

An Innovative HSS Algebra for Designing a Secure Like-NTRU Encryption

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Abstract

This paper aims to improve NTRU by creating a multi-dimensional public key cryptosystem called NTRSHH by replacing the algebra $Z[x] \setminus (x^N - 1)$ used in NTRU with a new algebra called HSS. This cryptosystem provides high security and speed compared to some cryptosystems similar to NTRU as HXDTRU and TOTRU by increasing the number of private keys in the public keys and the cipher text. The property of the multi-dimensionality of this cryptosystem gives it an advantage in the possibility of its application in many scopes.

Keywords: NTRU, HXDTRU, TOTRU, Security of key, and security of message.

Introduction

In 1996, Hoffstein et al. proposed the NTRU public key cryptosystem in 1996, which works with the ring of truncated polynomials $Z[x] \setminus (x^N - 1)$ [1]. It's the first system that doesn't rely on discrete logarithm problem or factorization integer numbers. One of the notable characteristics of NTRU is its speed, its much faster than RSA and ECC and has a much smaller key. Since it was proposed, many studies have improved NTRU performance.

In 2002, Gaborit et al. introduced analog of NTRU called CTRU depending on $F_2[x] \setminus (x^N - 1)$ [2]. In 2005, Coglianese and Goi proposed MaTRU an like-NTRU, this system depends on a ring of $k \times k$ matrices of polynomials in $Z[x] \setminus (x^N - 1)$ [3]. In 2009, Malekian et al. introduced QTRU, a

four-dimensional cryptosystem based on quaternion algebra [4]. In 2010, Malekian et al. proposed OTRU, a multi-dimensional cryptosystem based on alternative octonion algebra [5]. In 2011, Jarvis proposed ETRU based on the Eisenstein integer ring [6]. In 2016, Yassein and Al-Saidi introduced HXDTRU and BITRU, defined by the hexadecion and binary algebras as analog to NTRU cryptosystem [7–10]. In 2018, Yassein and Al-Saidi created BCTRU, a multidimensional NTRU-like cryptosystem that depends on Bi-cartesian algebra [11, 12]. Atani et al. [13] presented a new NETRU cryptosystem that operates over the ring of $k \times k$ matrices of polynomials in $Z[x] \setminus (x^N - 1)$. Yassein et al. [14] suggested a novel NTRU-analogue based on carternion algebra in 2020, dubbed QOBTRU. Yassein et al. [15] introduced NTRTE, a new multi-dimensional public key cryptosystem based on a commutative quaternion algebra with a novel structure.

In 2021, Yassein et al. designed a new mathematical structure to offer an analog QTRU cryptosystem, termed QMNTR [16]. In the same year, Yassein et al. introduced BOTRU, a new public key cryptosystem based on the bi-octonion subalgebra with a novel mathematical structure. Also, Yassein et al. designed NTRS and TRTSH cryptosystems like-NTRU depend on tripternion algebra, also, QOTRU depends on qu-octonion algebra [18-20]. In 2022, Yassein et al. improved NTRU through the design of NTRTRN and TOTRU which depend on tripternion algebra and octonion algebra respectively [21, 22].

In this paper, we have created a new public key cryptosystem, known as NTRHSS like-NTRU based on a new algebra called HSS and a new mathematical structure that gives it higher security and speed. Also, we compared its performance with NTRU, HXDTRU, and TOTRU.

ASS Algebra

Let F be a field with $\text{Char}(F) \neq 2$, then the algebra HSS defined over F as follows:

$HSS = \{(a_1, a_2, a_3)(1,1,1) + (a_4, a_5, a_6)(x, x, x) + (a_7, a_8, a_9)(y, y, y) \mid a_i \in F, \text{ for all } i=1,2,\dots,9\}$, where $\{(1,1,1), (x, x, x), (y, y, y)\}$ form the basis of this algebra. Now, if we have three truncated polynomial rings

$$K = Z[x] \setminus (x^N - 1), K_p = Z_p[x] \setminus (x^N - 1), \text{ and } K_q = Z_q[x] \setminus (x^N - 1).$$

We show three algebras \mathfrak{B} , \mathfrak{B}_p and \mathfrak{B}_q as follows:

$$\mathfrak{B} = \{(f_1, f_2, f_3)(1,1,1) + (f_4, f_5, f_6)(x, x, x) + (f_7, f_8, f_9)(y, y, y) \mid f_1, \dots, f_9 \in K\}$$

$$\mathfrak{B}_p = \{(f_1, f_2, f_3)(1,1,1) + (f_4, f_5, f_6)(x, x, x) + (f_7, f_8, f_9)(y, y, y) \mid f_1, \dots, f_9 \in K_p\}$$

$$\mathfrak{B}_q = \{(f_1, f_2, f_3)(1,1,1) + (f_4, f_5, f_6)(x, x, x) + (f_7, f_8, f_9)(y, y, y) \mid f_1, \dots, f_9 \in K_q\}$$

Let $\mu_1, \mu_2 \in \mathbb{B}_p$ or \mathbb{B}_q such that

$\mu_1 = (f_1, f_2, f_3)(1, 1, 1) + (f_4, f_5, f_6)(x, x, x) + (f_7, f_8, f_9)(y, y, y)$ and

$\mu_2 = (g_1, g_2, g_3)(1, 1, 1) + (g_4, g_5, g_6)(x, x, x) + (g_7, g_8, g_9)(y, y, y)$. The addition $\mu_1 + \mu_2$ is

performed by adding corresponding coefficients. The multiplication $\mu_1 * \mu_2$ can be determined by

$\mu_1 * \mu_2 = (f_1g_1 + f_4g_7 + f_7g_4, f_2g_2 + f_5g_8 + f_8g_5, f_3g_3 + f_6g_9 + f_9g_6)(1, 1, 1) + (f_4g_4 + f_1g_7 + f_7g_1, f_5g_5 + f_2g_8 + f_8g_2, f_6g_6 + f_3g_9 + f_9g_3)(x, x, x) + (f_7g_7 + f_1g_4 + f_4g_1, f_8g_8 + f_5g_2 + f_2g_5, f_9g_9 + f_3g_6 + f_6g_3)(y, y, y)$.

This multiplication is not associative, but it is commutative and alternative. Then the identity multiplication is

$$\varrho = (1, 1, 1)(1, 1, 1) + (0, 0, 0)(x, x, x) + (0, 0, 0)(y, y, y).$$

Inverse multiplication of μ_1 is defined by

$\mu_1^{-1} = (e_1, e_2, e_3)(1, 1, 1) + (e_4, e_5, e_6)(x, x, x) + (e_7, e_8, e_9)(y, y, y)$ where

$$e_1 = \frac{f_4f_7 - (f_1)^2}{3f_1f_4f_7 - (f_1)^2 - (f_4)^2 - (f_7)^2}, e_2 = \frac{f_5f_8 - (f_2)^2}{3f_2f_5f_8 - (f_2)^2 - (f_5)^2 - (f_8)^2}, e_3 = \frac{f_6f_9 - (f_3)^2}{3f_3f_6f_9 - (f_3)^2 - (f_6)^2 - (f_9)^2},$$

$$e_4 = \frac{f_1f_4 - (f_7)^2}{3f_1f_4f_7 - (f_1)^2 - (f_4)^2 - (f_7)^2}, e_5 = \frac{f_2f_5 - (f_8)^2}{3f_2f_5f_8 - (f_2)^2 - (f_5)^2 - (f_8)^2}, e_6 = \frac{f_3f_6 - (f_9)^2}{3f_3f_6f_9 - (f_3)^2 - (f_6)^2 - (f_9)^2},$$

$$e_7 = \frac{f_1f_7 - (f_4)^2}{3f_1f_4f_7 - (f_1)^2 - (f_4)^2 - (f_7)^2}, e_8 = \frac{f_2f_8 - (f_5)^2}{3f_2f_5f_8 - (f_2)^2 - (f_5)^2 - (f_8)^2}, e_9 = \frac{f_3f_9 - (f_6)^2}{3f_3f_6f_9 - (f_3)^2 - (f_6)^2 - (f_9)^2}.$$

NTRHSS Cryptosystem

Public Parameters

The public parameters in NTRHSS are similar to NTRU and subsets $T_F, T_V, T_Z, T_G, T_\theta, T_R$ and $T_M \subset \mathbb{B}$ as defined in Table 1.

Table 1: subsets of NTRHSS

Notation	Definition
T_F	$F = \{(f_1, f_2, f_3)(1, 1, 1) + (f_4, f_5, f_6)(x, x, x) + (f_7, f_8, f_9)(y, y, y) \in \mathbb{B} f_i \in K, i = 1, 2, \dots, 9 \text{ satisfy } \ell(d_f, d_f - 1)\}$
T_V	$V = \{(v_1, v_2, v_3)(1, 1, 1) + (v_4, v_5, v_6)(x, x, x) + (v_7, v_8, v_9)(y, y, y) \in \mathbb{B} v_i \in K, i = 1, 2, \dots, 9 \text{ satisfy } \ell(d_v, d_v - 1)\}$

T_Z	$Z = \{(z_1, z_2, z_3)(1,1,1) + (z_4, z_5, z_6)(x, x, x) + (z_7, z_8, z_9)(y, y, y) \in \mathbb{B} \mid z_i \in K, \\ i = 1, 2, \dots, 9 \text{ satisfy } \ell(d_z, d_z - 1)\}$
T_G	$G = \{(g_1, g_2, g_3)(1,1,1) + (g_4, g_5, g_6)(x, x, x) + (g_7, g_8, g_9)(y, y, y) \in \mathbb{B} \mid g_i \in K, \\ i = 1, 2, \dots, 9 \text{ satisfy } \ell(d_g, d_g)\}$
T_θ	$\theta = \{(\theta_1, \theta_2, \theta_3)(1,1,1) + (\theta_4, \theta_5, \theta_6)(x, x, x) + (\theta_7, \theta_8, \theta_9)(y, y, y) \in \mathbb{B} \mid \theta_i \in K, \\ i = 1, 2, \dots, 9 \text{ satisfy } \ell(d_\theta, d_\theta)\}$
T_R	$R = \{(r_1, r_2, r_3)(1,1,1) + (r_4, r_5, r_6)(x, x, x) + (r_7, r_8, r_9)(y, y, y) \in \mathbb{B} \mid r_i \in K, \\ i = 1, 2, \dots, 9 \text{ satisfy } \ell(d_r, d_r)\}$
T_M	$M = \{(m_1, m_2, m_3)(1,1,1) + (m_4, m_5, m_6)(x, x, x) + (m_7, m_8, m_9)(y, y, y) \in \mathbb{B} \mid m_i \in K, \text{coefficients } i = 1, 2, \dots, 9 \text{ are the chosen modulo between } -p/2 \text{ and } p/2 \}$

Where $\ell(d_1, d_2) = \{f \in K \mid f \text{ has } d_1 \text{ coefficients equal } 1, d_2 \text{ coefficient equal } -1, \text{ the remaining equal } 0\}$. The constants $d_f, d_v, d_z, d_g, d_\theta, d_r$ and d_m are defined in a similar role as in NTRU.

NTRHSS Phases

The NTRHSS cryptosystem can be denoted through the following three phases

I. Key Creation

The recipient generates the public keys H and K by choosing invertible element $F \in T_F \bmod p$ and q denoted by F_p^{-1} and F_q^{-1} respectively, invertible element $V \in T_V \bmod p$ denoted by V_p^{-1} , invertible element $Z \in T_Z \bmod q$ denoted by Z_q^{-1} and $G \in T_G$ and are calculated as follows:

$$H = F_q^{-1} * G(\bmod q); K = V * Z_q^{-1}(\bmod q)$$

II. Encryption

To obtain the cipher text E of the plaintext M , the sender chooses $\theta \in T_\theta$ and $R \in T_R$ and converts the plaintext into an element in HSS and uses the following formula

$$E = p(H * \theta + R) + M * K(\bmod q).$$

III. Decryption

To find the plaintext of the cipher text, the recipient performs the following steps:

$$B = F * E * Z(\bmod q)$$

$$\begin{aligned}
&= F * (p(H * \theta + R) + M * K) * Z \pmod{q} \\
&= p(F * H * \theta * Z) + p(F * R * Z) + F * (M * K) * Z \pmod{q} \\
&= p(F * (F_q^{-1} * G) * \theta * Z) + p(F * R * Z) + F * (M * (V * Z_q^{-1}) * Z) \pmod{q} \\
&= (F * F_q^{-1}) * (G * \theta * F) + p(F * R * Z) + F * ((M * V) * (Z_q^{-1} * Z)) \pmod{q} \\
&= p(G * \theta * F) + p(F * R * F) + F * M * V \pmod{q},
\end{aligned}$$

Such that the coefficient of $p(G * \theta * V) + p(F * R * V) + F * (M * V)$ lie in the interval $(-q/2, q/2]$.

Convert $B = p(G * \theta * V) + p(F * R * V) + F * (M * V) \pmod{q}$ to \pmod{p}

$$\begin{aligned}
B \pmod{p} &= p(G * \theta * V) + p(F * R * V) + F * (M * V) \pmod{p} \\
&= F * M * V \pmod{p}.
\end{aligned}$$

$F_p^{-1} * B * V_p^{-1} \pmod{p} = M \pmod{p}$ and the resulting coefficients are adjusted to lie in the interval $(-p/2, p/2]$.

Security Analysis

In a brute force attack, any attacker that obtains the public parameters and public keys $H = F_q^{-1} * G \pmod{q}$ and $K = V * Z_q^{-1} \pmod{q}$ of NTRHSS can access the plaintext by finding $(F \in T_F$ or $G \in T_G)$ and $(V \in T_V$ or $Z \in T_Z)$. The size of the subsets T_F, T_G, T_V and T_Z are calculated as follows:

$$\begin{aligned}
|T_F| &= \left(\frac{N!}{(d_f!)^2 (N-2d_f)!} \right)^9, |T_G| = \left(\frac{N!}{(d_g!)^2 (N-2d_g)!} \right)^9, |T_V| = \left(\frac{N!}{(d_v!)^2 (N-2d_v)!} \right)^9, \text{ and } |T_Z| = \\
&\left(\frac{N!}{(d_z!)^2 (N-2d_z)!} \right)^9
\end{aligned}$$

Therefore, the security of key is equal to the following:

$$\left(\frac{(N!)^{18}}{(d_g! d_v!)^{18} ((N-2d_g)! (N-2d_v)!)^9} \right)^{\frac{1}{2}}$$

Similarly, to find the original message, an attacker must search in T_θ and T_R . The size of the subsets T_θ and T_R are calculated as follows:

$$|T_\theta| = \left(\frac{N!}{(d_\theta!)^2 (N-2d_\theta)!} \right)^9 \text{ and } |T_r| = \left(\frac{N!}{(d_r!)^2 (N-2d_r)!} \right)^9$$

Therefore, the security of message is equal to the following:

$$\left(\frac{(N!)^{18}}{(d_\theta! d_r!)^{18} ((N - 2d_\theta)! (N - 2d_r)!)^9} \right)^{\frac{1}{2}}$$

Table 2 show security of key and security of message for NTRHSS according to generic parameters $d_v, d_g, d_\theta, d_r, N$.

Table 2: NTRHSS 's security of key and security of message

N	d_v	d_g	d_θ	d_r	Security of Key	Security of Message
107	12	12	5	5	1.5387×10^{271}	3.3459×10^{143}
107	20	20	10	10	1.9512×10^{360}	6.4256×10^{239}
149	12	12	10	10	1.1609×10^{299}	7.5988×10^{267}
149	25	25	20	20	1.1628×10^{488}	6.9376×10^{428}
167	18	18	18	18	1.6919×10^{414}	5.3505×10^{419}
167	27	27	22	22	6.1899×10^{537}	8.3259×10^{476}
211	20	20	18	18	2.2155×10^{489}	1.4695×10^{456}
211	34	34	22	22	2.5306×10^{682}	4.2822×10^{522}
257	20	20	18	18	1.3088×10^{522}	1.7715×10^{486}
257	24	24	24	24	3.1457×10^{594}	3.1457×10^{594}

Comparison of NTRU, TOTRU, HXDTRU, and NTRHSS

Relying on the values in Table 1, Tables 3 and 4 show the comparison between key security and message security between NTRU, TOTRU, HXDTRU, and NTRHSS cryptosystem respectively. Accordingly, the security of the key and the message of NTRHSS is more secure than NTRU, TOTRU, and HXDTRU.

Table 3: Security key of the NTRU and its Improvements

Security Key of NTRU	Security Key of TOTRU	Security Key of HXDTRU	Security Key of NTRHSS
1.1640×10^{15}	1.135×10^{241}	1.135×10^{241}	1.5387×10^{271}
2.3836×10^{20}	1.0859×10^{326}	1.0859×10^{326}	1.9512×10^{360}
9.3902×10^{16}	3.6545×10^{271}	3.6545×10^{271}	1.1609×10^{299}
1.3024×10^{27}	6.8558×10^{433}	6.8558×10^{433}	1.1628×10^{488}

2.0808×10^{23}	1.2357×10^{478}	1.2357×10^{478}	1.6919×10^{414}
7.5390×10^{29}	1.0891×10^{478}	1.0891×10^{478}	6.1899×10^{537}
1.7433×10^{27}	7.2895×10^{435}	7.2895×10^{435}	2.2155×10^{490}
8.1525×10^{29}	3.8075×10^{606}	3.8075×10^{606}	2.5306×10^{682}
1.3088×10^{29}	7.4107×10^{465}	7.4107×10^{465}	1.3088×10^{522}
1.0657×10^{33}	1.2915×10^{432}	1.2915×10^{432}	3.1457×10^{594}

Table 4: Security message of the NTRU and its Improvements

Message security of NTRU	Message security of TOTRU	Message security of HXDTRU	Message security of NTRHSS
2.9757×10^8	3.7788×10^{127}	3.7788×10^{127}	3.3459×10^{143}
2.1021×10^{13}	1.4541×10^{213}	1.4541×10^{213}	6.4256×10^{239}
2.4114×10^{15}	1.3068×10^{238}	1.3068×10^{238}	7.5988×10^{267}
2.1111×10^{24}	1.5568×10^{381}	1.5568×10^{381}	6.9376×10^{428}
2.0809×10^{23}	1.2357×10^{373}	1.2357×10^{373}	5.3505×10^{419}
3.13036×10^{26}	8.4972×10^{423}	8.4972×10^{423}	8.3259×10^{476}
2.2009×10^{25}	3.0335×10^{405}	3.0335×10^{405}	1.4695×10^{456}
1.0842×10^{29}	3.6438×10^{464}	3.6438×10^{464}	4.2822×10^{522}
1.0323×10^{27}	1.6621×10^{432}	1.6621×10^{432}	1.7715×10^{486}
1.0656×10^{33}	2.7825×10^{528}	2.7825×10^{528}	3.1457×10^{594}

Table 5 shows the number of mathematical operations from addition (α) and convolution multiplication (λ) in the three phases of NTRU, HXDTRU, TOTRU and NTRHSS. Therefore, the speed of key creation, encryption, and decryption of NTRHSS are faster than those of HXDTRU and TOTRU.

Table 5: Arithmetic operations of NTRU, TOTRU, HXDTRU and NTRHSS.

	NTRU	TOTRU	HXDTRU	NTRHSS
Key Generation	1λ	128λ	265λ	54λ
Encryption	1λ and 1α	128λ and 16α	265λ and 16α	54λ and 18α

Decryption	2λ and 1α	1536λ and 16α	4096λ and 16α	486λ and 18α
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Table 6 shows the speed NTRU, TOTRU, HXDTRU, and NTRHSS based on Table 5 such that τ is the time of convolution multiplication and τ_1 is the time of polynomial addition. Therefore, NTRHSS is faster than TOTRU and HXDTRU and slower than NTRU.

Table 6: Speed of NTRU and its Improvements

	NTRU	HXDTRU	TOTRU	NTRHSS
speed	$4\tau + 2\tau_1$	$4608\tau + 32\tau_1$	$1792\tau + 32\tau_1$	$594\tau + 36\tau_1$

Conclusion

NTRHSS is multi-dimensional public key cryptosystem based on a new commutative and alternative algebra HSS with high security and acceptable speed when compared to some of NTRU improvements. It can encrypt nine different messages from one source or from multiple sources, this feature can be very useful in designing protocols or similar applications.

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This template provides authors with most of the formatting specifications needed for preparing electronic versions of their papers. All standard paper components have been specified for three reasons: (1) ease of use when formatting individual papers, (2) automatic compliance to electronic requirements that facilitate the concurrent or later production of electronic products, and (3) conformity of style throughout a conference proceedings. Margins, column widths, line spacing, and type styles are built-in; examples of the type styles are provided throughout this document and are identified in italic type, within parentheses, following the example. PLEASE DO NOT RE-ADJUST THESE MARGINS. Some components, such as multi-leveled equations, graphics, and tables are not prescribed, although the various table text styles are provided. The formatter will need to create these components, incorporating the applicable criteria that follow.

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Wherever Times is specified, Times Roman or Times New Roman may be used. If neither is available on your word processor, please use the font closest in appearance to Times. Avoid using bit-mapped fonts if possible. True-Type 1 or Open Type fonts are preferred. Please embed symbol fonts, as well, for math, etc.

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Before you begin to format your paper, first write and save the content as a separate text file. Keep your text and graphic files separate until after the text has been formatted and styled. Do not use hard tabs, and limit use of hard returns to only one return at the end of a paragraph. Do not add any kind of pagination anywhere in the paper. Do not number text heads-the template will do that for you.

Finally, complete content and organizational editing before formatting. Please take note of the following items when proofreading spelling and grammar:

Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, sc, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as “3.5-inch disk drive”.
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.

- Do not mix complete spellings and abbreviations of units: “Wb/m²” or “webers per square meter”, not “webers/m²”. Spell out units when they appear in text: “. . . a few henries”, not “. . . a few H”.
- Use a zero before decimal points: “0.25”, not “.25”.

Equations

The equations are an exception to the prescribed specifications of this template. You will need to determine whether or not your equation should be typed using either the Times New Roman or the Symbol font (please no other font). To create multileveled equations, it may be necessary to treat the equation as a graphic and insert it into the text after your paper is styled.

Number equations consecutively. Equation numbers, within parentheses, are to position flush right, as in (1), using a right tab stop. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in

Note that the equation is centered using a center tab stop. Be sure that the symbols in your equation have been defined before or immediately following the equation. Use “(1)”, not “Eq. (1)” or “equation (1)”, except at the beginning of a sentence: “Equation (1) is . . .”

Some Common Mistakes

- The word “data” is plural, not singular.
- The subscript for the permeability of vacuum μ_0 , and other common scientific constants, is zero with subscript formatting, not a lowercase letter “o”.
- In American English, commas, semi-/colons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an “inset”, not an “insert”. The word alternatively is preferred to the word “alternately” (unless you really mean something that alternates).
- Do not use the word “essentially” to mean “approximately” or “effectively”.
- In your paper title, if the words “that uses” can accurately replace the word “using”, capitalize the “u”; if not, keep using lower-cased.

- Be aware of the different meanings of the homophones “affect” and “effect”, “complement” and “compliment”, “discreet” and “discrete”, “principal” and “principle”.
- Do not confuse “imply” and “infer”.
- The prefix “non” is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the “et” in the Latin abbreviation “et al.”.
- The abbreviation “i.e.” means “that is”, and the abbreviation “e.g.” means “for example”.

An excellent style manual for science writers is [7].

Using the Template

After the text edit has been completed, the paper is ready for the template. Duplicate the template file by using the Save As command, and use the naming convention prescribed by your conference for the name of your paper. In this newly created file, highlight all of the contents and import your prepared text file. You are now ready to style your paper.

Authors and Affiliations

The template is designed so that author affiliations are not repeated each time for multiple authors of the same affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization). This template was designed for two affiliations.

For author/s of only one affiliation (Heading 3): To change the default, adjust the template as follows.

Selection (Heading 4): Highlight all author and affiliation lines.

Change number of columns: Select Format >

Columns > Presets > One Column.

Deletion: Delete the author and affiliation lines for the second affiliation.

For author/s of more than two affiliations: To change the default, adjust the template as follows.

Selection: Highlight all author and affiliation lines.

Change number of columns: Select Format >

Columns > Presets > One Column.

Highlight author and affiliation lines of affiliation 1 and copy this selection.

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Then paste the copy of affiliation 1. Repeat as necessary for each additional affiliation.

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Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is “Heading 5”. Use “figure caption” for your Figure captions, and “table head” for your table title. Run-in heads, such as “Abstract”, will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced. Styles named “Heading 1”, “Heading 2”, “Heading 3”, and “Heading 4” are prescribed.

Figures and Tables

Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation “Fig. 1”, even at the beginning of a sentence.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity “Magnetization”, or “Magnetization, M”, not just “M”. If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write “Magnetization (A/m)” or “Magnetization {A[m(1)]}”, not just “A/m”. Do not label axes with a ratio of quantities and units. For example, write “Temperature (K)”, not “Temperature/K”.

Footnotes

Use footnotes sparingly (or not at all) and place them at the bottom of the column on the page on which they are referenced. Use Times 8-point type, single-spaced. To help your readers, avoid using footnotes altogether and include necessary peripheral observations in the text (within parentheses, if you prefer, as in this sentence).

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Acknowledgment

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression, "One of us (R.B.G.) thanks . . ." Instead, try "R.B.G. thanks". Put applicable sponsor acknowledgments here; DO NOT place them on the first page of your paper or as a footnote.

References

List and number all bibliographical references in 9-point Times, single-spaced, at the end of your paper. When referenced in the text, enclose the citation number in square brackets, for example [1]. Where appropriate, include the name(s) of editors of referenced books. The template will number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use "Ref. [3]" or "reference [3]" except at the beginning of a sentence: "Reference [3] was the first . . ."

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors' names; do not use "et al.". Papers that have not been published, even if they have been submitted for publication, should be cited as "unpublished" [4]. Papers that have been accepted for publication should be cited as "in press" [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

- [1] G. Eason, B. Noble, and I. N. Sneddon, "On certain integrals of Lipschitz-Hankel type involving products of Bessel functions," *Phil. Trans. Roy. Soc. London*, vol. A247, pp. 529–551, April 1955. (*references*)
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