}{\left\{\gamma\frac{\alpha x^{\alpha-1}}{\beta^{\alpha}}+(1-\gamma)\frac{\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}}\right\}} * \left[\frac{(1-\gamma)\lambda x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}}-\frac{(1-\gamma)\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}\ln\left(1+x^{-\lambda}\right)}{\left(1+\beta^{-\lambda}\right)^{-\theta}} + \frac{(1-\gamma)\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta}\ln\left(1+\beta^{-\lambda}\right)}{\left(1+\beta^{-\lambda}\right)^{-\theta}}\right]\right\}=0 \quad \dots(12-4)$$

$$\frac{d\ln lg}{d\gamma}=\left\{\sum_{i=1}^{n}\frac{1}{\left\{\gamma\frac{\alpha x^{\alpha-1}}{\beta^{\alpha}}+(1-\gamma)\frac{\lambda\theta x^{-\lambda-1}\left(1+x^{-\lambda}\right)^{-\theta-1}}{\left(1+\beta^{-\lambda}\right)^{-\theta}}\right\}} * \left[\frac{\alpha x^{\alpha-1}}{\beta^{\alpha}}-\lambda\theta x^{-\lambda-1}(1+\beta^{-\lambda})^{\theta}\left(1+x^{-\lambda}\right)^{-\theta-1}\right]\right\} = 0 \quad \dots(12-5)$$

For equations that represent a system of nonlinear equations that cannot be solved except by using one of the numerical method in order to obtain the estimations, the Newton-Raphson method has been used.

2- Percentiles Estimators Method:[6]

This method is based on the cumulative distribution function. Assuming that Pi is the estimator of the cumulative distribution function and making the function at its minimum limit by finding the estimator using the following formula:

$$Q=\sum_{i=1}^{n}[p_i-G(x_i)]^2$$

Where pi is the nonparametric estimator and takes the following form:

$$p_i=\frac{i-0.3}{n+0.25}$$

$$Q=\sum_{i=1}^{n}\left[\frac{i-0.3}{n+0.25}-\frac{\gamma x^{\alpha}}{\beta^{\alpha}}-\frac{(1-\gamma)}{(1+\beta^{-\lambda})^{-\theta}} * \left(1+x^{-\lambda}\right)^{-\theta}\right]^2 \quad \dots(12-6)$$

We derive the above equation for the parameters, we get:

$$\frac{dQ}{d\alpha}=\left\{2\sum_{i=1}^{n}\left[\frac{i-0.3}{n+0.25}-\frac{\gamma x^{\alpha}}{\beta^{\alpha}}-\frac{(1-\gamma)}{(1+\beta^{-\lambda})^{-\theta}}*\left(1+x^{-\lambda}\right)^{-\theta}\right] * \left[\frac{-\gamma x^{\alpha}ln(x)+\gamma x^{\alpha}ln(\beta)}{\beta^{\alpha}}\right]\right\} \quad ...(12-7)$$

$$\frac{dQ}{d\beta}=\left\{2\sum_{i=1}^{n}\left[\frac{i-0.3}{n+0.25}-\frac{\gamma x^{\alpha}}{\beta^{\alpha}}-\frac{(1-\gamma)}{(1+\beta^{-\lambda})^{-\theta}}*\left(1+x^{-\lambda}\right)^{-\theta}\right] * \left[\frac{\gamma\alpha x^{\alpha}}{\beta^{\alpha+1}}+\frac{(1-\gamma)\theta\lambda\beta^{-\lambda-1}(1+x^{-\lambda})^{-\theta}}{(1+\beta^{-\lambda})^{-\theta+1}}\right]\right\} \quad ..(12-8)$$

$$\frac{dQ}{d\lambda}=\left\{2\sum_{i=1}^{n}\left[\frac{i-0.3}{n+0.25}-\frac{\gamma x^{\alpha}}{\beta^{\alpha}}-\frac{(1-\gamma)}{(1+\beta^{-\lambda})^{-\theta}}*\left(1+x^{-\lambda}\right)^{-\theta}\right] * \left[\frac{-(1-\gamma)\theta\left(1+x^{-\lambda}\right)^{-\theta}x^{-\lambda}ln(x)}{(1+x^{-\lambda})(1+\beta^{-\lambda})^{-\theta}}+\frac{(1-\gamma)\theta\left(1+x^{-\lambda}\right)^{-\theta}\beta^{-\lambda}ln(\beta)}{(1+\beta^{-\lambda})^{-\theta+1}}\right]\right\} \quad ....(12-9)$$

$$\frac{dQ}{d\theta}=\left\{2\sum_{i=1}^{n}\left[\frac{i-0.3}{n+0.25}-\frac{\gamma x^{\alpha}}{\beta^{\alpha}}-\frac{(1-\gamma)}{(1+\beta^{-\lambda})^{-\theta}}*\left(1+x^{-\lambda}\right)^{-\theta}\right] * \left[\frac{(1-\gamma)\left(1+x^{-\lambda}\right)^{-\theta}ln(1+x^{-\lambda})}{(1+\beta^{-\lambda})^{-\theta}}-\frac{(1-\gamma)\left(1+x^{-\lambda}\right)^{-\theta}ln(1+\beta^{-\lambda}}{(1+\beta^{-\lambda})^{-\theta}}\right]\right\} \quad ....(12-10)$$

$$\frac{dQ}{d\gamma}=\left\{2\sum_{i=1}^{n}\left[\frac{i-0.3}{n+0.25}-\frac{\gamma x^{\alpha}}{\beta^{\alpha}}-\frac{(1-\gamma)}{(1+\beta^{-\lambda})^{-\theta}}*\left(1+x^{-\lambda}\right)^{-\theta}\right] * \left[\frac{-x^{\alpha}}{\beta^{\alpha}}+\frac{\left(1+x^{-\lambda}\right)^{-\theta}}{(1+\beta^{-\lambda})^{-\theta}}\right]\right\} \quad ...(12-11)$$

For equations that represent a system of nonlinear equations that cannot be solved except by using one of the numerical method in order to obtain the estimations, the Newton-Raphson method has been used.

13 Simulation Experiments:[6]

The simulation was implemented based on four sample sizes (30, 60,100,150) to see the impact of the sample size on the accuracy of the results of the estimation methods, in addition to relying on a set of models of default values for the (Power Function-Truncated Burr III) distribution parameters shown in the table below. The repetition of the experiment was 1000 for each model to obtain the highest possible homogeneity.

Table 1: The initial values of the parameters and the proposed models

Model	α	β	θ	λ	γ
Model 1	4	4	2	2	0.5
Model 2	4	4	2	2	0.1
Model 3	4	4	2	2	0.9
Model 4	3	1	1	3	0.7
Model 5	3	1	1	3	0.1
Model 6	1	3	1	3	0.1
Model 7	1	3	2	1	0.1
Model 8	1	3	2	1	0.8
Model 9	3	2	5	1	0.1
Model 10	1	1	1	1	0.5

The rejection and acceptance method was used to generate random numbers because it was not possible to obtain the inverse function of the cumulative distribution function.

14 Discuss the simulation results:

The rank method was used to compare the survival function estimation methods. Each estimation method will be given a rank for each value and for each sample size and takes the smallest IMSE value between the methods of the first and largest rank, the second rank, and this stage is called partial ranks, and then the sub-ranks are collected for each estimation method and new ranks are given to it. Here, the ranks are called the total ranks, which are compared based on it.

Table 2: Represents the total ranks of the mean IMSE for all estimation methods, for all models of default parameter values, and for all sample sizes.

n		MLE	PS
30	Sum of Ranks	18	26
	Overall Ranks	1	2
60	Sum of Ranks	18	23.5
	Overall Ranks	1	2
100	Sum of Ranks	18	20
	Overall Ranks	1	2
150	Sum of Ranks	22	14.5
	Overall Ranks	2	1

From the above table we can conclude the following:

1- The Maximum Likelihood method is best for estimating the survival function at
2- Sample size (30,60,100) and it take ranked second when the sample size is 150.
3- Percentiles Estimators method ranked second when the sample sizes (30,60,100) and ranked first when the sample size was 150 وThis means that it is the best method for large sample sizes.

15 Practical application on local data:

A sample of size n=140 was taken randomly and represented by measuring the length of stay in weeks under treatment until death for patients with Coronavirus (COVID-19).

The following table shows the main statistics of the actual data sample:

mean	1.58673
Variance	0.875713
mean	0.488767
Variance	2.67191
mean	1.57143
Variance	0.935796

16 Real data analysis:

Real data were analyzed by PS method. The Kolmogorov-Smirnov test was used to find out if the data follow the distribution (Power Function- Truncated Burr III) and according to the hypothesis:

H0: (Power Function- Truncated Burr III) the data follow a distribution.

H1: (Power Function- Truncated Burr III) the data does not follow a distribution.

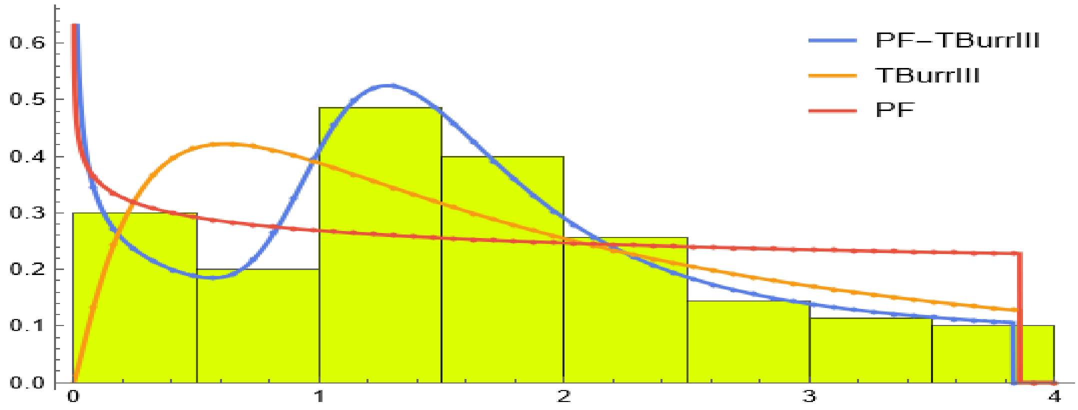
	Statistic	P-Value
Kolmogorov-Smirnov	0.080964	0.301357

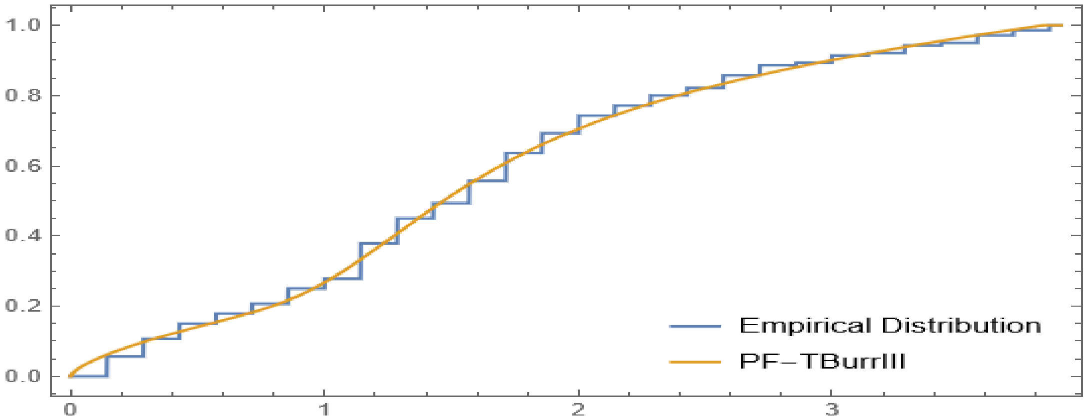
We notice from the above table that the P-value is greater than the level of significance (0.05), so we accept the null hypothesis and the data are distributed according to the Power Function- Truncated Burr III distribution.

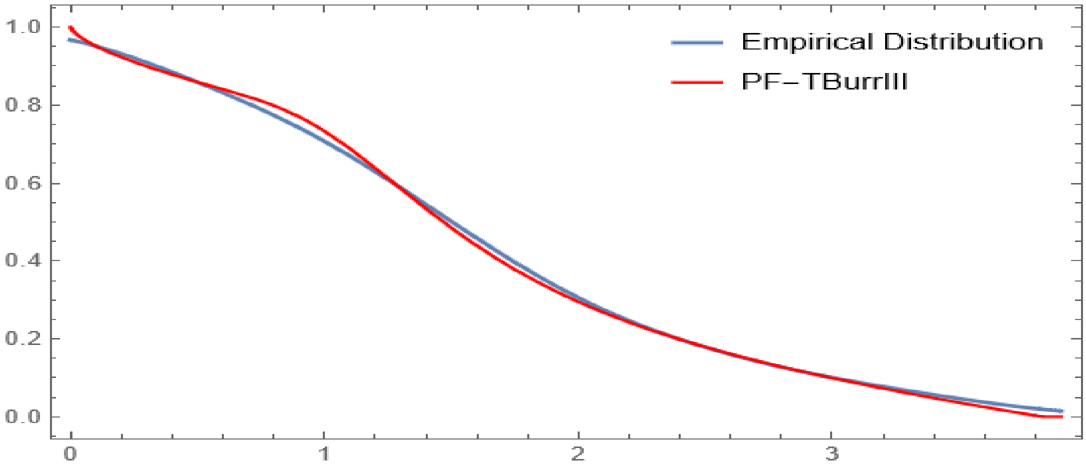
Dist.	Parameter					Log (l)	AIC	AIC_C
PF-TBurrIII	0.66191	3.7994	3.42206	3.39451	0.532881	174.909	359.818	360.266
TBurrIII	_	3.85714	2.37274	0.94299	_	178.254	362.509	362.685
PF	0.878144	3.85714	_	_	_	187.755	379.51	379.597

From the above table, it is clear that the best distribution is (Power Function- Truncated Burr III) because it has the lowest value of the parameters.

The following figure shows the appropriateness of the distribution of (Power Function- Truncated Burr III):







After analyzing the data, the values of survival function, cumulative distribution function and risk function were extracted as shown in the following table:

i	t_i	Cdf	S(x)	h(x)	i	t_i	Cdf	S(x)	h(x)
1	0.14	0.060748	0.939252	0.299672	32	0.86	0.215605	0.784395	0.377489
2	0.14	0.060748	0.939252	0.299672	33	0.86	0.215605	0.784395	0.377489
3	0.14	0.060748	0.939252	0.299672	34	0.86	0.215605	0.784395	0.377489

4	0.14	0.060748	0.939252	0.299672	35	0.86	0.215605	0.784395	0.377489
5	0.14	0.060748	0.939252	0.299672	36	1.00	0.266252	0.733748	0.562842
6	0.14	0.060748	0.939252	0.299672	37	1.00	0.266252	0.733748	0.562842
7	0.14	0.060748	0.939252	0.299672	38	1.00	0.266252	0.733748	0.562842
8	0.14	0.060748	0.939252	0.299672	39	1.00	0.266252	0.733748	0.562842
9	0.29	0.096114	0.903886	0.246353	40	1.14	0.332112	0.667888	0.748589
10	0.29	0.096114	0.903886	0.246353	41	1.14	0.332112	0.667888	0.748589
11	0.29	0.096114	0.903886	0.246353	42	1.14	0.332112	0.667888	0.748589
12	0.29	0.096114	0.903886	0.246353	43	1.14	0.332112	0.667888	0.748589
13	0.29	0.096114	0.903886	0.246353	44	1.14	0.332112	0.667888	0.748589
14	0.29	0.096114	0.903886	0.246353	45	1.14	0.332112	0.667888	0.748589
15	0.29	0.096114	0.903886	0.246353	46	1.14	0.332112	0.667888	0.748589
16	0.43	0.125724	0.874276	0.22269	47	1.14	0.332112	0.667888	0.748589
17	0.43	0.125724	0.874276	0.22269	48	1.14	0.332112	0.667888	0.748589
18	0.43	0.125724	0.874276	0.22269	49	1.14	0.332112	0.667888	0.748589
19	0.43	0.125724	0.874276	0.22269	50	1.14	0.332112	0.667888	0.748589
20	0.43	0.125724	0.874276	0.22269	51	1.14	0.332112	0.667888	0.748589
21	0.43	0.125724	0.874276	0.22269	52	1.14	0.332112	0.667888	0.748589
22	0.57	0.152525	0.847475	0.217382	53	1.14	0.332112	0.667888	0.748589
23	0.57	0.152525	0.847475	0.217382	54	1.29	0.406135	0.593865	0.886049
24	0.57	0.152525	0.847475	0.217382	55	1.29	0.406135	0.593865	0.886049
25	0.57	0.152525	0.847475	0.217382	56	1.29	0.406135	0.593865	0.886049
26	0.71	0.180095	0.819905	0.25681	57	1.29	0.406135	0.593865	0.886049
27	0.71	0.180095	0.819905	0.25681	58	1.29	0.406135	0.593865	0.886049
28	0.71	0.180095	0.819905	0.25681	59	1.29	0.406135	0.593865	0.886049
29	0.71	0.180095	0.819905	0.25681	60	1.29	0.406135	0.593865	0.886049
30	0.86	0.215605	0.784395	0.377489	61	1.29	0.406135	0.593865	0.886049
31	0.86	0.215605	0.784395	0.377489	62	1.29	0.406135	0.593865	0.886049
63	1.29	0.406135	0.593865	0.886049	102	2.00	0.705872	0.294128	0.994348
64	1.43	0.480026	0.519974	0.965127	103	2.00	0.705872	0.294128	0.994348
65	1.43	0.480026	0.519974	0.965127	104	2.00	0.705872	0.294128	0.994348

66	1.43	0.480026	0.519974	0.965127	105	2.14	0.744813	0.255187	0.99552
67	1.43	0.480026	0.519974	0.965127	106	2.14	0.744813	0.255187	0.99552
68	1.43	0.480026	0.519974	0.965127	107	2.14	0.744813	0.255187	0.99552
69	1.43	0.480026	0.519974	0.965127	108	2.14	0.744813	0.255187	0.99552
70	1.57	0.548275	0.451725	0.998722	109	2.29	0.778813	0.221187	1.00866
71	1.57	0.548275	0.451725	0.998722	110	2.29	0.778813	0.221187	1.00866
72	1.57	0.548275	0.451725	0.998722	111	2.29	0.778813	0.221187	1.00866
73	1.57	0.548275	0.451725	0.998722	112	2.29	0.778813	0.221187	1.00866
74	1.57	0.548275	0.451725	0.998722	113	2.43	0.808853	0.191147	1.03796
75	1.57	0.548275	0.451725	0.998722	114	2.43	0.808853	0.191147	1.03796
76	1.57	0.548275	0.451725	0.998722	115	2.43	0.808853	0.191147	1.03796
77	1.57	0.548275	0.451725	0.998722	116	2.57	0.835737	0.164263	1.08797
78	1.57	0.548275	0.451725	0.998722	117	2.57	0.835737	0.164263	1.08797
79	1.71	0.608605	0.391395	1.00517	118	2.57	0.835737	0.164263	1.08797
80	1.71	0.608605	0.391395	1.00517	119	2.57	0.835737	0.164263	1.08797
81	1.71	0.608605	0.391395	1.00517	120	2.57	0.835737	0.164263	1.08797
82	1.71	0.608605	0.391395	1.00517	121	2.71	0.860101	0.139899	1.16485
83	1.71	0.608605	0.391395	1.00517	122	2.71	0.860101	0.139899	1.16485
84	1.71	0.608605	0.391395	1.00517	123	2.71	0.860101	0.139899	1.16485
85	1.71	0.608605	0.391395	1.00517	124	2.71	0.860101	0.139899	1.16485
86	1.71	0.608605	0.391395	1.00517	125	2.86	0.882444	0.117556	1.27837
87	1.71	0.608605	0.391395	1.00517	126	3.00	0.903152	0.096848	1.44548
88	1.71	0.608605	0.391395	1.00517	127	3.00	0.903152	0.096848	1.44548
89	1.71	0.608605	0.391395	1.00517	128	3.00	0.903152	0.096848	1.44548
90	1.86	0.660855	0.339145	1.00002	129	3.14	0.92253	0.07747	1.69829
91	1.86	0.660855	0.339145	1.00002	130	3.29	0.940812	0.059188	2.10519
92	1.86	0.660855	0.339145	1.00002	131	3.29	0.940812	0.059188	2.10519
93	1.86	0.660855	0.339145	1.00002	132	3.29	0.940812	0.059188	2.10519
94	1.86	0.660855	0.339145	1.00002	133	3.43	0.958185	0.041815	2.84075
95	1.86	0.660855	0.339145	1.00002	134	3.57	0.974794	0.025206	4.51817
96	1.86	0.660855	0.339145	1.00002	135	3.57	0.974794	0.025206	4.51817

97	1.86	0.660855	0.339145	1.00002	136	3.57	0.974794	0.025206	4.51817
98	2.00	0.705872	0.294128	0.994348	137	3.71	0.990757	0.009243	11.8699
99	2.00	0.705872	0.294128	0.994348	138	3.71	0.990757	0.009243	11.8699
100	2.00	0.705872	0.294128	0.994348	139	3.86	1	0	0
101	2.00	0.705872	0.294128	0.994348	140	3.86	1	0	0
Sum							70.3915	69.6085	147.002
Mean							0.5028	0.4972	1.05001

From the above table it is clear that:

1- The survival function is decreasing with time, that is, inversely proportional to time, and this is consistent with the statistical theory.
2- The cumulative distribution function values lie between zero and one and are increasing with time, and this is consistent with the statistical theory.
3- The average survival function is 0.4972, meaning that the probability of survival of the patient infected with the virus is approximately 50%.
4- The risk function is uneven, as it begins to increase gradually at the beginning of hospitalization, then stabilizes somewhat, and then returns to increasing to reach a maximum, and then gradually decreases its value.

17 Conclusions and Recommendations:

In this research, the distribution of (Power Function- Truncated Burr III) was found with the study of its properties. The distribution parameters were also estimated using two methods and then the best method was chosen for estimation using the mean integral error squares (IMSE), and the Percentiles Estimators Method (PS) was the best method.

One of the results we obtained is that the survival function is decreasing with time and this is consistent with the statistical theory. The values of the cumulative function lie between zero and one and are directly proportional to time. The sum of the values of the survival function and the values of the cumulative function is equal to one, the average values of the survival function are 0.4972, meaning that the probability of the patient's survival Survival of the injured is approximately 50%.

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