Modeling of Oscillatory Processes with Fading Effect and Their Accurate Description

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Abstract

The initial boundary value problem for the hyperbolic equation of the second order with nontrivial boundary condition is discussed in the paper. This problem is a mathematical model of different oscillatory processes. Thus, for example, the model of voltage distribution in a telegraph line emerges for the one-dimensional equation of oscillations. The models of oscillations of a round homogeneous solid membrane and membrane with an opening, and the model of gas oscillations in the sphere and spherical region emerge for two- and three-dimensional operators, but taking into account the radial symmetry of oscillations. The unified algorithm for reducing the corresponding problems to the initial boundary value problem with trivial boundary conditions is proposed. The description of solution development in the form of Fourier series by eigen functions of the corresponding Sturm-Liouville problem is presented.

Keywords: hyperbolic equation, modeling of oscillations with fading effect, boundary value problem, accurate solutions .

Introduction

Let us consider the differential equation in partial derivatives:

$$\alpha^2(t)u_{tt} + 2\beta(t)u_t + \gamma^2(t)u = Lu, \qquad (1)$$

$$u|_{x=0} = v(t), \qquad u|_{x=l} = 0, \qquad (2)$$

$$u|_{t=0} = u_t|_{t=0} = 0, \qquad (3)$$

where $\alpha(t), \beta(t), \gamma(t) -$ prescribed functions of time whose physical sense depends on the consideration of a specific physical process, $u = u(x,t) -$ required function. The differential expression defined by differential operator L by spatial variables, which has the second order or higher, is in the right side of the equation (1). In this paper we will consider the case of one spatial variable or multi-dimensional problems, which can be reduced to a one-dimensional one in one way or another.

Further, let us introduce the differential operator by time variable:

$$D_t \coloneqq \alpha^2(t)\frac{\partial^2}{\partial t^2} + 2\beta(t)\frac{\partial}{\partial t} + \gamma^2(t), \qquad (4)$$

in this case, the equation (1) will be as follows:

$$D_t u = Lu, \qquad (5)$$

the boundary conditions (2) and the initial conditions (3) remain unchanged.

Let us point out that many authors pay attention to such problems. Thus, the problem of the round film membrane oscillation with the electric current conductors distributed on it is discussed in [1]. It is possible to obtain the method of formulating the accurate law of membrane oscillations with the assumption of the solution linear dependence on radial variable and method of Fourier variable separation. The paper [2] considers the problem of controlling a string oscillation process without friction. The mathematical models of dynamic processes in heterogeneous structures are described in [3] according to the hypotheses of complex rigidity and internal friction. The accurate solutions obtained in the paper emerged when investigating ordinary differential equations with constant coefficients being the mathematical models of oscillations with friction. The method of obtaining the accurate solution for the linear model of oscillations with friction is proposed in [4]. The approximate-analytical method for calculating small free and forced oscillations of one-dimensional systems with dry friction is considered in [5].

It should be also pointed out that the equation (5) can be considered not only as a model of oscillatory processes in mechanics but also in biology. It has been found that the oscillatory process of an internal ear is described by the same equation (for example, [6]). It is known that radial-symmetric oscillations are modeled by one-dimensional hyperbolic-type equations. Thus, the method of developing the solution of heterogeneous equation of the type (1) with harmonic type of heterogeneous component is proposed in [7]. In [8], a “very weak” solution for the equation similar to (1) is defined and the acoustic problem of shallow water, for which the qualitative effect of echo emergence was found by numerical methods is considered.

It is possible to introduce substitution for the required function in such a way that boundary conditions (2) will become trivial. For this, let us introduce the new function $w(x,t)$:

$$u = w + v(t)\left(1 - \frac{x}{l}\right), \qquad (6)$$

for which the equation (5) and the conditions (2)-(3) will be as follows:

$$D_t w + \left(1 - \frac{x}{l}\right) D_t v(t) = Lw, \qquad (7)$$

$$w|_{x=0} = w|_{x=l} = 0, \qquad (8)$$

$$w|_{t=0} = -v(0)\left(1 - \frac{x}{l}\right), \qquad w_t|_{t=0} = -v'(0)\left(1 - \frac{x}{l}\right). \qquad (9)$$

Then we introduce the substitution for function w in such a way that the summand disappears from the first variable by time variable in the equation (7). Let

$$w = A(t)W, \qquad v(t) = A(t)s(t), \qquad (10)$$

where function $A(t)$ is determined from Cauchy problem:

$$\alpha^2(t)A'(t) + \beta(t)A(t) = 0, \qquad A(0) = 1, \qquad (11)$$

that is

$$A(t) = exp\left(-\int_0^t \frac{\beta(\tau)}{\alpha^2(\tau)} d\tau\right).$$

Then according to (10) we have that

$$w = exp\left(-\int_0^t \frac{\beta(\tau)}{\alpha^2(\tau)} d\tau\right)W, \qquad s(t) = exp\left(-\int_0^t \frac{\beta(\tau)}{\alpha^2(\tau)} d\tau\right)v.$$

With such substitution we get the equation for the new function W:

$$\alpha^2(t)W_{tt} + Q(t)W + \left(1 - \frac{x}{l}\right)(\alpha^2(t)s''(t) + Q(t)s(t)) = LW, \qquad (12)$$

where $Q(t)$ is defined by the formula (13):

$$Q(t) = -\frac{\beta'(t)\alpha^2(t) - \beta(t)\left(\alpha^2(t)\right)'}{\alpha^2(t)} - \frac{\beta^2(t)}{\alpha^2(t)} + \gamma^2(t). \qquad (13)$$

It should be noted that the boundary conditions (8) for the new function W do not change:

$$W|_{x=0} = W|_{x=l} = 0, \qquad (14)$$

and the initial conditions will be as follows:

$$W|_{t=0} = -v(0)\left(1 - \frac{x}{l}\right), \qquad W_t|_{t=0} = -\left(v'(0) + \frac{\beta(0)}{\alpha^2(0)}v(0)\right)\left(1 - \frac{x}{l}\right). \quad (15)$$

It should be indicated that the solution of the equation (1) is expressed through the solution of the equation (12) by the formula:

$$u(x,t) = v(t)\left(1-\frac{x}{l}\right) + exp\left(-\int_0^t \frac{\beta(\tau)}{\alpha^2(\tau)}d\tau\right)W(x,t). \quad (16)$$

Let us find the solution of the problem (12), (14), (15) with the help of variable separation method. Let us consider the equation (12), which does not have a heterogeneous summand:

$$\alpha^2(t)W_{tt} + Q(t)W = LW, \qquad (17)$$

we define $W = TX$, then after substituting this expression into the equation (17) we have:

$$\alpha^2(t)\frac{T''}{T} + Q(t) = \frac{LX}{X}.$$

And Sturm-Liouville problem arises for function X:

$$LX = \lambda X, \qquad X(0) = X(L) = 0. \qquad (18)$$

As it is known, the problem (18) has the computational set of eigen numbers λ_n and eigen functions X_n with the corresponding differential operator L. Let us further expand function $\left(1-\frac{x}{l}\right)$ as Fourier series by the system of eigen functions X_n of the problem (18). Let

$$\left(1-\frac{x}{l}\right) = \sum_n b_n X_n, \qquad (19)$$

where $b_n -$ Fourier coefficients defined by the formula (20):

$$b_n = \frac{\left(1-\frac{x}{l}, X_n\right)}{(X_n, X_n)}. \quad (20)$$

Taking into account (19) and (20), the problem (12), (14), (15) will be as follows:

$$\sum_n \left(\alpha^2(t)T_n'' + Q(t)T_n + b_n\left(\alpha^2(t)s''(t) + Q(t)s(t)\right)\right)X_n = \sum_n \lambda_n T_n X_n,$$

$$W|_{t=0} = -v(0)\sum_n b_n X_n, \qquad W_t|_{t=0} = -\left(v'(0) + \frac{\beta(0)}{\alpha^2(0)}v(0)\right)\sum_n b_n X_n.$$

Due to the linear independence of eigen functions X_n, we have the class of Cauchy problems for correcting $T_n(t)$:

$$\alpha^2(t)T_n'' + (Q(t)-\lambda_n)T_n = -b_n\left(\alpha^2(t)s''(t) + Q(t)s(t)\right), \qquad (21)$$

$$T_n|_{t=0} = -v(0)b_n, \qquad (T_n)_t|_{t=0} = -\left(v'(0) + \frac{\beta(0)}{\alpha^2(0)}v(0)\right)b_n, \qquad (22)$$

Having obtained their solutions, we get the accurate solution with the help of the formula (16) in the form of Fourier series of the initial problem (1)-(2)-(3):

$$u(x,t) = v(t)\left(1-\frac{x}{l}\right) + exp\left(-\int_0^t \frac{\beta(\tau)}{\alpha^2(\tau)}d\tau\right)\sum_n T_n(t)X_n(x). \qquad (23)$$

Certain models of oscillatory processes

Following [9], [10], let us consider the limited telegraph line with length l. With the distributed parameters C, L, R, G, where $C -$ capacity per length unit, $L -$ inductance per length unit, $R -$ resistance per length unit, $G -$ inductivity per length unit (see [11], [12]). Let differential operator $L = \frac{\partial^2}{\partial x^2}$, $\alpha^2 = \sqrt{CL}, \beta = \frac{CR+LG}{2}, \gamma^2 = RG$. Let us assume that the line right end is grounded, and the left end is connected with the power source applying the voltage following the harmonic law:

$$v(t) = V\sin\omega t,$$

where $V -$ voltage amplitude, $\omega -$ frequency. Let us also suppose that there is neither current nor voltage in the line at the initial time moment. Let $u = u(x,t) -$ voltage distribution in such telegraph line. It is found that this function is the solution for the one-dimensional problem (1)-(2)-(3) (for example, [13], [14], [15]).

With the parameters introduced it is easy to obtain that

$$Q(t) \equiv \gamma^2 - \frac{\beta^2}{\alpha^2}, \qquad \lambda_n = -\left(\frac{\pi n}{l}\right)^2, \qquad X_n = \sin\frac{\pi n x}{l}, \qquad b_n = \frac{2}{\pi n},$$

at $n \in \mathbb{N}$. The initial conditions (22) will be as follows:

$$T_n(0) = 0, \qquad T_n'(0) = -2\frac{\omega V}{\pi n}. \qquad (24)$$

In the equation (21) the expression $\left(\gamma^2 - \frac{\beta^2}{\alpha^2} + \left(\frac{\pi n}{l}\right)^2\right)$ can be both positive and negative, but for all possible natural n it will be negative only for the finite set of n values.

Let $N_1 := \left\{n | n \in \mathbb{N}\ and\ \gamma^2 - \frac{\beta^2}{\alpha^2} + \left(\frac{\pi n}{l}\right)^2 < 0\right\}$. Then let $-\sigma_n^2 = \gamma^2 - \frac{\beta^2}{\alpha^2} + \left(\frac{\pi n}{l}\right)^2$. Then $\forall n \in N_1$, we have

$$T_n = c_{1n}e^{\frac{\sigma_n t}{\alpha}} + c_{2n}e^{-\frac{\sigma_n t}{\alpha}} + \frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 + \sigma_n^2)}V\sin\omega t, \qquad (25)$$

and from the initial conditions we find that

$$c_{1n} = -\frac{\alpha}{2\sigma_n}\left(\frac{\omega V(\alpha^2\omega^2 - \gamma^2)}{\pi n(\alpha^2\omega^2 + \sigma_n^2)} + \frac{2\omega V}{\pi n}\right), \qquad c_{2n} = \frac{\alpha}{2\sigma_n}\left(\frac{\omega V(\alpha^2\omega^2 - \gamma^2)}{\pi n(\alpha^2\omega^2 + \sigma_n^2)} + \frac{2\omega V}{\pi n}\right)$$

and

$$W = \sum_{n\in N_1}\left(\frac{\alpha}{2\sigma_n}\left(\frac{\omega V(\alpha^2\omega^2 - \gamma^2)}{\pi n(\alpha^2\omega^2 + \sigma_n^2)} + \frac{2\omega V}{\pi n}\right)\left(-e^{\frac{\sigma_n t}{\alpha}} + e^{-\frac{\sigma_n t}{\alpha}}\right) + \frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 + \sigma_n^2)}V\sin\omega t\right)\sin\frac{\pi n x}{l},$$

Then based on the formulas (4) and (12) we have that

$$u = \left(1 - \frac{x}{l}\right) V \sin \omega t + \sum_{n \in N_1} \left(\frac{\alpha \omega V}{2\sigma_n} \left(\frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 + \sigma_n^2)} + \frac{2}{\pi n} \right) \left(-e^{\left(\frac{\sigma_n}{\alpha} - \frac{\beta}{\alpha^2}\right)t} + e^{-\left(\frac{\sigma_n t}{\alpha} + \frac{\beta}{\alpha^2}\right)t} \right) + \frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 + \sigma_n^2)} V \sin \omega t \right) \sin \frac{\pi n x}{l}. \quad (26)$$

Let now $N_2 := \left\{ n | n \in \mathbb{N} \; and \; \gamma^2 - \frac{\beta^2}{\alpha^2} + \left(\frac{\pi n}{l}\right)^2 > 0 \right\}$. Let us introduce the designation $\zeta_n^2 = \gamma^2 - \frac{\beta^2}{\alpha^2} + \left(\frac{\pi n}{l}\right)^2$. Then $\forall n \in N_2$ and $\omega^2 \neq \zeta_n^2$, we have

$$T_n = c_{1n} \sin \frac{\zeta_n}{\alpha} t + c_{2n} \cos \frac{\zeta_n}{\alpha} t + \frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 - \zeta_n^2)} V \sin \omega t, \quad (27)$$

and from the initial conditions we find that

$$c_{1n} = -\frac{\alpha \omega V}{2\zeta_n} \left(\frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 - \zeta_n^2)} + \frac{2}{\pi n} \right), \qquad c_{2n} = 0$$

and

$$W = \sum_{n \in N_2} \left(-\frac{\alpha \omega V}{2\zeta_n} \left(\frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 - \zeta_n^2)} + \frac{2}{\pi n} \right) \sin \frac{\zeta_n}{\alpha} t + \frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 - \zeta_n^2)} V \sin \omega t \right) \sin \frac{\pi n x}{l}.$$

Then based on the formulas (23) we have that

$$u = \left(1 - \frac{x}{l}\right) V \sin \omega t + \sum_{n \in N_2} \left(-\frac{\alpha \omega V}{2\zeta_n} \left(\frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 + \zeta_n^2)} + \frac{2}{\pi n} \right) \sin \frac{\zeta_n}{\alpha} t + \frac{\alpha^2\omega^2 - \gamma^2}{\pi n(\alpha^2\omega^2 + \zeta_n^2)} V \sin \omega t \right) e^{-\frac{\beta}{\alpha^2}t} \sin \frac{\pi n x}{l}. \quad (28)$$

For the resonance case $\omega^2 = \zeta_n^2$ the solution defined by the formula (27) is substituted for

$$T_n = c_{1n} \sin \omega t + c_{2n} \cos \omega t + t(A \sin \omega t + B \cos \omega t), \quad (29)$$

constants from (29) can be also found from the initial conditions and the formula for the accurate solution in the form of Fourier series, similar to (28), can be obtained.

Let us point out that the solution of the problem of small radial oscillations of gas with fading effect [16] with the availability of nonstationary disturbance at the boundary can be reduced to the discussed problem (1)-(3). Let $u = u(x, y, z, t), L = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. And the region, in which the oscillations take place, is radially symmetrical. Thus, for example, it is possible to take the sphere with radius R or spherical layer $\Omega = \{(x,y,z)|(x,y,z) \in \mathbb{R}^3, R_0^2 < x^2 + y^2 + z^2 < R_1^2\}$. Let us indicate that the solution $u(x,y,z,t)$ will depend on variable $r = \sqrt{x^2 + y^2 + z^2}$, and by spatial variables operator L will be as follows [17]:

$$L := \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}. \quad (30)$$

Let us execute a formal problem setting for the spherical region. It is necessary to find the solution for the equation

$$D_t u = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)u, \qquad (31)$$

which satisfies the boundary conditions (32):

$$u|_{r=0} = 0, \qquad u|_{r=R_0} = v(t), \qquad (32)$$

and the initial conditions (33):

$$u|_{t=0} = u_t|_{t=0} = 0, \qquad (33)$$

It is known that function $Z = Z(r,t)$, which is found by the rule (34):

$$u = \frac{Z}{r}, \qquad (34)$$

It reduces the equation (31) to

$$D_t Z = Z_{rr}, \qquad (35)$$

the boundary conditions

$$Z|_{r=0} = 0, \qquad Z|_{r=R_0} = R_0 v(t), \qquad (36)$$

and the initial conditions

$$Z|_{t=0} = Z_t|_{t=0} = 0. \qquad (37)$$

The problem (35)-(37) is almost the same as the problem (1)-(3), but the difference is in the boundary condition. However, in this case, the substitution of the required function by the formula (38):

$$Z = w + rv(t) \qquad (38)$$

allows nullifying the boundary conditions and, repeating the solution of the problem (7)-(9), obtaining the accurate expression for function Z and then writing down the solution of the initial problem (31)-(33).

Only the boundary conditions will change for the spherical layer. Let us assume that there are no oscillations on the spherical layer external surface, and some mode, which depends only on time variable, is set on the internal surface. In this case, we have the boundary conditions (39):

$$u|_{r=R_0} = v(t), \qquad u|_{r=R_1} = 0. \quad (39)$$

So, the substitution (34) remains, the equation (35) does not change, and the boundary conditions (36) will be as follows (40):

$$Z|_{r=R_0} = R_0 v(t), \qquad Z|_{r=R_1} = 0, \qquad (40)$$

and the initial conditions (37) will be unchanged. Let us introduce the substitution to nullify the boundary conditions (41):

$$Z = w + \left(\frac{r - R_1}{R_0 - R_1}\right) R_0 v(t) \;\; (41)$$

that again provides the reduction to the previously solved problem (1)-(3)

Let us further consider oscillations of the round membrane. This model emerges when studying a human cochlear, as well as in technical acoustics [18]. As it is known, the equation $u = u(x, y, t)$ and we will examine radially symmetrical oscillations of the circular membrane with radius R_0. At the same time, let us assume that the function $u = u(r, t)$, where $r = \sqrt{x^2 + y^2}$. And differential operator L is defined by the formula (42):

$$L \coloneqq \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}, \qquad (42)$$

and the boundary conditions (2) are as follows:

$$u|_{r=R_0} = v(t), \qquad |u|\big|_{r \leq R_0} < \infty. \quad (43)$$

The initial conditions (3) remain unchanged. The boundary conditions (43) are nullified by the standard procedure. It is known that Bessel functions of the first kind, zero order are the eigen functions of the operator (42) [16, 17]:

$$X_n = J_0\left(\frac{\kappa_n r}{R_0}\right), \qquad (44)$$

where $\kappa_n -$ equation root

$$J_0(\kappa_n) = 0, \qquad (45)$$

Fourier series, which provides the solution of the corresponding boundary problem, is defined by the formulas similar to (26) and (28), the difference consists in the fact that Bessel functions will be the basis functions, and sets N_1 and N_2 will be defined by the roots of the equation (45). The formula (46) demonstrates the solution:

$$u = rv(t) + \sum_{n \in M_1} T_{1n}(t) e^{-\frac{\beta}{\alpha^2}t} J_0\left(\frac{\kappa_n r}{R_0}\right) + \sum_{n \in M_2} T_{2n}(t) e^{-\frac{\beta}{\alpha^2}t} J_0\left(\frac{\kappa_n r}{R_0}\right), \qquad (46)$$

where $M_1 := \left\{n | n \in \mathbb{N} \; and \; \gamma^2 - \frac{\beta^2}{\alpha^2} + \left(\frac{\kappa_n}{R_0}\right)^2 < 0\right\}, M_2 := \left\{n | n \in \mathbb{N} \; and \; \gamma^2 - \frac{\beta^2}{\alpha^2} + \left(\frac{\kappa_n}{R_0}\right)^2 > 0\right\}$, $T_{1n}(t) -$ function not containing periodic summands relative to t, $T_{2n}(t) -$ function, which contains periodic summands relative to t. Let us point out that set M_1 is finite or empty, set M_2 is countable.

Let us now consider oscillations of a circular membrane. In this case, $R_1 -$ membrane radius, $R_0 -$ radius of the opening in the membrane. Let us assume that the membrane is fixed along the external edge, and some mode, which depends only on time variable, is set on the internal boundary, i.e. the boundary conditions are fulfilled

$$u|_{r=R_0} = v(t), \qquad u|_{r=R_1} = 0. \quad (47)$$

Let us point out that, in this case, the substitution of the variable (6) does not work and it is necessary to use the following formula:

$$u = w + v(t)\left(\frac{r - R_1}{R_0 - R_1}\right). \quad (48)$$

The boundary conditions will be fulfilled for the function $w = w(r,t)$:

$$w|_{r=R_0} = w|_{r=R_1} = 0, \qquad (49)$$

as well as the initial conditions (50):

$$w|_{t=0} = -v(0)\left(\frac{r - R_1}{R_0 - R_1}\right), \qquad w_t|_{t=0} = -v'(0)\left(\frac{r - R_1}{R_0 - R_1}\right). \quad (50)$$

It should be noted that the heterogeneous summand will change when substituting (33) in the equation (7). For the case in question we have:

$$D_t w + \left(\frac{r - R_1}{R_0 - R_1}\right) D_t v(t) - \frac{1}{R_0 - R_1} v(t) = Lw. \quad (51)$$

Bessel functions of the first and second order are the eigen functions of operator L. At the same time, eigen functions for the boundary conditions (49) are defined by the formula (52):

$$X_n = N_0(\kappa_n R_0) J_0(\kappa_n r) - J_0(\kappa_n R_0) N_0(\kappa_n r), \qquad (52)$$

where $\kappa_n -$ roots of the characteristic equation:

$$J_0(\kappa R_0) N_0(\kappa R_1) - J_0(\kappa R_1) N_0(\kappa R_0) = 0. \qquad (53)$$

Fourier series by the system of eigen functions (52) is the required solution of the problem of the circular membrane oscillation. Fourier series is defined by the formula similar to (46).

Conclusion

The paper considers the mathematical model of oscillations with friction and set mode at the boundary, which depends only on time. The oscillation law is described by the initial boundary value problem for the hyperbolic equation with nontrivial boundary condition. The differential operator of the second order by space variables discussed in the paper is initially one-dimensional or, based on the oscillatory process symmetry, can be reduced to the one-dimensional one. The successful substitution and variable separation method helped to reduce the solution of this problem to the known problem of one-dimensional limited string oscillation that allowed proposing the method of solution development in the form of Fourier series. At the same time, Fourier series obtained, depending on the differential operator type by spatial variables, set the voltage distribution for the telegraph line, the law of gas oscillations in the sphere or in the spherical region, and the law of oscillations of the round membrane without an opening and with an opening. The work results can be applied in practical calculations of telegraph lines, modeling of oscillations in acoustics, biology and medicine.

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