

k-out-of-n Systems Growth Study focusing on Redundant Reliability Systems by using Heuristic Programming Approach

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Abstract

The most common problems with nonlinear programming that relate to system dependability optimization include integer variables. If each technology comprises 'k' out of 'n' factors, the configuration can be applied to 'k' out of 'n' systems. To precisely solve a dependability optimization problem using a heuristic approach, only specific structures of the objective function and limits can be used. Its usefulness decreases with the number of restrictions, making it ineffective for maximizing dependability in a large system. In this article the authors review the literature on system reliability optimization with redundancy and integrated reliability models with redundancy, and suggest further improvements.

Keywords: Reliability Theory, IRM, Lagrangean Approach, Stage Efficiency, Heuristic Approach, System Efficiency.

1. Introduction:

The complex's reliability can be improved by either placing superfluous units, applying the element of greater reliability or by adopting the two methods at a time and both of them use extra resources. Optimizing complex reliability, and conditions to resource availability viz. size-component, cost-component, load-component are examined. In general, reliability is tested as an element of cost; But, when tested with real-world problems, the invisible effect of other restraints such as load-component, size-component, etc. has a special effect on

improving structural reliability [1]. The specific functionality of the over-reliability model with several limitations to optimize the recommended setup was examined to maximize the recommended setup.

To establish and enhance the integrated reliability models [4,5] for redundant systems with multiple constraints, the heuristic approach [11] for the proposed mathematical functions under consideration is utilized. To design optimization, the present paper considers the case problem for the mathematical functions (refer to the equations like 2a, 2b, and 2c) using the component reliability values (r_{tj}) and the number of components in each stage (X_{tj}) as inputs for a heuristic approach. This method is useful for optimizing the design [2] with integer values for (X_{tj}), which is highly applicable to the implementation of real-world problems.

In literature, Integrated reliability models [15, 17] are enhanced by applying value restraints where there is a fixed association between cost-component and its reliability. A unique pattern of planned work is a deliberation of the load-component and size-component as supplementary restraints along with value to form and improve the superfluous reliability system for 'k' out of 'n' complex composition [9].

2. Methodology:

2.1. Assumptions and Notations:

- Each stage's elements are believed to be identical, i.e., all elements have the same level of reliability.
- All elements are supposed to be statistically independent, meaning that their failure has no bearing on the performance of other elements in the complex.

R_{CR} = Complex Reliability

R_{RM} = Reliability of Moment, $0 < R_{RM} < 1$

r_{tj} = Reliability of each element in phase tj ; $0 < r_{tj} < 1$

X_{tj} = Number of components in phase tj

CC_{tj} = Cost-Component in phase tj

LC_{tj} = Load-Component in phase tj

SC_{tj} = Size-Component in phase tj

C_{t0} = Greatest allowable complex for Cost-Component

L_{t0} = Greatest allowable complex for Load-Component

S_{t0} = Greatest allowable complex for Size-Component

C_{tj} ; $\emptyset_{tj}$; L_{tj} ; μ_{tj} ; S_{tj} ; ω_{tj} are Constants.

2.2 Mathematical Analysis:

The efficiency of the system to the provided cost-component function

$$R_{CR} = \sum_{i=1}^n B(m, i) p^i (1 - p)^{m-i} \quad (1)$$

The following relationship between cost-component and efficiency is used to calculate the cost coefficient of each unit in phase t_j .

$$CC_{tj} = C_{tj} \cdot e^{\frac{[1-\phi_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \quad (2a)$$

$$\text{Therefore } LC_{tj} = L_{tj} \cdot e^{\frac{[1-\mu_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \quad (2b)$$

$$SC_{tj} = S_{tj} \cdot e^{\frac{[1-\omega_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \quad (2c)$$

Since cost-components are linear in t_j ,

$$\sum_{j=1}^n CC_{tj} \cdot t_j \leq CC_{t0} \quad (3a)$$

Similarly load-components and size-components are also linear in t_j ,

$$\sum_{j=1}^n LC_{tj} \cdot t_j \leq LC_{t0} \quad (3b)$$

$$\sum_{j=1}^n SC_{tj} \cdot t_j \leq SC_{t0} \quad (3c)$$

Substituting (2a), (2b) & (2c) in (3a), (3b) & (3c) respectively.

$$\sum_{j=1}^n C_{tj} \cdot e^{\frac{[1-\phi_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot t_j - CC_{t0} \leq 0 \quad (4a)$$

$$\sum_{j=1}^n L_{tj} \cdot e^{\frac{[1-\mu_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot t_j - LC_{t0} \leq 0 \quad (4b)$$

$$\sum_{j=1}^n S_{tj} \cdot e^{\frac{[1-\omega_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot t_j - SC_{t0} \leq 0 \quad (4c)$$

$$\text{The transformed equation through the relation } t_j = \frac{\ln R_{RM}}{\ln r_{tj}} \quad (5)$$

$$\text{Where } R_{CR} = \sum_{k=2}^n B(t_j, k) (r_{tj})^k (1 - r_{tj})^{tj-k} \quad (6)$$

Subject to the constraints

$$\sum_{j=1}^n C_{tj} \cdot e^{\frac{[1-\phi_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} - CC_{t0} \leq 0 \quad (7a)$$

$$\sum_{j=1}^n L_{tj} \cdot e^{\frac{[1-\mu_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} - LC_{t0} \leq 0 \quad (7b)$$

$$\sum_{j=1}^n S_{tj} \cdot e^{\frac{[1-\omega_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} - SC_{t0} \leq 0 \quad (7c)$$

Positivity restrictions $t_j \geq 0$, A Lagrangean function is defined as

$$L_F = R_{CR} + \beta_1 \left[\sum_{j=1}^n C_{tj} \cdot e^{\frac{[1-\phi_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} - CC_{t0} \right] +$$

$$\beta_2 \left[\sum_{j=1}^n L_{tj} \cdot e^{\frac{[1-\mu_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} - LC_{t0} \right] + \beta_3 \left[\sum_{j=1}^n S_{tj} \cdot e^{\frac{[1-\omega_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} -$$

$$SC_{t0} \right] \quad (8)$$

The Lagrangean function can be used to find the ideal point and separating it by R_{RM} , r_{tj} , δ_1 , δ_2

and δ_3 .

$$\frac{\partial L_F}{\partial R_{SR}} = 1 + \beta_1 \left[\sum_{j=1}^n C_{tj} \cdot e^{\frac{[1-\phi_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{1}{\ln r_{tj}} \frac{1}{R_{RM}} \right]$$

$$+ \beta_2 \left[\sum_{j=1}^n L_{tj} \cdot e^{\frac{[1-\mu_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{1}{\ln r_{tj}} \frac{1}{R_{RM}} \right]$$

$$+ \beta_3 \left[\sum_{j=1}^n S_{tj} \cdot e^{\frac{[1-\omega_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{1}{\ln r_{tj}} \frac{1}{R_{RM}} \right]$$

(09)

$$\frac{\partial L_F}{\partial r_{tj}} = \beta_1 \left[\sum_{j=1}^n C_{tj} \cdot e^{\frac{[1-\phi_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} \right] \left[\frac{(r_{tj,max} + r_{tj,min})(1-\phi_i)}{(r_{tj,max} - r_{tj})^2} - \frac{1}{r_{tj} \ln r_{tj}} \right] +$$

$$\beta_2 \left[\sum_{j=1}^n L_{tj} \cdot e^{\frac{[1-\mu_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} \right] \left[\frac{(r_{tj,max} + r_{tj,min})(1-\mu_i)}{(r_{tj,max} - r_{tj})^2} - \frac{1}{r_{tj} \ln r_{tj}} \right] +$$

$$\beta_3 \left[\sum_{j=1}^n S_{tj} \cdot e^{\frac{[1-\omega_{tj}](|r_{tj}-r_{tj,min}|)}{r_{tj,max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} \right] \left[\frac{(r_{tj,max} + r_{tj,min})(1-\omega_i)}{(r_{tj,max} - r_{tj})^2} - \frac{1}{r_{tj} \ln r_{tj}} \right] \quad (10)$$

$$\frac{\partial L_F}{\partial \beta_1} = \sum_{j=1}^n C_{tj} \cdot e^{\frac{[1-\phi_{tj}](|r_{tj}-r_{tj,\min}|)}{r_{tj,\max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} - C_{t0} \quad (11)$$

$$\frac{\partial L_F}{\partial \beta_2} = \sum_{j=1}^n L_{tj} \cdot e^{\frac{[1-\mu_{tj}](|r_{tj}-r_{tj,\min}|)}{r_{tj,\max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} - L_{t0} \quad (12)$$

$$\frac{\partial L_F}{\partial \beta_3} = \sum_{j=1}^n S_{tj} \cdot e^{\frac{[1-\omega_{tj}](|r_{tj}-r_{tj,\min}|)}{r_{tj,\max}-r_{tj}}} \cdot \frac{\ln R_{RM}}{\ln r_{tj}} - S_{t0} \quad (13)$$

Where β_1, β_2 and β_3 are Lagrangean multipliers.

The number of elements in each phase (X_{tj}), the best element reliability (r_{tj}), the reliability of moment (R_{RM}) and the complex reliability (R_{CR}) [7] are derived by using the Heuristic approach [12, 13]. This method provides a real (valued) solution concerning cost, weight and volume.

2.3 Case Problem:

To derive the multiple parameters of a given mechanical system [10] using optimization techniques, where all the assumptions like cost-component, load-component and size-component are directly proportional to system reliability has been considered in this research work. The same logic may not be true in the case of electronic systems. Hence, the optimal element accuracy (r_{tj}) [3], phase reliability (R_{RM}), Number of elements in each phase (X_{tj}), and complex accuracy (R_{CR}) [6] can be evaluated in any given mechanical system. In this work, an attempt has been made to evaluate the Complex accuracy of a special purpose machine that is used for single phase turbo-charged generators assembly.

The machine is used for the assembly of 3 or 4 components on the base of the turbo-charged generators. The machine's approximate worth was \$15000, which is considered a complex cost, the load of the machine is 1000 pounds which is the load-component of the complex, and the space occupied by the machine is 500 cm³, which is the volume or size-component of the complex. To attract the authors from different cross sections, the authors attempted to use hypothetical numbers, which can be changed according to the environment.

2.4 Constants:

The data required for the constants for the case problem are provided in Table 1.

Table 1: Worth, Load and Size Pre-fixed Constant Values

Phase	Worth Constants		Load Constants		Size Constants	
	C_{tj}	ϕ_{tj}	L_{tj}	μ_{tj}	S_{tj}	ω_{tj}
1	1000	0.72	60	0.69	30	0.85
2	1200	0.84	70	0.87	40	0.88
3	1400	0.94	80	0.94	50	0.92

The efficiency of each factor, phase, and number of factors in each stage, as well as the structural efficiency, are shown in the tables below.

2.4.1 The Details of Cost-Component Constraint by using Lagrangean Multiplier Method without Rounding-Off

The value-related efficiency design is described in the Table 2.

Table 2: Cost Constraint Analysis by using Lagrangean Multiplier Method

Phase	C_{tj}	ϕ_{tj}	r_{tj}	$\text{Log } r_{tj}$	R_{RM}	$\text{Log } R_{RM}$	X_{tj}	$CC_{tj} = C_{tj} \cdot e^{\frac{[1-\phi_{tj}](r_{tj}-r_{tj,min})}{r_{tj,max}-r_{tj}}}$	$CC_{tj} \cdot X_{tj}$
01	1000	0.85	0.8741	-0.0584	0.6777	-0.1690	2.89	1343	3883
02	1200	0.88	0.8445	-0.0734	0.6487	-0.1880	2.56	1207	3091
03	1400	0.91	0.8456	-0.0728	0.5461	-0.2627	3.61	1403	5061
Final Worth-Component									12035

2.4.2 The Details of Load-Component Constraint by using Lagrangean Multiplier Method without Rounding-Off

The equivalent results for the load are shown in the Table 3.

Table 3: Load Constraint Analysis by using Lagrangean Multiplier Method

Phase	L_{tj}	μ_{tj}	r_{tj}	$\text{Log } r_{tj}$	R_{RM}	$\text{Log } R_{RM}$	X_{tj}	$LC_{tj} = L_{tj} \cdot e^{\frac{[1-\mu_{tj}](r_{tj}-r_{tj,min})}{r_{tj,max}-r_{tj}}}$	$LC_{tj} \cdot X_{tj}$
01	100	0.92	0.8741	-0.0584	0.6777	-0.1690	2.89	83	240
02	80	0.88	0.8445	-0.0734	0.6487	-0.1880	2.56	70	179
03	60	0.91	0.8456	-0.0728	0.5461	-0.2627	3.61	82	296
Final Load-Component									715

2.4.3 The Details of Size-Component Constraint by using Lagrangean Multiplier Method without Rounding-Off

The equivalent results for size are described in the Table 4.

Table 4: Size Constraint Analysis by using Lagrangean Multiplier Method

Phase	S_{tj}	ω_{tj}	r_{tj}	$\text{Log } r_{tj}$	R_{RM}	$\text{Log } R_{RM}$	X_{tj}	$SC_{tj} = S_{tj} \cdot e^{\frac{[1-\omega_{tj}](r_{tj}-r_{tj,min})}{r_{tj,max}-r_{tj}}}$	$SC_{tj} \cdot X_{tj}$
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01	30	0.94	0.8741	-0.0584	0.6777	-0.1690	2.89	35	101
02	40	0.89	0.8445	-0.0734	0.6487	-0.1880	2.56	40	102
03	50	0.86	0.8456	-0.0728	0.5461	-0.2627	3.61	52	188
Final Size-Component									391

3. Efficiency Design by using Lagrangean Multiplier Method:

The efficiency design [14] summarizes the e_j values as integers (rounding the value of t_j to the nearest integer), and the acceptable outcomes for the worth, load, and size are listed in the tables. Calculate variance due to cost-component, load-component and size-component, construction capacity (before and after rounding off t_j to the nearest integer) to obtain information.

3.1 Efficiency Design by using Lagrangean Multiplier Method Concerning Cost, Load and Size with Rounding-Off

Table 5: Cost, Load and Size Constraint Analysis by using Lagrangean Multiplier Method with Rounding Off

Phase	r _{tj}	R _{RM}	X _{tj}	CC _{tj}	C _{tj} · X _{tj}	LC _{tj}	LC _{tj} · X _{tj}	SC _{tj}	SC _{tj} · X _{tj}
01	0.8741	0.6777	3	1343	4029	83	249	35	105
02	0.8445	0.6487	3	1207	3621	70	210	45	120
03	0.8456	0.5461	4	1403	5612	82	328	52	208
Total Cost, Load and Size				13262		787		433	
Complex Reliability (R _{CR})								0.9562	

$$\text{Deviation in Cost-Component} = \frac{\text{Total Cost with rounding off} - \text{Total Cost without rounding off}}{\text{Total Cost without rounding off}} = 10.19\%$$

$$\text{Deviation in Load-Component} = \frac{\text{Total Load with rounding off} - \text{Total Load without rounding off}}{\text{Total Load without rounding off}} = 10.07\%$$

$$\text{Deviation in Size-Component} = \frac{\text{Total Size with rounding off} - \text{Total Size without rounding off}}{\text{Total Size without rounding off}} = 10.69\%$$

4. Heuristic Approach:

In the majority of cases, a heuristic technique [9] provides the optimal solution with minimal additional computational effort, but the result may not be optimal. In 1971, Sharma J. and Venkateswaran K.V. [18] devised a simple computational method, which need not be linear, to distribute redundancy across subsystems in order to maximize the reliability of a multistage system subject to numerous constraints. Aggarwal K.K. et al. published a new algorithm for the heuristic solution of redundancy optimization problems in 1975. 1976 also

saw the creation of a redundancy allocation mechanism for generic systems by Aggarwal K.K [19].

The Lagrangean method [13] may be difficult to apply due to several drawbacks, such as the requirement to specify the number of components required at each step (X_{tj}) in real numbers. The widely used method of rounding down the value causes changes in the cost-component, weight-component, and volume-component, compromising system reliability and having a significant impact on the model's efficiency design. This issue could be considered, in which case the author suggests a different empirical implementation that uses the Heuristic approach [16] instead of the Lagrangean technique and leverages the latter to generate an integer solution by using the latter's solutions as parameters.

4.1 New Heuristic Algorithm:

Step1: Initialize the necessary input parameters, then enter their values.

Step2: Enter the most components possible (t_j).

Step3: Set the first stage's component count to one and compute the first stage's system cost (CC_{tj}), load (LC_{tj}), and size (SC_{tj}).

Step4: Set the second stage's component count at one and compute the second stage's system cost (CC_{tj}), load (LC_{tj}) and size (SC_{tj}).

Step5: Set the third stage's component count to 1 and determine the third stage's system cost (CC_{tj}), load (LC_{tj}) and size (SC_{tj}).

- (i) Totalize all the values. CC_{tj} , LC_{tj} , , SC_{tj} for all three phases.
- (ii) Determine the Reliability of the System (R_{CR}).
- (iii) Examine the restrictions.

Step6:

- i. If the constraints are satisfied, output the corresponding values for the number of components and system reliability (R_{CR}).
- ii. Proceed to STEP 5 and increase the number of stage three components by one if the constraints are not met
- iii. Continue in this manner until the total number of components in all three phases is equal to or less than the maximum number of components (t_j).

5. Results:

5.1 The Details of Cost-Component Constraint by using Heuristic Approach:

The value-related efficiency design is described in the Table 6.

Table 6: The Details of Worth-Component constraint by using Heuristic Approach

Phase	C_{tj}	ϕ_{tj}	r_{tj}	$\text{Log } r_{tj}$	R_{RM}	$\text{Log } R_{RM}$	X_{tj}	$CC_{tj} = C_{tj} \cdot e^{\frac{[1-\phi_{tj}](r_{tj}-r_{tj,min})}{r_{tj,max}-r_{tj}}}$	$CC_{tj} \cdot X_{tj}$
01	1000	0.85	0.9256	-0.0008	0.7928	-0.0024	3	1175	3525
02	1200	0.88	0.9347	-0.0116	0.8164	-0.0348	3	1212	3636
03	1400	0.91	0.9199	-0.0048	0.7163	-0.0190	4	1428	5712
Final Worth-Component									12873

5.2 The Details of Load-Component Constraint by using Heuristic Approach:

The equivalent results for the load are shown in the Table 7.

Table 7: The Details of Load- Component constraint by using Heuristic Approach

Phase	L_{tj}	μ_{tj}	r_{tj}	$\text{Log } r_{tj}$	R_{RM}	$\text{Log } R_{RM}$	X_{tj}	$LC_{tj} = L_{tj} \cdot e^{\frac{[1-\mu_{tj}](r_{tj}-r_{tj,min})}{r_{tj,max}-r_{tj}}}$	$LC_{tj} \cdot X_{tj}$
01	100	0.92	0.9256	-0.0008	0.7928	-0.0024	3	72	216
02	80	0.88	0.9347	-0.0116	0.8164	-0.0348	3	87	261
03	60	0.91	0.9199	-0.0048	0.7163	-0.0190	4	101.5	406
Final Load-Component									883

5.3 The Details of Size-Component Constrains by using Heuristic Approach:

The equivalent results for size are described in the Table 8.

Table 8: The Details of Size- Component constraint by using Heuristic Approach

Phase	S_{tj}	ω_{tj}	r_{tj}	$\text{Log } r_{tj}$	R_{RM}	$\text{Log } R_{RM}$	X_{tj}	$SC_{tj} = S_{tj} \cdot e^{\frac{[1-\omega_{tj}](r_{tj}-r_{tj,min})}{r_{tj,max}-r_{tj}}}$	$SC_{tj} \cdot X_{tj}$
01	100	0.94	0.9256	-0.0008	0.7928	-0.0024	3	33	98
02	90	0.89	0.9347	-0.0116	0.8164	-0.0348	3	49	148
03	80	0.86	0.9199	-0.0048	0.7163	-0.0190	4	52	209
Final Size-Component									455
Complex Reliability (R_{CR})									0.9864

5.4 Comparison of Optimization of Integrated Redundant Reliability 'k' out of 'n' systems – LMM with rounding-off and Heuristic Approach for Cost-Component

Table 9: Results Correlated LMM with rounding off approach and Heuristic approach for Worth

		With Rounding Off				Heuristic Approach			
Phase	X_{tj}	r_{tj}	R_{RM}	CC_{tj}	$CC_{tj} \cdot X_{tj}$	r_{tj}	R_{RM}	CC_{tj}	$CC_{tj} \cdot X_{tj}$
01	3	0.8741	0.6777	1343	4029	0.9256	0.7928	1175	3525
02	3	0.8445	0.6487	1207	3621	0.9347	0.8164	1212	3636
03	4	0.8456	0.5461	1403	5612	0.9199	0.7163	1428	5712
Total Worth		13262				12873			
Complex Efficiency		With Rounding Off (R_{CR}) 0.9987				Heuristic Approach (R_{CR}) 0.9999			

5.5 Comparison of Optimization of Integrated Redundant Reliability 'k' out of 'n' systems – LMM with rounding-off and Heuristic approach for Load-Component

Table 10: Results Correlated with LMM rounding off approach and Heuristic approach for Load

		With Rounding Off				Heuristic Approach			
Phase	X_{tj}	r_{tj}	R_{RM}	LC_{tj}	$LC_{tj} \cdot X_{tj}$	r_{tj}	R_{RM}	LC_{tj}	$LC_{tj} \cdot X_{tj}$
01	3	0.8741	0.6777	83	249	0.9256	0.7928	72	216
02	3	0.8445	0.6487	70	210	0.9347	0.8164	87	261
03	4	0.8456	0.5461	82	328	0.9199	0.7163	101.5	406
Total Load		787				883			
Complex Reliability		With Rounding Off (R_{CR}) 0.9987				Heuristic Approach (R_{CR}) 0.9999			

5.6 Comparison of Optimization of Integrated Redundant Reliability 'k' out of 'n' systems – LMM with rounding-off and Heuristic approach for Size-Component

Table 11: Results Correlated LMM with rounding off approach and Heuristic approach for Size

		With Rounding Off				Heuristic Approach			
Phase	X_{tj}	r_{tj}	R_{RM}	SC_{tj}	$SC_{tj} \cdot X_{tj}$	r_{tj}	R_{RM}	SC_{tj}	$SC_{tj} \cdot X_{tj}$
01	3	0.8741	0.6777	35	105	0.9256	0.7928	33	98
02	3	0.8445	0.6487	45	120	0.9347	0.8164	49	148
03	4	0.8456	0.5461	52	208	0.9199	0.7163	52	209
Total Size		433				455			
Complex Reliability		With Rounding Off (R_{CR}) 0.9987				Heuristic Approach (R_{CR}) 0.9999			

6. Discussion:

An integrated reliability model for a ' k ' out of ' n ' configuration system with numerous efficiency requirements is proposed in this work. The Lagrangean multiplier approach is used to calculate the number of components (X_{ij}), efficiencies (r_{ij}), phase efficiency (R_{RM}), and system efficiency (R_{CR}) once it is realized that the data are in reals. In order to achieve practical applicability, the Lagrangean method inputs are used in conjunction with the Heuristic approach to provide an integer solution.

When a ' k ' out of ' n ' configuration IRM with reliability engineer redundancy is required in real-world circumstances, the IRM produced in this way is highly valuable. The suggested approach is extremely useful for the dependability design engineer to construct high-quality and efficient materials in situations where the system value is low.

References:

1. Derman, C., Lieberman, G. J., & Ross, S. M. (1982). On the consecutive-k-of-n: F system. *IEEE Transactions on Reliability*, 31(1), 57-63.
2. Wei, V. K., Hwang, F. K., & Sos, V. T. (1983). Optimal sequencing of items in a consecutive-2-out-of-n system. *IEEE Transactions on Reliability*, 32(1), 30-33.
3. Risse, T. (1987). On the evaluation of the reliability of k-out-of-n systems. *IEEE transactions on Reliability*, 36(4), 433-435.
4. Pham, H., & Upadhyaya, S. J. (1988). The efficiency of computing the reliability of k-out-of-n systems. *IEEE transactions on Reliability*, 37(5), 521-523.
5. Bai, D. S., Yun, W. Y., & Chung, S. W. (1991). Redundancy optimization of k-out-of-n systems with common-cause failures. *IEEE Transactions on Reliability*, 40(1), 56-59.
6. Chao, M. T., Fu, J. C., & Koutras, M. V. (1995). Survey of reliability studies of consecutive-k-out-of-n: F and related systems. *IEEE Transactions on reliability*, 44(1), 120-127.
7. Sarhan, A. M., & Abouammoh, A. M. (2001). Reliability of k-out-of-n nonrepairable systems with non-independent components subjected to common shocks. *Microelectronics Reliability*, 41(4), 617-621.
8. Koucký, M. (2003). Exact reliability formula and bounds for general k-out-of-n systems. *Reliability Engineering & System Safety*, 82(2), 229-231.
9. Flynn, J., & Chung, C. S. (2004). A heuristic algorithm for determining replacement policies in consecutive k-out-of-n systems. *Computers & Operations Research*, 31(8), 1335-1348.
10. Li, X., Zuo, M. J., & Yam, R. C. (2006). Reliability analysis of a repairable k-out-of-n system with some components being suspended when the system is down. *Reliability engineering & System safety*, 91(3), 305-310.
11. Sankaraiah, G., Sarma, B. D., Umashankar, C. and Sarma, Y. V. S. (2011). Designing, modelling and optimizing of an integrated reliability redundant system. *South African Journal of Industrial Engineering*, 22(2):100-106.
12. Sridhar, A., Kumar, S. P., Reddy, Y. R., Sankaraiah, G. and Umashankar, C. (2013). The k-out-of-n Redundant IRM Optimization with multiple constraints. *International Journal*

- for Research in Science & Advanced Technologies, 3(4):436-438.
13. Sasikala, P., Sridhar, A., Kumar, S. P. and Umashankar, C. (2013). Optimization of IRM Parallel-Series Redundant System, International Journal of Engineering Research & Technology, 2(2):1-6.
 14. Kumar, S. P., Sridhar, A. and Sasikala, P. (2020). Optimization of Integrated Redundant Reliability Coherent Systems – Heuristic Method. Journal of Critical Reviews, 7(5):2480-2491.
 15. Sasikala, P., Sridhar, A. and Kumar, S. P. (2020). Optimization of IRM Series-Parallel and Parallel-Series Redundant System. Journal of Critical Reviews, 7(17):1801-1811.
 16. Sridhar, A., Sasikala, P., Kumar, S. P. and Sarma, Y. V. S. (2021). Design and evaluation of coherent redundant system reliability. [Systems Reliability Engineering](#), edited by Amit Kumar and Mangey Ram, De Gruyter, 137-152.
 17. Sridhar, A., Sasikala, P., Kumar, S. P. and Sarma, Y. V. S. (2022). Design and evaluation of parallel-series IRM system. System Assurances Modeling and Management, edited by Prashant Johri, Adarsh Anand, Juri Vain, Jagvinder Singh, and Mohammad Quasim, Academic Press (Elsevier), 189-208.
 18. Sharma, J., & Venkateswaran, K. V. (1971). A direct method for maximizing the system reliability. IEEE Transactions on Reliability, 20(4), 256-259.
 19. Aggarwal, K. K. (1976). Redundancy optimization in general systems. IEEE Transactions on Reliability, 25(5), 330-332.