Memory responses on the Reflection of Electro-Magneto-Thermoelastic Plane Waves from Impedance Boundary of an Initially Stressed Thermoelastic Solid in Triple Phase lag Thermo-Elasticity.

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Abstract
This research is an investigation of propagation of the plane waves in a pre-stressed electrically conducting thermoelastic solid half-space in apropos of triple phase lag memory-dependent derivatives (MDDs) magneto-thermo-elasticity. The governing equations in the x-z plane are framed to get the velocity equations whose plane-wave solution bespeaks the existence of three plane waves. The thermally insulated/ isothermal impedance boundary reflection problem is investigated. The formulation for reflection coefficients of reflected waves is derived, enumerated for a particular material and the effects of impedance boundary, magnetic field, initial stress and MDD parameters on the reflection amplitude ratio are depicted graphically.
Keywords: Memory-dependent derivatives (MDDs), triple phase lag thermo-elasticity, magnetic field, reflection coefficients, Impedance boundary, initial stress.

1. Introduction

A subset of elasticity called thermos-elasticity deals with the combined impact of temperature and strain fields. Engineering issues frequently include Fourier's law of heat conduction in continuous mediums. Many scholars, including Biot [1], Lord and Shulman [2], have explored thermal effects in continuous elastic substances. As a result of their research, they developed a new concept of movement of heat, called a generalised theory of thermo-elasticity. Incorporating material and temperature factors, relaxation time lag, and memory-dependent derivatives (MDD) effects in the Fourier abstraction of heat movement further improved the researchers' generalised theory of thermo-elasticity. Tzou [3] fostered a new generalized thermo-elasticity inculcating two different parameters is referred to as the dual phase lag model (DPL). Subsequently, Roy Choudhuri [4] fostered the concept of three-phase-lags (TPL) heat movement in which the Fourier rule of the heat movement was replaced by gauging of TPL by introducing three different relaxation times to develop this theory and made modifications to the traditional Fourier’s model. Othman and Mansour [5] discussed two-dimensional generalized two-temperature problem in magneto-thermoelastic half-space using TPL concept.

Mondal and Kanoria [6] modified the Fourier law in sense of MMD for TPL model for elastic medium.

In reality, memory dependency is considered to be non-locality in time. Diethelm [7] updated the Caputo-concept of non-integer-order derivative and introduced a kernel function. The time lag value and the kernel may be taken at random, but they have a higher likelihood of capturing the material's true characteristics. For the first time, MDDs were included to Fourier's theory of heat movement by Wang and Li [8]. and provided a new finite-type generalised heat exchange equation with counting memory. Later, Ezzat et al. [9,10] inculcated the MDD in spite of non-integer derivative into the rate of heat flux in LS theory [2] of generalized thermo-elasticity to signify memory-dependence as:

$$\left(1+\tau_0 D_{\tau_0}\right) q_i = -KT_{,i} \Rightarrow q_i + \tau_0 D_{\tau_0} q_i = -KT_{,i}, \tag{1}$$

$$D_{\tau_0} q_i = D_{\tau_0}^{(1)} q_i = \frac{1}{\tau_0} \int_{t-\tau_0}^{t} k(t,\xi) q_i^{(1)}(\xi) d\xi \tag{2}$$

where, $k(t,\xi)$ *is* the kernel function, D_{τ_0}, is the MMD with respect to time, τ_0 is the time lag parameter, for actual time as *t*, and $[t-\tau_0, t)$ is as the former-time interval. The result of the MDD $D_{\tau_0} f(t)$ is usually smaller than that of ordinary $f^{/}(t)$ and may be more practical because they are monotonic functions. Ezzat et al. [11] submitted memory kernel as follows:

$$k(t-\xi) = 1 - \frac{2b}{\tau_0}(t-\xi) + \frac{a^2}{\tau_0^2}(t-\xi)^2, \tag{3}$$

A, B are the constants, the values of which are to be taken freely. Four kernels are chosen as

$$k(t-\xi) = 1 - \frac{2b}{\tau_0}(t-\xi) + \frac{a^2}{\tau_0^2}(t-\xi)^2 = \begin{cases} 1, & if\ a=0, b=0, \\ 1-\dfrac{(t-\xi)}{\tau_0}, & if\ a=0, b=\dfrac{1}{2}, \\ 1-(t-\xi), & if\ a=0, b=\dfrac{\tau_0}{2}, \\ \left(1-\dfrac{(t-\xi)}{\tau_0}\right)^2, & if\ a=1, b=1, \end{cases} \tag{4}$$

Mondal et al. [12] modified the Fourier law for TPL model in sense of MMD for elastic materials. Willson [13] inspected magneto-thermoelastic plane waves' propagation. Dey and Addy [14] looked over the beginning stress's impact on the problem of reflection. In an elastic medium that had been initially pressured, Sidhu and Singh [15] examined P and SV waves. Montanaro [16] probed initial stress effect in isotropic linear thermo-elasticity. Yadav [17] examined the magneto-thermoelastic waves in a orthotropic diffusion rotator. Godoy et al. [18] investigated impedance boundary conditions for surface waves in an elastic half-space. Yadav [19] illustrated the impact of impedance boundary and initial stress on the plane wave reflection in fraction-order thermo-elasticity in a rotating medium with a magnetic field. Lotfy and Sarkar

[20] probed the effect of MMD for photothermal semiconductor two-temperature theory. Abo-Dahab et al. [21] surved the effect of three phase lag and initial stress in a rotating thermo-magneto-microstretch medium. Sarkar et al. [22] examined the impact of time-delay on the reflection of thermoelastic waves with MMD from the insulated surface.

2. Formulation of the problem and its solution

In regards with the concept of L S [2], Willson [13], and Montanaro [16], equations that govern an isotropic electrically conducting initially stressed state of hydrostatic stress (compression) P_i, thermo-elastic medium linearly in generalized triple phase lag theory in the presence of a primary magnetic field **B**, $\mathbf{B}=\mu_e\mathbf{H}$ with memory-dependent heat transfer at reference temperature T_0 when body forces and heat sources remain absent, are

Equations of motion

$$\mu\mathbf{u}_{i,jj}+(\lambda+\mu)\mathbf{u}_{j,ij}-\alpha_T T_{,i}+\mu_e(\mathbf{J}\times\mathbf{H})_i+P_i=\rho\ddot{\mathbf{u}}_i, \tag{5}$$

Let the magnetic field is presumed as $\mathbf{H}=\mathbf{H}_0+\mathbf{h}$, $\mathbf{H}_0=\left(0,H_0,0\right)$, $\mathbf{h}\left(h_x,h_y,h_z\right)$, is a change to the fundamental magnetic field, an prompted magnetic field $\mathbf{h}=(0,h,0)$ and a generated electric field **E,** are evolved due $\mathbf{H}=\left(0,H_0,0\right)$ with conduction current **J.** For homogenous electrically conducting elastic solid , the linear equations of electrodynamic in a simplified form may be written as

$$\nabla\times\mathbf{h}=\mathbf{J}+\epsilon_0\dot{\mathbf{E}},\ \nabla\times\mathbf{E}=-\mu_e\dot{\mathbf{h}},\nabla.\mathbf{h}=0,\mathbf{E}=-\mu_e(\dot{\mathbf{u}}\times\mathbf{H}),\dot{\mathbf{E}}=-\mu_e(\ddot{\mathbf{u}}\times\mathbf{H}), \tag{6}$$

Lorentz force components in x-z plane are

$$\left.(\mathbf{J}\times\mathbf{B})_1=\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x\partial z}\right)-\epsilon_0\mu_e^2 H_0^2\ddot{u},(\mathbf{J}\times\mathbf{B})_3=\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial z^2}\right)-\epsilon_0\mu_e^2 H_0^2\ddot{w},\right\}, \tag{7}$$

Following Ezzat et al. [9, 10] memory-dependent derivatives generalized thermoelastic heat conduction equation is

$$\left(1+\tau_i D_{\alpha_i}\right)\left(\rho C_E\dot{T}+\beta_T T_0\dot{u}_{,i}\right)=-KT_{,ii}, \tag{8}$$

In accordance with Roy Choudhuri's TPL model [4],

$$-K(1+\tau_2\frac{\partial}{\partial t})\dot{T}_{,i}-K^*(1+\tau_3\frac{\partial}{\partial t})\dot{\upsilon}_{,i}=(1+\tau_1\frac{\partial}{\partial t}+\frac{\tau_1^2}{2}\frac{\partial^2}{\partial t^2})\dot{q}_i^*, \tag{9}$$

$$K\frac{\partial}{\partial t}(1+\tau_2\frac{\partial}{\partial t})\nabla^2 T+K^*(1+\tau_3\frac{\partial}{\partial t})\nabla^2 T=\left(1+\tau_1\frac{\partial}{\partial t}+\frac{\tau_1^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\alpha_T T_0\frac{\partial^2 e_{ij}}{\partial t^2}+\rho C_E\frac{\partial^2 T}{\partial t^2}\right),$$

(10)

Following Ezzat et al. [9, 10,11], Wang and Li [8] and Mondal, Pal, and Kanoria [12] using the definition of MMD in TPL theory the heat transport in TPL with MMD effect with arbitrary kernel $k(t-\xi)$ during a slipping period $[(t-\alpha_i),\ t]$ as follows:

$$D_{\alpha_i} f(t) = \frac{1}{\alpha_i} \int_{t-\alpha_i}^{t} k(t-\xi) f^{/}(\xi) d\xi, \tag{11}$$

$k\,(t-\xi)$ is differentiable w. r. t. variables t and ξ, which represent the impact of memory on the delay interval $[(t-\alpha_i),\ t]$, and $0 \le k\,(t-\xi) < 1$ *for* $\xi \in [(t-\alpha_i),\ t]$. Here α_i takes the values $\alpha_1, \alpha_2, \alpha_3$, for TPL heat equation.

$$K(1+\tau_2 D_{\alpha_2})\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)+K^*(1+\tau_3 D_{\alpha_3})\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)$$
$$=\left(1+\tau_1 D_{\alpha_1}+\frac{\tau_1^2}{2}D_{\alpha_1}^2\right)\left\{\alpha_T T_0\left(\frac{\partial \ddot{u}}{\partial x}+\frac{\partial \ddot{w}}{\partial z}\right)+\rho C_E \frac{\partial^2 T}{\partial t^2}\right\},$$

(12)

τ_1, τ_2, τ_3 are phase delay of heat flux due thermal inertia, phase delay of temperature gradient, phase delay of displacement gradient caused by temperature, and $\alpha_1, \alpha_2, \alpha_3$ are the delay times due to TPL model for MDD respectively.

$$K\left(\frac{\partial^2 \dot{T}}{\partial x^2}+\frac{\partial^2 \dot{T}}{\partial z^2}\right)+K\frac{\tau_2}{\alpha_2}\int_{t-\alpha_2}^{t} k(t-\xi)\left(\frac{\partial^4 T}{\partial \xi^2 \partial x^2}+\frac{\partial^4 T}{\partial \xi^2 \partial z^2}\right)d\xi+K^*\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)+K^*\frac{\tau_3}{\alpha_3}$$
$$\int_{t-\alpha_3}^{t} k(t-\xi)\left(\frac{\partial^3 T}{\partial \xi \partial x^2}+\frac{\partial^3 T}{\partial \xi \partial z^2}\right)d\xi=\left\{\alpha_T T_0\left(\frac{\partial \ddot{u}}{\partial x}+\frac{\partial \ddot{w}}{\partial z}\right)+\rho C_E \frac{\partial^2 T}{\partial t^2}\right\}+\frac{\tau_1}{\alpha_1}\int_{t-\alpha_1}^{t} k(t-\xi)$$
$$\left\{\alpha_T T_0\left(\frac{\partial^4 u}{\partial \xi^3 \partial x}+\frac{\partial^4 w}{\partial \xi^3 \partial z}\right)+\rho C_E \frac{\partial^3 T}{\partial \xi^3}\right\}d\xi+\frac{\tau_1^2}{2\alpha_1}\int_{t-\alpha_1}^{t} k(t-\xi)$$
$$\left\{\alpha_T T_0\left(\frac{\partial^5 u}{\partial \xi^4 \partial x}+\frac{\partial^5 w}{\partial \xi^4 \partial z}\right)+\rho C_E \frac{\partial^4 T}{\partial \xi^4}\right\}d\xi,$$

(13)

$$K\left(1+\frac{\tau_2}{\alpha_2}M_2(t)\right)\left(\frac{\partial^2 \dot{T}}{\partial x^2}+\frac{\partial^2 \dot{T}}{\partial z^2}\right)+K^*\left(1+\frac{\tau_3}{\alpha_3}M_3(t)\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)$$
$$=\left(1+\frac{\tau_1}{\alpha_1}M_1(t)+\frac{\tau_1^2}{2\alpha_1}M_{12}(t)\right)\left\{\alpha_T T_0\left(\frac{\partial \ddot{u}}{\partial x}+\frac{\partial \ddot{w}}{\partial z}\right)+\rho C_E \frac{\partial^2 T}{\partial t^2}\right\},$$

(14)

$D_{\alpha_1}, D_{\alpha_2}, D_{\alpha_3}$, are memory dependent derivative, K, is thermal conductivities, K^*, is the additional material constant. We take the origin on the plane surface and restrict our analysis to plane strain parallel to x-z plane denoted by (z ≤ 0) having displacement vector $\mathbf{u} = (u, 0, w)$, $\frac{\partial}{\partial y} = 0$, and using Helmholtz's representation for displacement in x-z plane. The Constitutive equations are as

$$\sigma_{ij} = \lambda\delta_{ij}e_{kk} + 2\mu e_{ij} - \alpha_T T\delta_{ij}, \tag{15}$$

$$2e_{ij} = (u_{i,j} + u_{j,i}), -q^*_{i,i} = \rho T_0 \dot{S}, \rho T_0 S = \alpha_T T_0 e + \rho C_E T, -q^*_{i,i} = \alpha_T T_0 \dot{e} + \rho C_E \dot{T}, \dot{\upsilon} = T,$$

(16)

$$\left.\begin{aligned}&\sigma_{zz} = (\lambda+2\mu)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} - \alpha_T T, \sigma_{xx} = (\lambda+2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} - \alpha_T T, \sigma_{zx} = \mu\frac{\partial w}{\partial x} + \mu\frac{\partial u}{\partial z},\\ &\sigma_{xz} = \mu\frac{\partial u}{\partial z} + \mu\frac{\partial w}{\partial x}, u = \frac{\partial q}{\partial x} - \frac{\partial \psi}{\partial z},\ w = \frac{\partial q}{\partial z} + \frac{\partial \psi}{\partial x},\end{aligned}\right\},$$

(17)

$$\left.\begin{aligned}&\bar{\sigma}_{ij} = \mu_e\left[\mathbf{H}_i\mathbf{h}_j + \mathbf{H}_j\mathbf{h}_i - \mathbf{H}_k\mathbf{h}_k\delta_{ij}\right], \bar{\sigma}_{zz} = -\mu_e H_0^2\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) = -\mu_e H_0^2\nabla^2 q, \bar{\sigma}_{zx} = 0,\\ &P_i = -p_0\nabla^2 u_i,\ P_1 = -p_0\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right),\ P_3 = -p_0\left(\frac{\partial^2 w}{\partial x} + \frac{\partial^2 w}{\partial z^2}\right), i, j, k = 1, 2, 3,\end{aligned}\right\}, \tag{18}$$

Using constitutive equations (15) to (18) the x-z plane's field equations transform into

$$\begin{aligned}&(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 w}{\partial x\partial z} + \mu\frac{\partial^2 u}{\partial z^2} + \mu_e H_0^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x\partial z}\right)\\ &\qquad - p_0\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - \alpha_T\frac{\partial T}{\partial x} = (\rho + \in_0 \mu_e^2 H_0^2)\frac{\partial^2 u}{\partial t^2},\end{aligned} \tag{19}$$

$$\begin{aligned}&(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x\partial z} + \mu\frac{\partial^2 w}{\partial x^2} + \mu_e H_0^2\left(\frac{\partial^2 u}{\partial x\partial z} + \frac{\partial^2 w}{\partial z^2}\right) - p_0\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right)\\ &\qquad - \alpha_T\frac{\partial T}{\partial z} = (\rho + \in_0 \mu_e^2 H_0^2)\frac{\partial^2 w}{\partial t^2},\end{aligned}$$

(20)

$$\begin{aligned}&K\left(1 + \frac{\tau_2}{\alpha_2}M_2(t)\right)\left(\frac{\partial^2 \dot{T}}{\partial x^2} + \frac{\partial^2 \dot{T}}{\partial z^2}\right) + K^*\left(1 + \frac{\tau_3}{\alpha_3}M_3(t)\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) =\\ &\qquad \left(1 + \frac{\tau_1}{\alpha_1}M_1(t) + \frac{\tau_1^2}{2\alpha_1}M_{12}(t)\right)\left\{\alpha_T T_0\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{w}}{\partial z}\right) + \rho C_E\frac{\partial^2 T}{\partial t^2}\right\},\end{aligned}$$

(21)

Using the Helmholtz's representation in equations (19) to (21) and proceeding the solution in form of

$$(q, \psi, T) = (\bar{q}, \bar{\psi}, \bar{T})\, e^{ik(x\sin\theta + z\cos\theta - Vt)}, \tag{22}$$

we get

$$\frac{\mu - p_0}{\rho} - B_{11}V^2 = 0, \tag{23}$$

$$B_{11}V^4 - \left(B_{22} + B_{11}\bar{B}_{33} - \varepsilon\right)V^2 + B_{22}\bar{B}_{33} = 0, \tag{24}$$

the solution of equation (23) i. e. $V_3 = \sqrt{\dfrac{\mu - p_0}{\rho(1 + \in_0 \mu_e^2 H_0^2)}}$, is a measure of wave speed of SV wave. The speed of SV wave is influenced by the appearance of pre-stress. Initial stress causes a decrease in SV wave speed, making it to be slower than it would be in a typical thermoelastic material. It is concluded that presence of magnetic field also decreases the speed of SV wave. Change in temperature does not affect the speed of SV wave. Equation (24) is a quadratic in V^2, then the real parts of the roots of equation (24) represent the velocity of two coupled plane waves namely longitudinal (P) wave say V_1, thermal (T) wave say V_2, and if $V_i^{-1} = V_i^{*-1} - i\,\omega^{-1} q_i, i = 1, 2, 3,$ q_i represent the coefficient of attenuation

where,

$$V_1^2, V_2^2 = \frac{\left(B_{22} + B_{11}\bar{B}_{33} - \varepsilon\right) \pm \sqrt{\left(B_{22} + B_{11}\bar{B}_{33} - \varepsilon\right)^2 - 4B_{11}B_{22}\bar{B}_{33}}}{2B_{11}}, \varepsilon = \frac{\alpha_T^2 T_0}{\rho^2 C_E}, \bar{B}_{33} = \frac{B_{33}}{\rho C_E},$$

$$B_{11} = 1 + \in_0 \mu_e^2 H_0^2, B_{22} = \frac{\lambda + 2\mu + \mu_e H_0^2 - p_0}{\rho}, B_{33} = \frac{-i\omega K\left(1 + \dfrac{\tau_2}{\alpha_2} M_2(t)\right) + K^*\left(1 + \dfrac{\tau_3}{\alpha_3} M_3(t)\right)}{\left(1 + \dfrac{\tau_1}{\alpha_1} M_1(t) + \dfrac{\tau_1^2}{2\alpha_1} M_{12}(t)\right)},$$

$$M_{j=1,2,3}(\tau_j, \omega) = \frac{1}{\tau_j^2\omega^2}\left\{e^{i\omega\tau_j}\left(2a^2 - \tau_j^2\omega^2 - a^2\tau_j^2\omega^2 + 2b\tau_j^2\omega^2 - 2ia^2\tau_j\omega + 2ib\tau_j\omega\right) + \left(\tau_j^2\omega^2 - 2bi\tau_j\omega - 2a^2\right)\right\},$$

$$M_{12}(\tau_1, \omega) =$$
$$\frac{1}{4i\omega\tau_1^2}\left\{e^{2i\omega\tau_1}\left(-2\omega^2\tau_1^2 + 4b\omega^2\tau_1^2 + 2bi\omega\tau_1 - 2a^2\omega^2\tau_1^2 - 2ia^2\omega\tau_1 + a^2\right) + \left(-a^2 - 2bi\omega\tau_1 + 2\omega^2\tau_1^2\right)\right\},$$

where, $\bar{q}, \bar{\psi}, \bar{T}$ are located the constants, Phase velocity is denoted by V, and wave number by k. The wave's circular frequency is $\omega = kV$, λ, μ are elastic constant, $\alpha_T = (3\lambda + 2\mu)\alpha_t$, is thermal conductivities α_t, is the constant of linear thermal enlargement, q_i^*, is the heat flux component, C_E, is the constant of specific heat, δ_{ij} is Kronecker delta, u, w are the displacement vector's component, ρ, is the density of the medium, e_{ij} are constituent of the tensor of strain, τ_{ij} are components of the tensor of stress. $\in_0$, μ_e, $\mathbf{B}$, and $\mathbf{H}$ are the medium's electric conductivity, magnetic permeability, magnetic induction vector, and vector of the overall magnetic field respectively. As the half-space gets deeper, the negative z-axis is drawn along it.

3. Reflection from free surface

An isotropic pre-stressed magneto-thermoelastic electrically conducting at thermally insulated impedance boundary surface is investigated for reflection. The decreasing z-axis is taken along the increasing depth into the medium. As P wave is made incident at z = 0, consequently there will be reflected P, T and SV wave. The complete geometry shown in figure 1.

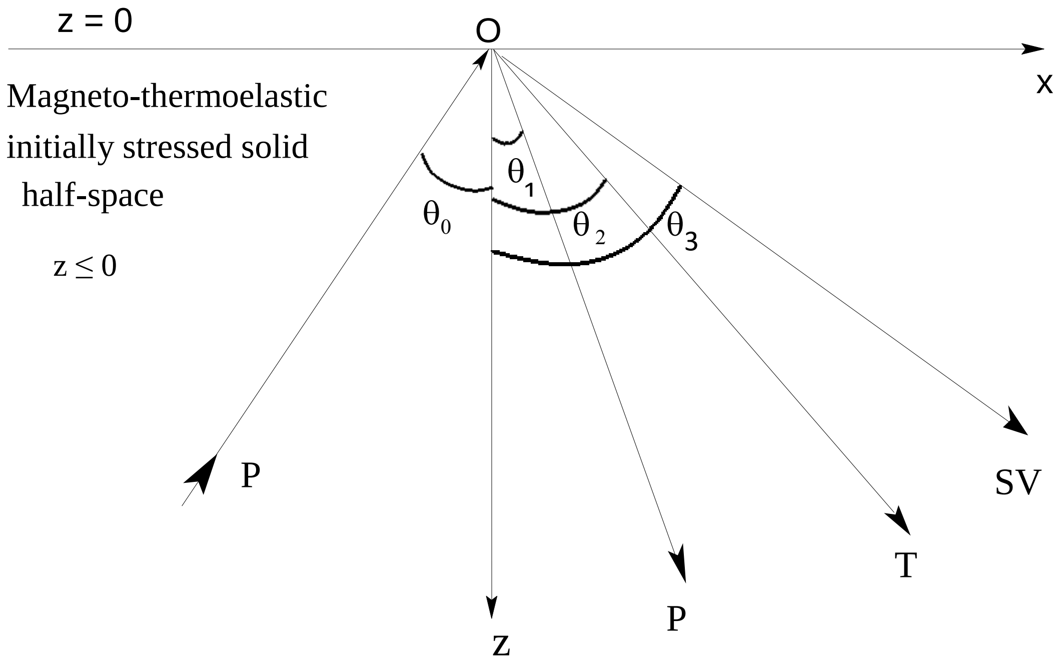


Figure 1. Geometry of the problem.

The suitable displacement and temperature field potentials in the half space are inferred as

$$q = X_0 \exp\left\{ik_1\left(x\sin\theta_0 + z\cos\theta_0 - V_1^* t\right)\right\} + \sum_{s=1}^{2} X_s \exp\left\{ik_s\left(x\sin\theta_s - z\cos\theta_s - V_s^* t\right)\right\},$$

(25)

$$T = F_1 X_0 \exp\left\{ik_1\left(x\sin\theta_0 + z\cos\theta_0 - V_1^* t\right)\right\} + \sum_{s=1}^{2} F_s X_s \exp\left\{ik_s\left(x\sin\theta_s - z\cos\theta_s - V_s^* t\right)\right\},$$

(26)

$$\psi = X_3 \exp\left\{ik_3\left(x\sin\theta_3 + z\cos\theta_3 - V_3^* t\right)\right\}, \tag{27}$$

where $(s=1,2,3), V_1^*$ is the incident and reflected velocity of P, and V_1^*, V_3^*, are the velocities of reflected T and SV waves respectively, $k_s, (s=1,2,3)$ are complex wave numbers and the coefficient of coupling $\frac{F_s}{k_s^2}$, $(s=1,2,)$ are given as

$$\frac{F_s}{k_s^2} = \frac{\rho(1+\in_0 \mu_e^2 H_0^2)V_s^{*2} - \left(\lambda + 2\mu + \mu_e H_0^2 - p_0\right)}{\alpha_T}, \quad s=1,2., \tag{28}$$

The boundary condition at z =0, are given by

$$\left.\begin{array}{l} \sigma_{zz} + \bar{\sigma}_{zz} + \omega\xi_3 w \Rightarrow \lambda\dfrac{\partial^2 q}{\partial x^2} + (\lambda+2\mu)\dfrac{\partial^2 q}{\partial z^2} + 2\mu\dfrac{\partial^2\psi}{\partial x\partial z} - \alpha_T T - \mu_e H_0^2\nabla^2 q + \omega\xi_3\left(\dfrac{\partial q}{\partial z} + \dfrac{\partial\psi}{\partial x}\right) = 0, \\ \sigma_{zx} + \bar{\sigma}_{zx} + \omega\xi_1 u \Rightarrow 2\mu\dfrac{\partial^2 q}{\partial x\partial z} + \mu\dfrac{\partial^2\psi}{\partial x^2} - \mu\dfrac{\partial^2\psi}{\partial z^2} + \omega\xi_1\left(\dfrac{\partial q}{\partial x} - \dfrac{\partial\psi}{\partial z}\right) = 0, \dfrac{\partial T}{\partial z} + h_1 T = 0, \end{array}\right\},$$

(29)

and $h_1 \to 0$ for thermally insulated and $h_1 \to \infty$ for isothermal case, ξ_1, ξ_3 are real impedance parameters. The potentials (25) to (27) corresponding to the incident and reflected waves must satisfy the boundary conditions (29). Since the phases of the waves must be the same for each value of x. For the wavenumber k_1, k_2, k_3 and the angles $\theta_0, \theta_1, \theta_2, \theta_3$ we to determine Snell's law for the for the present problem at the surface z = 0 as

$$\frac{\sin\theta_0}{V_1^*} = \frac{\sin\theta_i}{V_s^*}, \quad k_s V_s^* = \omega\left(say\right), \quad (s=1,2,3), \tag{30}$$

and using equation (25) to (27) in equation (29) and with the assist of equation (30), the system of three non-homogeneous is collected as,

$$\sum_{j=1}^{3} \bar{a}_{ij}\bar{Z}_j = \bar{b}_i, \ \left(i=1,2,3\right),$$

(31)

where,

$$a_{1j}=\left[\lambda-\mu_e H_0^2+2\mu\left\{1-\left(\frac{V_j^*}{V_1^*}\right)^2\sin^2\theta_0\right\}+\frac{\alpha_T F_j}{k_j^2}\right]\left(\frac{V_1^*}{V_j^*}\right)^2+i\xi_3 V_1^*\frac{V_1^*}{V_j^*}\sqrt{1-\left(\frac{V_j^*}{V_1^*}\right)^2\sin^2\theta_0},(j=1,2),$$

$$a_{13}=-2\mu\sin\theta_0\frac{V_1^*}{V_3^*}\sqrt{1-\left(\frac{V_3^*}{V_1^*}\right)^2\sin^2\theta_0}-i\xi_3 V_1^*\sin\theta_0\ ,$$

$$b_1=-[\lambda-\mu_e H_0^2+2\mu\cos^2\theta_0+\frac{\alpha_T F_1}{k_1^2}-i\xi_3 V_1^*\cos\theta_0],$$

$$a_{2j}=2\mu\sin\theta_0\frac{V_1^*}{V_j^*}\sqrt{1-\left(\frac{V_j^*}{V_1^*}\right)^2\sin^2\theta_0}+i\xi_1 V_1^*\sin\theta_0,(j=1,2),$$

$$a_{23}=-\mu\sin^2\theta_0+\mu\left(\frac{V_1^*}{V_3^*}\right)^2\left\{1-\left(\frac{V_3^*}{V_1^*}\right)^2\sin^2\theta_0\right\}+i\xi_1 V_1^*\frac{V_1^*}{V_j^*}\sqrt{1-\left(\frac{V_j^*}{V_1^*}\right)^2\sin^2\theta_0},$$

$$b_2=2\mu\sin\theta_0\cos\theta_0-i\xi_1 V_1^*\sin\theta_0,$$

(a) For thermally insulated

$$a_{3j}=\frac{F_j}{k_j^2}\left(\frac{V_1^*}{V_j^*}\right)^3\sqrt{1-\left(\frac{V_j^*}{V_1^*}\right)^2\sin^2\theta_0},\ (j=1,2),a_{33}=0,\ b_3=\frac{F_1}{k_1^2}\cos\theta_0,$$

(b) For isothermal

$$a_{3j}=\frac{F_j}{k_j^2}\left(\frac{V_1^*}{V_j^*}\right)^2\ ,(j=1,2),a_{33}=0,\quad b_3=-\frac{F_1}{k_1^2}.\quad Z_1=\frac{X_1}{X_0},\quad Z_2=\frac{X_2}{X_0},\ Z_3=\frac{X_3}{X_0},$$

where, $|Z_s|,(s=1,2,3)$ are the reflection coefficients of reflected P, T, and SV waves, respectively.

4. Numerical Results and Discussion

Opted copper material constants from Othman and Mansour [5] at T_0 = 293 K,

λ = 7.76 × 10^{10} N.m^{-2}, μ = 3.86 × 10^{10} N.m^{-2}, ρ = 8.954× 10^3 kg.m^{-3}, $K^*=2.97\times10^{13}$,

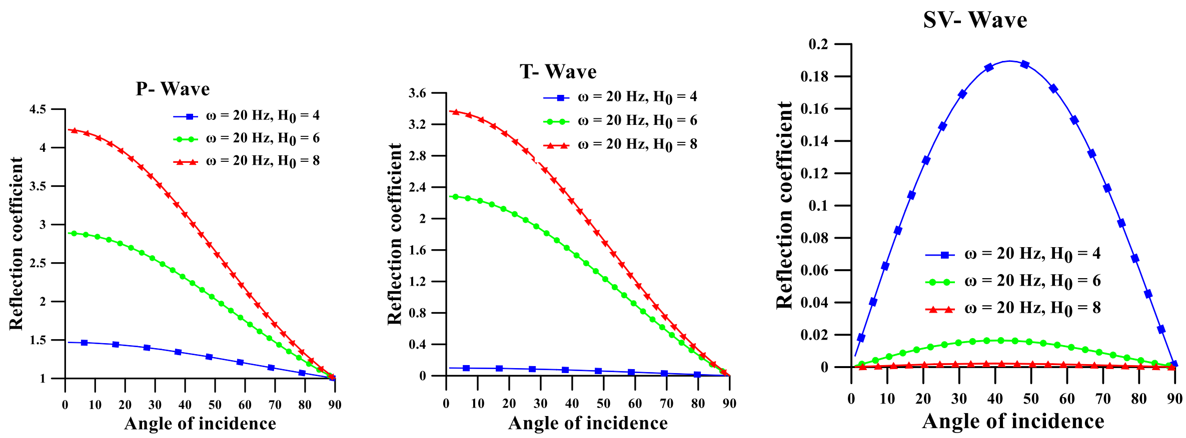
C_E = 0.3831× 10^3 J. Kg^{-1}. K^{-1}., K = 0.300× 10^3 N. K^{-1}. s^{-1}, P_0 = 1.0 × 10^{10} N.m^{-2},

$\alpha_t=1.78\times10^{-5}K^{-1}$, $\beta=0.5$ $H_0=6.0\times10^5 Oe$, $\mu_e=1\times10^{-7}Hm^{-1}$,

$\tau_1=0.02s,\tau_2=0.03s,\tau_3=0.01s,$ $\omega=20Hz,$

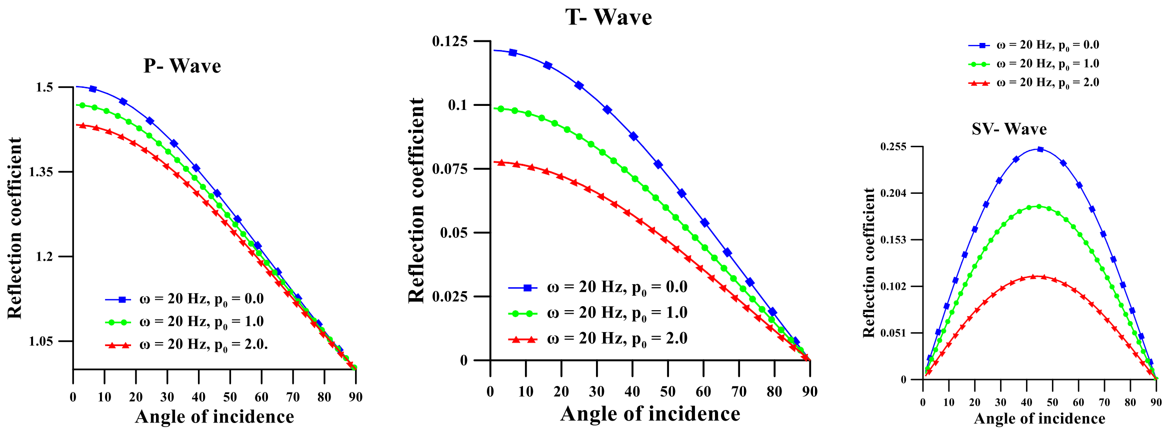
For the above data, reflection coefficients $|Z_1|, |Z_2|$ and $|Z_3|$ from the system of Eq. (31) of reflected P, T, and SV waves versus incident angle of P wave $0^0 < \theta_0 < 90^0$ are computed numerically by using a Fortran program for thermal insulation impedance boundary surface. The alterations in reflection coefficients due to various magnetic factors H_0, initial stress parameter P_0, MMD parameters a, b, displayed visually in figures 2(a) - 2(c) to 4(a) -4(c). The dependence of the constant of reflection versus impedance parameter $\xi_1 = -10 < \xi_1 < 10$, at different three values of impedance parameter $\xi_2 = 0, 5, 10$ is displayed visually in figures 5(a) - 5(c).

The effect of magnetic field on the amplitude ratio $|Z_1|$ of reflected P wave for different three values of magnetic parameter $H_0 = 4\times10^5 Oe, 6\times10^5 Oe, 8\times10^5 Oe$ by blue colour curve with solid square, green colour curve with solid circle and red colour curve with solid triangle respectively when $\mathrm{a} = 0, \mathrm{b} = 0.5$, $\xi_1 = 5, \xi_3 = 5$, $P_0 = 1$, $\tau_1 = 0.02$, $\tau_2 = 0.03, \tau_3 = 0.01$, $\omega = 20\,\mathrm{Hz}$. are shown graphically in figures 2(a) - 2(c). The value of reflection coefficient for P, T, wave decreases to minimum value one as angle of incidence changes from $0^0 < \theta_0 < 90^0$ while that of SV waves first value increases to its greatest value, then drops to its minimum value as angle of incidence changes from $0^0 < \theta_0 < 90^0$.



(a) **(b)** **(c)**

Figures 2(a) – 2(c). Dependence of reflected wave's coefficient of the reflection of P, T, and SV waves versus incident angle of P wave at distinct values of $H_0 = 4, 6, 8$.

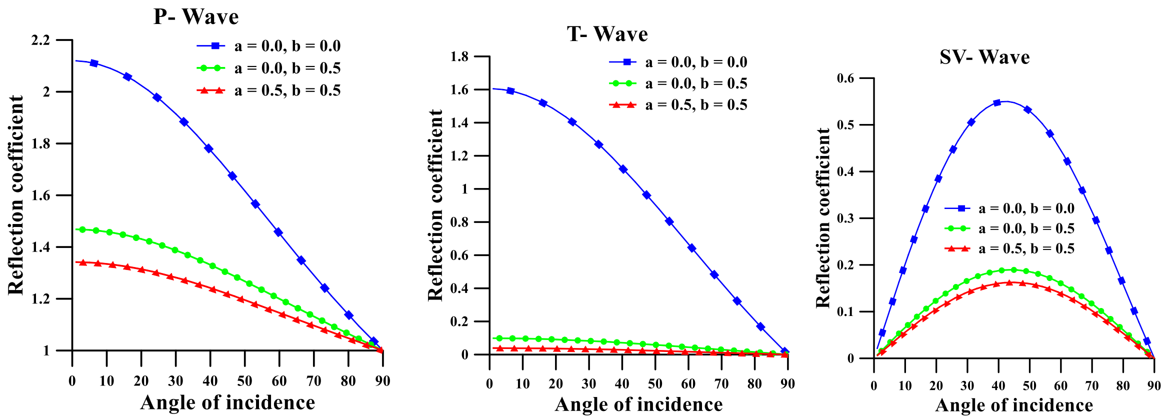


(a) **(b)** **(c)**

Figures 3(a) – 3(c). Dependence of reflected wave's coefficient of the reflection of P, T, and SV waves versus incident angle of P wave at distinct values of initial stress $P_0 = (0.0, 1.0, 2.0) \times 10^{10}\,\mathrm{N.m^{-2}}$.

Similar the dependence of reflected wave's coefficient of the reflection of P, T, and SV waves against angle of incidence of P wave at different values of initial stress $P_0 = (0.0, 1.0, 2.0) \times 10^{10}\,\mathrm{N.m^{-2}}$, when $\mathrm{a} = 0, \mathrm{b} = 0.5, \xi_1 = 5, \xi_3 = 5, H_0 = 4,$ $\tau_1 = 0.02, \tau_2 = 0.03, \tau_3 = 0.01,$ $\omega = 20\,\mathrm{Hz}$ is observed as shown in figures 3(a) - 3(c). It is observed that as pre-stress increases the value of reflection coefficients $|Z_1|$ of reflected P inflate while that of T, and SV waves decreases as angle of incidence changes from $0^0 < \theta_0 < 90^0$.

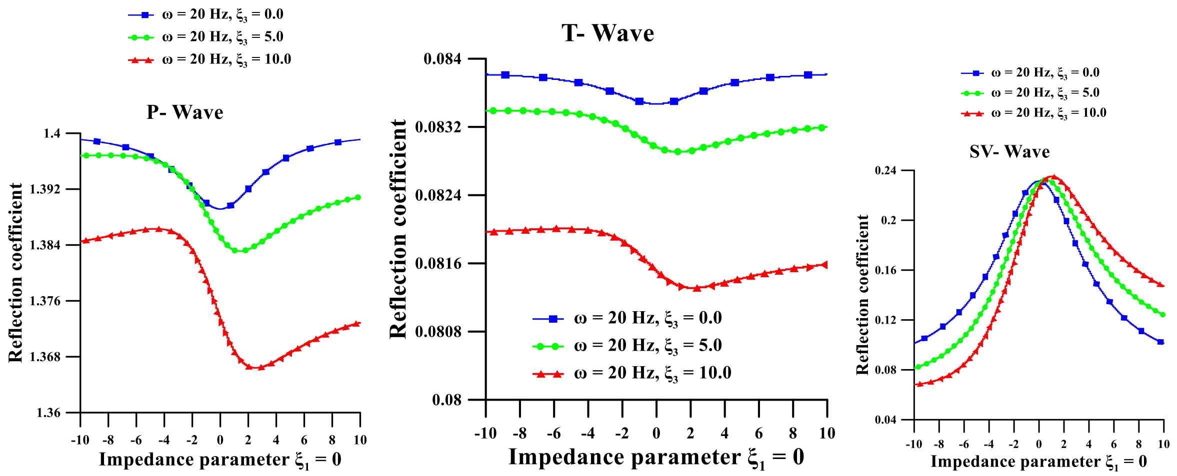
The impact of memory dependent derivative parameter $(\mathrm{a} = 0, \mathrm{b} = 0)$, $(\mathrm{a} = 0, \mathrm{b} = 0.5)$, $(\mathrm{a} = 0.5, \mathrm{b} = 0.5)$, at $\xi_1 = 5,$ $\xi_3 = 5,$ $H_0 = 4,$ $P_0 = 1,$ is shown in figures 3(a) - 3(c) for time delay $\tau_1 = 0.02,$ $\tau_2 = 0.03,$ $\tau_3 = 0.01$. The dependence of reflected wave's coefficient of the reflection of P, T, and SV waves versus incident angle of P wave at distinct of kernel function $k(t-\xi) = 1,$ for $a = 0, b = 0,$ $k(t-\xi) = 1 - \dfrac{(t-\xi)}{\tau_0}$, for $a = 0, b = 0.5$ and $k(t-\xi) = \left(\dfrac{2\tau_0 - (t-\xi)}{2\tau_0}\right)^2$, for $a = 0.5, b = 0.5,$ is studied. It is observed that as memory dependent derivative parameters (a, b) increases the value of reflection coefficients $|Z_1|$ of reflected P decreases while that of T, and SV waves increases as angle of incidence changes from $0^0 < \theta_0 < 90^0$.



(a) **(b)** **(c)**

Figures 4(a) – 4(c). Fluctuation of reflected wave's coefficient of the reflection of P, T, and SV waves versus incident angle of P wave at distinct values of memory dependent derivative parameter $(a=0, b=0)$, $(a=0, b=0.5)$, $(a=0.5, b=0.5)$,

The change of the reflection coefficient of reflected P, T, and SV waves versus impedance parameter $\xi_1 = -10 < \xi_1 < 10$, at different values of impedance parameter $\xi_2 = 0, 5, 10$ at an



(a) **(b)** **(c)**

Figures 5(a) – 5(c). Dependence of the reflection coefficient of reflected P, T, and SV waves against impedance parameter $\xi_1 = -10 < \xi_1 < 10$, at different values of impedance parameter $\xi_2 = 0, 5, 10$.

angle of incidence $\theta_0 = 30^0$, when $H_0 = 4$, $P_0 = 1$, $\tau_1 = 0.02$, $\tau_2 = 0.03$, $\tau_3 = 0.01$, $\omega = 20\,\text{Hz}$ is shown in figures 5(a) - 5(c).

5. Conclusions

The reflection coefficients $|Z_1|,|Z_2|$ and $|Z_3|$ of reflected P, T, and SV waves are significantly affected by magnetic parameter, initial stress parameter, memory dependent derivative parameter $\mathrm{a,b}$, and impedance parameter versus incident angle of P wave $0^0 < \theta_0 < 90^0$.

1) It is observed that magnetic field parameter $H_0 = 4,6,8$ increases, the value of reflection coefficients $|Z_1|,|Z_2|$ of reflected P and T waves increases while $|Z_3|$ that of SV waves decreases as angle of incidence changes from $0^0 < \theta_0 < 90^0$.

2) It is observed that as initial stress $P_0 = (0.0,1.0,2.0)\times 10^{10}\,\mathrm{N.m^{-2}}$, increases the value of reflection coefficients $|Z_1|,|Z_2|$ and $|Z_3|$ of reflected P T, and SV waves decreases as angle of incidence changes from $0^0 < \theta_0 < 90^0$.

3) It is observed that as memory dependent derivative parameters (a,b) increases from $(\mathrm{a}=0,\mathrm{b}=0)$, $(\mathrm{a}=0,\mathrm{b}=0.5)$, $(\mathrm{a}=0.5,\mathrm{b}=0.5)$, the value of reflection coefficients $|Z_1|,|Z_2|$ and $|Z_3|$ of reflected P T, and SV waves decreases as angle of incidence changes from $0^0 < \theta_0 < 90^0$.

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