A Deteriorating Inventory Model with Ramp Type Demand and Time Dependent Holding Cost

Sanjay Singh[1*], Garima Garg[2], Vinita Singh[3] and Kapil Dave[4]
[1,2,3]Department of Applied Sciences and Humanities (Mathematmatics),
Raj Kumar Goel Institute of Technology, Ghaziabad, U.P., India.
[4] Department of Mathematics (Research Scholar)
SRMIST, Modinagar, U.P. India
Email: ssingh_hapur@yahoo.co.in, garima6982@gmail.com , vinitasingh000@gmail.com,
and kapil.d99@gmail.com

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Abstract
In this paper, an inventory model has been developed with ramp type demand and time-dependent holding cost. At the beginning of the cycle, the holding cost increases linearly with time and then becomes constant. The effect of deterioration has been taken into account and considered as a linearly increasing function of time. The main objective of the proposed paper is to derive the inventory policies for minimizing the total cost involved in developing inventory model. The convexity of the total cost has been shown graphically. Also, a numerical example has been solved to illustrate and validate the model.
Keywords- Inventory, deterioration, holding cost, ramp type demand

1. Introduction

A considerable amount of literature has been available with regard to inventory models. Most of the inventory models have been developed with the assumption of a constant demand rate. The constant demand shows realistic behavior in the case of some items like newspapers, LPG, milk, etc. But the demand for most of the items is not constant, it depends on time. It has been noticed that when the product of a popular brand is launched in the market its demand increases with time till a certain point and after that it becomes constant. This type of behavior of demand is very common and it is known as ramp type demand.

Deterioration of the items is an important feature of any inventory model. Generally, the items may deteriorate or damage or decay with time in any inventory system. Also to hold the items fresh or in order or safe, a structure is required and needs money which is known as holding cost. Holding cost may be constant or time-dependent. Therefore, the combination of deterioration of items, ramp type demand, and time-dependent holding, in the process of developing an inventory model, is very close to the realistic environment of society and industries.

In the last decade, the researchers have developed a number of research papers in the field of inventory management taking into account various assumptions. Singh et al. (2011) developed a production inventory model for deteriorating items in which the demand rate is a function of price. Sanni and Chukwu (2013) analyzed an inventory model for deteriorating items with ramp type demand. They proposed the necessary and sufficient conditions for the policy of optimal replenishment. Sarkar and Sarkar (2013) analyzed a deteriorating inventory model assuming the stock-dependent demand. They determined the optimal cycle length of each product taken in the model. Haidar et al. (2014) introduced an inventory model for defective items considering that a percentage of the on-hand inventory is wasted due to deterioration. Dutta and Kumar (2015) developed an inventory model for deteriorating items with time dependent holding cost and demand. Singh et al. (2016) established an inventory model for deteriorating items assuming the multivariate demand depending upon the selling price, time, and on-hand inventory level under the effect of customer returns and inflation. Wu et al.

(2017) developed an inventory model for a trapezoidal-type demand pattern including the constant, increasing, decreasing, and ramp-type demand rates as special cases. Singh et al. (2017) pondered a production inventory model for deteriorating items with time varying demand and production rate. Saha et al. (2018) evolved an inventory model for deteriorating items with ramp type demand wherein they discussed the effect of price discounts. Sharma et al. (2018) proposed a deteriorating inventory model with an expiry date of the items to determine optimal order quantity such that the total profit per unit time is maximized. Singh et al. (2018) analysed an inventory model for deteriorating items with dynamic demand considering the effect of inflation. San-Jose et al. (2019) formulated a deterministic inventory model in which demand is a function of selling price and time. Garg et al. (2020) developed a two-warehouse inventory model with ramp type demand to maximize the total profit and optimal ordering quantity. Halim et al. (2021) determined a production inventory model for deteriorating items taking into consideration the overtime production process. They have assumed that demand is a function of Price and available stock. Cardenas-Barron et al. (2021) developed an inventory model wherein the demand rate depends simultaneously on both the selling price and time according to a power pattern. Tiwari et al. (2022) studied a two warehouse inventory model and discussed the effect of deterioration and trade credit.
The outline of the considered inventory model is as follows. Notations and assumptions used throughout the article have been provided in Section 2. In section 3, a deteriorating inventory model with ramp type demand and time dependent holding cost has been formulated and developed. The procedure for solving the problem has been provided in Section 4. In Section 5, a numerical example has been solved by using MATHEMATICA-8. Sensitivity analysis with respect to changes in various model parameters has been provided in Section 6 and observations are presented in Section 7. Finally, the paper is ended with the conclusions given in Section 8.

2. Notations and assumptions

The following notations are used for proposed inventory model:

$\theta(t)$	time dependent deterioration rate
$d(t)$	time dependent demand
T	length of the one cycle
$I_1(t)$	inventory level at time $t \in [0, t_1]$
$I_2(t)$	inventory level at time $t \in [t_1, T]$
A	ordering cost
c	cost of an item
c_d	deterioration cost per item
α	Deterioration coefficient
a_o, a_1 & a_2	demand coefficients
b_o, b_1 & b_2	holding cost coefficients

The following assumptions have been adopted while developing the inventory model:

1. A single item is used in the developed inventory model
2. Lead time is zero
3. The demand rate is ramp type and defined as $d(t) = \begin{cases} a_0 + a_1 t & 0 \le t \le t_1 \\ a_2 & t_1 \le t \le T \end{cases}$.
4. The inventory deteriorates as a linear function time and given by $\theta(t) = \alpha t$.
5. The holding cost is time dependent and given by $h(t) = \begin{cases} b_o + b_1 t & 0 \le t \le t_1 \\ b_2 & t_1 \le t \le T \end{cases}$.

7. Time horizon is finite.

3. Mathematical formulation and analysis

At the beginning of the cycle, a lot size of I_o units is received. Due to time-dependent demand and deterioration, the inventory level decreases till $t = t_1$. After that instant the inventory level decreases due to constant demand and time-dependent deterioration and vanishes at $t = T$.

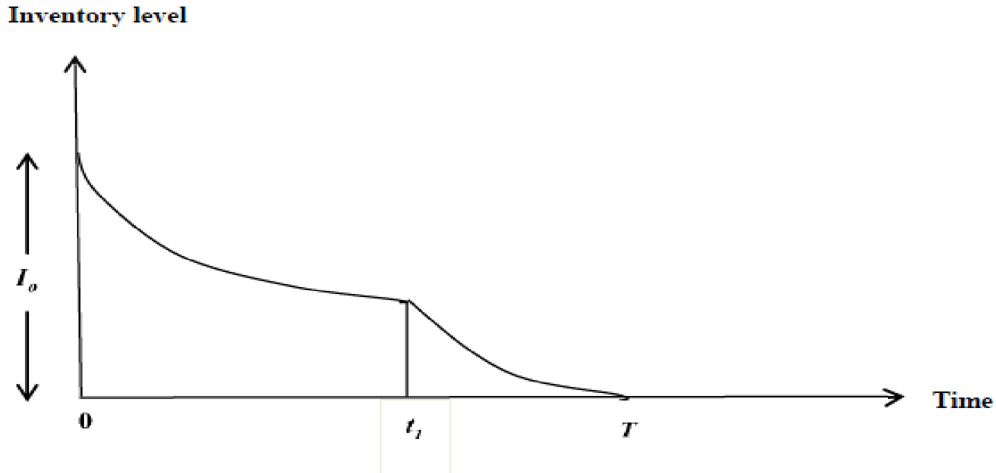

Figure 1: Inventory system

Thus, the inventory level $I_i(t)$ at any time t in the interval $[0,T]$ can be represented by the following differential equations:

$$\frac{dI_1(t)}{dt} = -d(t) - \theta(t) I_1(t), \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -d(t) - \theta(t) I_2(t), \qquad t_1 \le t \le T \tag{2}$$

with boundary conditions $I_1(0) = I_o$, $I_1(t_1) = I_2(t_1)$ & $I_2(T) = 0$.

The solutions of differential equations (1) and (2) with boundary conditions are as follows:

$$I_1(t) = I_o\left(1 - \frac{\alpha t^2}{2}\right) - a_o\left(t - \frac{\alpha t^2}{3} - \frac{\alpha^2 t^5}{12}\right) - a_1\left(\frac{t^2}{2} - \frac{\alpha t^4}{8} - \frac{\alpha^2 t^6}{16}\right). \tag{3}$$

$$I_2(t) = a_2\left[(T - t) + \frac{\alpha}{6}\left(T^3 - t^3\right) + \frac{\alpha t^2}{2}(t - T) + \frac{\alpha^2 t^2}{12}(t^3 - T^3)\right]. \tag{4}$$

Ordering cost

$$OC = A. \tag{5}$$

Purchase cost

$$PC = cI_0. \tag{6}$$

Deterioration Cost

$$DC = c_d\left[I_o - \int_0^{t_1}(a_0 + a_1 t)dt + \int_{t_1}^{T} a_2 dt\right]$$

$$= c_d\left[I_o - a_o t_1 - \frac{1}{2}a_1 t_1^2 - a_2(T - t_1)\right]. \tag{7}$$

Holding Cost

$$HC=\int_0^{t_1}(b_0+b_1t)\,I_1(t)\,dt+\int_{t_1}^{T}b_2I_2(t)\,dt$$

$$\begin{aligned}&=b_0I_0\left(t_1-\frac{\alpha t_1^3}{6}\right)+b_1I_0\left(\frac{t_1^2}{2}-\frac{\alpha t_1^4}{8}\right)+b_0a_0\left(-\frac{t_1^2}{2}+\frac{\alpha t_1^4}{12}+\frac{\alpha^2t_1^6}{72}\right)+b_0a_1\left(-\frac{t_1^3}{6}+\frac{at_1^5}{40}+\frac{\alpha^2t_1^7}{112}\right)\\&+b_1a_0\left(-\frac{t_1^3}{3}+\frac{\alpha t_1^5}{15}+\frac{\alpha^2t_1^7}{84}\right)+b_1a_1\left(-\frac{t_1^4}{8}+\frac{\alpha t_1^6}{48}+\frac{\alpha^2t_1^8}{128}\right)\\&+b_2a_2\left(\frac{T^2}{2}+\frac{\alpha T^4}{12}-\frac{\alpha^2T^6}{72}-t_1T-\frac{\alpha t_1^4}{12}-\frac{\alpha^2t_1^6}{72}-\frac{\alpha t_1T^3}{6}+\frac{t_1^2}{2}+\frac{\alpha Tt_1^3}{6}+\frac{\alpha^2T^3t_1^3}{36}\right).\end{aligned}\tag{8}$$

Thus, total cost per unit time of inventory system is given by

$$TC=\frac{1}{T}\left[OC+PC+DC+HC\right]$$

$$=\frac{1}{T}\left[\begin{aligned}&A+cI_0+c_d\left[I_o-a_ot_1-\frac{1}{2}a_1t_1^2-a_2(T-t_1)\right]+b_0I_0\left(t_1-\frac{\alpha t_1^3}{6}\right)\\&+b_1I_0\left(\frac{t_1^2}{2}-\frac{\alpha t_1^4}{8}\right)+b_0a_0\left(-\frac{t_1^2}{2}+\frac{\alpha t_1^4}{12}+\frac{\alpha^2t_1^6}{72}\right)+b_0a_1\left(-\frac{t_1^3}{6}+\frac{at_1^5}{40}+\frac{\alpha^2t_1^7}{112}\right)\\&+b_1a_0\left(-\frac{t_1^3}{3}+\frac{\alpha t_1^5}{15}+\frac{\alpha^2t_1^7}{84}\right)+b_1a_1\left(-\frac{t_1^4}{8}+\frac{\alpha t_1^6}{48}+\frac{\alpha^2t_1^8}{128}\right)\\&+b_2a_2\left(\frac{T^2}{2}+\frac{\alpha T^4}{12}-\frac{\alpha^2T^6}{72}-t_1T-\frac{\alpha t_1^4}{12}-\frac{\alpha^2t_1^6}{72}-\frac{\alpha t_1T^3}{6}+\frac{t_1^2}{2}+\frac{\alpha Tt_1^3}{6}+\frac{\alpha^2T^3t_1^3}{36}\right)\end{aligned}\right].\tag{9}$$

4. Optimal solution procedure

The main objective of the formulated model is to determine the optimal values of t_1 and T that must minimize the total cost per unit time TC of one complete cycle over the time T. The necessary conditions for minimization of the total cost per unit time TC over the cycle time T are

$$\frac{\partial(TC)}{\partial t_1}=0,\tag{10}$$

$$\frac{\partial(TC)}{\partial T}=0.\tag{11}$$

The sufficient condition for minimization of total cost per unit time TC over the cycle time T is that TC is convex function or

$$\frac{\partial^2(TC)}{\partial t_1^2}>0,\quad\frac{\partial^2(TC)}{\partial T^2}>0\quad\&\quad\begin{vmatrix}\dfrac{\partial^2(TC)}{\partial t_1^2}&\dfrac{\partial^2(TC)}{\partial t_1\partial T}\\\dfrac{\partial^2(TC)}{\partial T\partial t_1}&\dfrac{\partial^2(TC)}{\partial T^2}\end{vmatrix}>0.\tag{12}$$

The optimal values t_1^* and T^* of t_1 and T can be obtained by solving the equations (10) and (11). The optimal value TC^* of TC can be obtained by putting optimal values t_1^* and T^* of t_1 and T in equation (9).

5. Numerical results

To illustrate the solution of proposed inventory model developed above, a numerical example has been solved. Data have been assumed randomly from the literature in appropriate units to obtain optimal values of t_1, T & TC, and perform the analysis

Example

The input data for various parameters have been taken as

$A=10,\ I_o=1000,\ \alpha=0.01, a_0=5,\ a_1=2,\ a_2=10,\ b_0=0.5,\ b_1=1.5,\ b_2=1,\ c=10,\ c_d=0.5.$

The optimal values of t_1, T and TC are obtained by using MATHEMATICA 8.0. The obtained optimal values are: $t_1^*=13.8528, T=22.4678$ and $TC^*=11228.$

To show the convexity of total cost per unit of inventory system by theoretical results is very difficult because the equation (9) is highly nonlinear function of decision parameters. Thus, the convexity of total cost TC with respect to decision variables is shown graphically in figure 2.

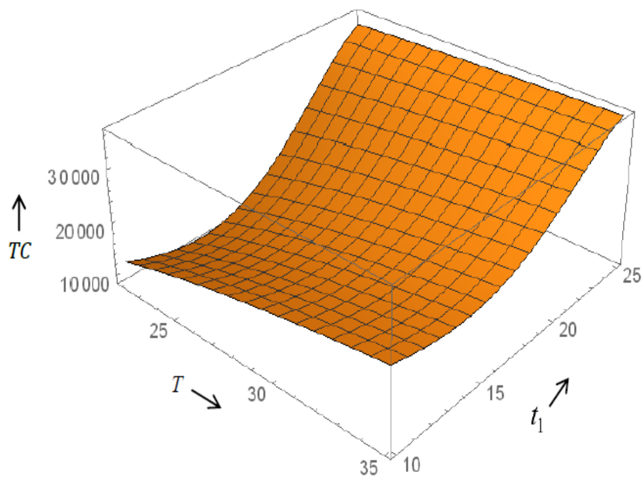

Figure 1: Convexity of total cost TC

6. Sensitivity analysis

The sensitivity analysis has been performed by changing the values of the key parameters $a_o, a_1, a_2, b_o, b_1, b_2$ & α. The values of key parameters are changed by $\pm10\%$ and $\pm20\%$ to show the effect of their values on optimal values t_1^*, T^* and TC^* of t_1, T and TC, respectively. Only one of the key parameters decreases or increases by 10% and 20% at time and other kept unchanged to obtain the optimal values t_1^*, T^* and TC^*. The changes in optimal values t_1^*, t_2^*, TC^* and percentage change in TC^* have been presented in tables 1-7.

Table 1: Sensitivity analysis when a_o varies

a_o	t_1^*	T^*	TC^*	% change in TC^*
4	12.8872	22.9802	11662	3.87
4.5	13.3456	22.6434	11434	1.83
5.5	14.1121	22.2234	11020	1.85
6	14.5642	22.0312	10898	2.94

Table 2: Sensitivity analysis when a_1 varies

a_1	t_1^*	T^*	TC^*	% change in TC^*
1.6	11.9876	22.8832	11998	6.86
1.8	12.7645	22.6134	11632	3.60
2.2	14.6245	22.3014	10788	3.92
2.4	15.4502	22.1432	10345	7.86

Table 3: Sensitivity analysis when a_2 varies

a_2	t_1^*	T^*	TC^*	% change in TC^*
8	13.2786	23.0830	12448	10.87
9	13.5367	22.7914	11845	5.50
11	14.0240	22.1202	10638	5.25
12	14.3256	21.8350	10020	10.76

Table 4: Sensitivity analysis when b_o varies

b_0	t_1^*	T^*	TC^*	% change in TC^*
0.4	13.7478	22.0256	9387	16.40
0.45	13.8012	22.2454	10298	8.28
0.55	13.9122	22.6423	12434	10.74
0.6	13.9632	22.8378	13332	18.74

Table 5: Sensitivity analysis when b_1 varies

b_1	t_1^*	T^*	TC^*	% change in TC^*
1.20	13.7075	22.3142	7456	33.59
1.35	13.7898	22.3830	9586	14.62
1.65	13.9201	22.5224	13686	21.89
1.80	13.9998	22.6012	15780	40.54

Table 6: Sensitivity analysis when b_2 varies

b_2	t_1^*	T^*	TC^*	% change in TC^*
0.8	12.8266	21.4896	10812	3.71
0.9	13.3445	21.9565	11002	2.01
1.1	14.2312	22.8876	11558	2.94
1.2	14.7134	23.3910	11886	5.86

Table 7: Sensitivity analysis when α varies

α	t_1^*	T^*	TC^*	% change in TC^*
0.008	15.8712	24.5346	10486	6.61
0.009	14.7932	23.4987	10844	3.42
0.011	12.9884	21.5390	11602	3.33
0.012	11.8892	20.6145	11998	6.86

7. Observations

From Tables 1 - 7, the following observations can be obtained.

i) The optimal value of total cost per unit time that is TC^* decreases with increasing the values demand parameters a_o, a_1 & a_2, whereas decreases with increase of holding cost parameters b_o, b_1 & b_2 and deterioration parameter α. It is revealed that TC^* is less sensitive to changes in a_o & b_2, while fairly sensitive to changes in a_1, a_2 & α. Further, TC^* is highly sensitive with respect to changes in b_o & b_1.

ii) The optimal value t_1^* increases with increase in the values of a_o, a_1, a_2, b_o, b_1 & b_2. Also, it decreases by increasing the value of α. It is observed that t_1^* is less sensitive to changes in a_2, b_o & b_1, while t_1^* is fairly sensitive to changes in a_o, a_1 & b_2. Furthermore, it is highly sensitive to changes in α.

iii) The optimal value T^* decreases by increasing values of a_o, a_1, a_2 & α, while increases with increase the values of b_o, b_1 & b_2. It is seen that T^* is less sensitive to changes in a_o, a_1, b_o, b_1 & b_2. Also, It is fairly sensitive to changes in a_2 & α.

8. Conclusions

In this study, a deteriorating inventory model with ramp type demand and time dependent holding cost has been formulated and solved. The ramp type demand is the most common demand in all real aspects of the business. In addition, time dependent holding cost is very close to the real environment. A numerical example has been solved and sensitivity analysis has been performed to illustrate the model. The convexity of the total cost has been depicted graphically. It is noticed that the total cost per unit TC is highly sensitive to changes in b_o & b_1, fairly sensitive to changes in a_1, a_2 & α, and less sensitive to changes in a_o & b_2. The range of values of percentage changes in TC is from 1.83 % to 40.54%.

The developed model can be extended further for shortages, trade credit, and the time value of money.

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