Hybrid gls-ridge Modelling in the Presence of Multicollinearity and Autocorrelation Phenomena.

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Abstract

Multicollinearity and autocorrelation are two common issues in multiple regression analysis that have a negative impact on estimation and prediction. The presence of multicollinearity causes estimation instability and model misspecification, whereas the presence of autocorrelation causes underestimation of the estimator's variance and inefficient prediction. The Generalised Least Squares (GLS) regression model was developed to solve the problem of autocorrelation, whereas the ridge regression model was developed to solve the problem of multicollinearity. However, according to the literature review, no method has solved these two problems in a model at the same time. As a result, this study employs a hybrid Generalized Least Squares-Ridge (GLS-R) regression model to solve problems caused by multicollinearity and autocorrelation in a single model. Data on GDP, inflation rate, exchange rate, and money supply were obtained from the Central Bank of Nigeria's (CBN) Statistical Bulletin and used for analysis from 1981 to 2020. The regression coefficients for each estimator were estimated, and the model with the lowest Mean Square Error (MSE) and Akaike Information Criteria was chosen (AIC). The AIC of GLS-R regression techniques was lower than the MSE and AIC of Least Squares, Ridge regression, and Lasso regression techniques. When there is a problem of multicollinearity and autocorrelation simultaneously, the GLS-R regression techniques outperform the Least Squares, Ridge, and Lasso regression techniques, implying that the GLS-R regression model is preferred over the Least Squares, Ridge, and Lasso regression models.

Keywords: Autocorrelation, Lasso, Monte Carlo simulation, Multicollinearity, Ridge regression

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1.1 Introduction

Multicollinearity and Autocorrelation are two very common problems in regression analysis. The term multicollinearity is used to denote the presence of linear relationships among explanatory variables [1]. This is a clear violation of the Ordinary Least Squares (OLS) assumption that state that the explanatory variables are not perfectly linearly correlated [2], 2002). In the view of Agunbiade [3], he emphasized that multicollinearity is matter of degree that is inherent in any dataset. Multicollinearity may arise for various reasons. Firstly, there is the tendency of economic variables to move along over a period of time. Secondly, the use of lagged values of some explanatory variables as separate independent factors in the relationship. With strongly interrelated regressors, the regression coefficients provided by the OLS estimator are no longer stable even when they are still unbiased as long as multicollinearity is not perfect. Furthermore, the regression coefficient may have large sampling errors which affect both the inference and forecasting that is based on the model. Various other estimation methods have been developed to tackle this problem. This estimation include Ridge regression estimator developed by Hoerl [4] and Hoerl and Kennard [5], estimator based on principal component regression suggested by Massy [6], Marquardt [7] and Bock, *et al*. [8], Naes and Marten [9] and method of partial least squares developed by Hermon Wold in the 1960's, Holland [10], Phatak and Jony [11], Ayinde [12] and Alhassan *et al.* [13].

As its well-known, the presence of some degree of multicollinearity results in estimation instability and model mis-specification while the presence of serial correlated errors leads to underestimation of the variance of parameter estimates and inefficient prediction. The adverse effect of these two phenomena on estimation and prediction of a model is yet to be addressed collectively. Though several efforts were made in the literature addressing individually. Thus, this study explores the predictive ability of the hybrid GLS-Ridge regression on multicollinearity and autocorrelation problems simultaneously, using real life dataset.

Data on GDP, inflation rate, exchange rate, and money supply were obtained from the Central Bank of Nigeria's (CBN) Statistical Bulletin and used for analysis from 1981 to 2020.

2.0 Literature Review

Chopra et al. [14] used two types of regression techniques: traditional regression and RR. The traditional method of forecasting the compressive strength of concrete did not prove to be reliable. When performing RR, there was a frequent minimum effect that had no or negligible impact on the coefficients.

Muniz et al. [15] reviewed 19 existing estimators and proposed new ones for ridge parameter estimation. Monte Carlo simulations were used to conduct the investigation. In all cases, the suggested estimators produced lower MSE values than OLS regression.

Zaka and Akhter [16] investigated the Relative Least Squares Method (RLSM), an RR method parameter power function distribution, in their study. Based on different sample sizes and parameter values, this study determined the best estimation method.

Cule and DeIorio [17] investigated the use of RR, a common penalised regression method. The motivation was the quandary of out-of-sample prediction. The data were high-density genotype data from a re-sequencing study or from a genome-wide association study.

For linear limitations binding regression coefficients that are stochastic, Chandra and Sarkar [18] proposed a new estimator known as the restricted r-k class estimator. The proposed estimator's dominance over the other two estimators was investigated in this study.

Khalaf [19] used the MSE to evaluate two proposed RR parameters. The results showed that the proposed estimators outperformed the OLS and other estimators. There is still no agreement on the best or most general method for determining k.

El-Dereny and Rashwan [20] investigated the problem of multicollinearity. They created a variety of RR models based on standard deviation, MSE, and R2 for each model's estimators. The RR models were then compared to OLS methods based on data simulations (2000 iterations).

Yanagihara [21] considered optimising the ridge parameters in GRR. Because GRR can solve the minimization problem for one model selection measure, it has a significant advantage over RR. Such a solution for any model selection measure (for example, GCV, CV, or the C p measure) cannot be obtained explicitly with RR.

Agunbiade [3] investigated the effects of multicollinearity on the exogenous variables of a simultaneous equation model in a study. To accomplish this, a Monte Carlo approach was used, which involved creating a two- equation with five structural parameters and employing six different estimation techniques.

Agunbiade and Iyaniwura [22] investigated estimation in the presence of multicollinearity. Their findings revealed that the estimates for three estimators: LIML, 2SLS, and ILS are nearly identical. The three levels of multicolliearity considered have an uneven impact on the performance of the other categories.

Alhassan et al. [13] investigated the effects and consequences of multicollinearity in multiple regression on both standard error and explanatory variables. The correlation between X1 and X6 (independent variables) measures their individual effect and performance on Y (response variable), and it is carefully observed how those explanatory variables intercorrelated with each other and with the response variable.

Ayinde et al. [23] investigated the effect of multicollinearity and autocorrelation on the predictive ability of some linear regression model estimators. In linear regression, violations of the assumptions of independent regressors and error terms have resulted in multicolinearity and autocorrlation problems. The results show that the performance patterns of COR, ML, OLS, and PC estimators are generally and clearly convex. At higher levels of accommodation, the COR and ML estimators perform similarly and even better.

Lukman et al. [24] propose a robust ridge regression estimator for simultaneously addressing multicollinearity and outliers in a classical linear regression model. The estimator technique necessitates the use of robust estimators (M, MM, S, LTS, LAD, LMS) rather than the Ordinary Least Squares (OLS) estimator to estimate the ridge parameter.

In the presence of multicollinearity, Oyeyemi et al. [25] present the performance of LASSO-type estimators. A 1000-trial Monte Carlo experiment with various sample sizes n was conducted (50,100 and 150). The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) criteria are used to compare the results.

Ayinde et al. [26] investigated the performance of the Ordinary Least Square (OLS) estimator, Cochrane-Orcutt (COR), Maximum Likelihood (ML), and Principal Component (PC) analysis in predicting linear regression models when the assumptions of non-stochastic regressors, independent regressors, and error terms were violated. The research also identifies the best estimator that can be used for prediction.

3.0 Materials and Methods

3.1 Linear Regression

Linear regression allows us to study the relationship between k explanatory variables, $X_1, \ldots, X_k$, and a continuous response variable, Y.

Let us assume that a linear relationship exists between a variable Y and $k$explanatory variables $X_1, X_2, X_3, \ldots, X_K$ and a disturbance term,e. If we have a sample of n-observation on Y and the X's, we can write

$$Y_i = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki} + e_i i = 1,2,\ldots,n \quad (1)$$

The beta coefficients and the parameter of the e distribution are known and our problem is to obtain estimates of these unknowns, where;

β_0 = intercept.

$\beta_1, \ldots, \beta_k$ = partial slope coefficients.

e=stochastic disturbance term and i^{th} observations being the size of the population.

Equation (1) is interpreted: It gives the mean or expected value of Y conditional upon the fixed (in repeated sampling) values of X_1, X_2, . . . ,X_K, that is:

$$E(Y/X_{1i}, X_{2i}, \ldots, X_{ki})$$

The n-equation in (1) is written below;

$$Y_1 = \beta_0 + \beta_1 X_{11} + \cdots + \beta_k X_{k1} + e_1$$

$$Y_2 = \beta_0 + \beta_1 X_{12} + \cdots + \beta_k X_{k2} + e_2 \quad (2)$$

$$\vdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$Y_n = \beta_0 + \beta_1 X_{1n} + \cdots + \beta_k X_{kn} + e_n$$

Alternatively, in matrix form;

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & \cdots & X_{k1} \\ 1 & X_{12} & \cdots & X_{k1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & X_{1n} & \cdots & X_{kn} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \quad (3)$$

$n \times 1 \qquad n \times (k+1) \quad (k+1) \times 1 \quad n \times 1$

$Y \quad = \qquad X\beta \qquad + \qquad e$

Where;

$Y = n \times 1$, column vector of observation on the dependent Y.

$X = n \times (k+1)$ matrix giving n observations on $k+1$ variables X_1 to X_K, the first column of 1's representing the intercept term (this matrix is also known as data matrix).

$\beta = (k+1) \times 1$ column vector of the unknown parameters $\beta_0, \beta_{1,} \ldots, \beta_k$.

$e = n \times 1$ column vector of n disturbances, e_i.

The n-equations (2) can be set out compactly in matrix notation as

$Y = X\beta + e$ (4)

Our objective is to estimate the parameters of the multiple regression (1) and to draw inferences about them from the data at hand. In matrix notation, this amounts to estimating β and drawing inferences about this β. For the purpose of estimation, we may use the method of ordinary least squares (OLS).

3.2 Ridge Regression

Ridge regression solves the multicollinearity problem through shrinkage parameter k.

Ridge regression model can be written as the following:

$$\hat{Y} *= \hat{\beta}_{R_0} + \hat{\beta}_{R_1}X_1 + \hat{\beta}_{R_2}X_2 + \hat{\beta}_{R_3}X_3 + \cdots + \hat{\beta}_{R_{p-1}}X_{p-1} \qquad (5)$$

Where $\hat{\beta}_{R_{p-1}}$: Ridge regression parameter estimate, X_{p-1}: Standardized predictor variable. Besides, Ridge regression can be formulated through matrix with the purpose of minimizing the sum square of error, $\hat{\varepsilon} = Y * X\hat{\beta}_R$.

The Lagrange multiplier is used for minimizing the sum square of error, $\hat{\varepsilon}'\hat{\varepsilon} = (Y * X\hat{\beta}_R)'\left(Y * X\hat{\beta}_R\right)$ with constraint of $\hat{\beta}_R{}'\hat{\beta}_R = q^2, \hat{\beta}_R{}'\hat{\beta}_R - q^2 = 0$. Then parameter estimate of Ridge regression model, with bias constant involved, can be written as

$$\hat{\beta}_R(k) = (X'X + kI)^{-1}(X'Y *), 0 \leq k \leq 1 \qquad (6)$$

where: $\hat{\beta}_R$: vector of Ridge regression estimates, with dimension of $(p \times 1)$; k: bias constant, Z: matrix of standardized predictor variables $(n \times p), I$: identity matrix with ordo p, and $Y *$:vector of standardized response variable $(n \times 1)$.

Eq. (5) can be obtained thus,

$$PRSS(\beta)\ell_2 = \sum_{i=1}^{n}(y_i - x_i^T\beta)^2 + k\sum_{j=1}^{p}\beta_j^2 \qquad (7) \qquad = (y - X\beta)^T(y - X\beta) + \; k||\beta||_j^2 \qquad (8)$$

Its solution may have smaller average PE than β^{ls}

$PRSS(\beta)\ell_2$ is convex and hence has a unique solution

Taking derivatives, we obtain:

$\frac{PRSS(\beta)\ell_2}{\partial\beta} = -2X^T(\mathrm{y} - X\beta) + 2\mathrm{k}\beta$ (9)

$\partial PRSS(\beta)\ell_2$= --2$X^T$(y - $X\beta$)+ 2$k\beta$ (10)

The solution to PRSS ($\beta\ell_2$) is now seen to be:

$\hat{\beta}_k^{ridge} = \left(X^TX + kI_p\right)^{-1}X^Ty$ (10)

k is the shrinkage parameter. k controls the size of the coefficients. k controls amount of regularization.

As $k \downarrow 0$, we obtain the least squares solution. As $k \uparrow \infty$, we have $\beta_{k=\infty}^{ridge} = 0$ (intercept only model). The $k's$ trace out a set of ridge solutions.

3.3 Least Absolute Selection and Shrinkage Operator

The least absolute selection and shrinkage operator (LASSO) is another method used to obtain sparse models. The estimate $\breve{\beta}$ˆ is found by minimizing the L1-penalized least squares criterion, $(Y - X\beta)'(Y - X\beta) + kPp|\beta j|$ for a given value of k. The parameter k is a penalty which prevents the coefficient estimates from growing to their full OLS estimates and is chosen through cross-validation. Using this criterion results in parameter estimates that are shrunk relative to the OLS solutions. Some estimates are shrunk to 0, resulting in a sparser model. It is therefore used both as a shrinkage estimator and as a variable selection technique.

The LASSO estimator uses the penalized least squares criterion to obtain a sparse solution to the following optimization problem:

$\hat{\beta}(lasso) = {}^{argmin}_{\beta} \|y - X\beta\|_2^2 + k\|\beta\|_1$ (24)

Where $\|\beta\|_1 = \sum_i^p |\beta_j|$ is the ℓ_1-norm penalty on β, which induces sparsity in the solution, and ≥ 0 is a tuning parameter.

The ridge estimator minimizes the *ridge loss function*, which is defined as:

$\mathcal{L}_{ridge}(\beta; k) = \|Y - X\beta\|_2^2 + k\|\beta\|_2^2$(25)

$= \sum_{i=1}^n (Y_i - X_{i*}\beta)^2 + k\sum_{j=1}^p \beta_j^2$ (26)

To verify that the ridge estimator indeed minimizes the ridge loss function, proceed as usual. Take the derivative with respect to β:

$$\frac{\partial}{\partial\beta}\mathcal{L}_{ridge}(\beta; k) = -2X'(Y - X\beta) + 2kI_{p\times p}\beta$$

$$= -2X'Y + 2\left(X'X + kI_{p\times p}\right)\beta \qquad (27)$$

Equate the derivative to zero and solve for β. This yields the ridge regression estimator.

The ridge estimator is thus a stationary point of the ridge loss function. A stationary point corresponds to a minimum if the Hessian matrix with second order partial derivatives is positive definite. The Hessian of the ridge loss function is

$$\frac{\partial^2}{\partial\beta\partial\beta'}\mathcal{L}_{ridge}(\beta;k) = 2\left(X'X + kI_{p\times p}\right) \qquad (28)$$

This Hessian is the sum of the semi-positive definite matrix $X'X$ and the positive definite matrix $kI_{p\times p}$. Harville (2008) then states that the sum of these matrices is itself a positive definite matrix. Hence, the Hessian is positive definite and the ridge loss function has a stationary point at the ridge estimator, which is a minimum.

3.5 Hybrid Generalised Least Squares-Ridge Regression Model

The hybrid regression model can be written as:

$$\hat{Y} \circledast = \hat{\beta}_{RGLS_0} + \hat{\beta}_{RGLS_1}X_1 + \hat{\beta}_{RGLS_2}X_2 + \hat{\beta}_{RGLS_3}X_3 + \cdots + \hat{\beta}_{RGLS\,p-1}X_{p-1} \qquad (32)$$

where,

$\hat{\beta}_{RGLS_{p-1}}$: Ridge-GLS regression parameter estimate,

X_{p-1}: Standardized predictor variable.

Consider a general linear regression model (1) with errors satisfying relation and the regressors exhibiting near multicollinearity. As seen earlier, in case of autocorrelation. Hence autocorrelation is a particular case of heteroscedasticity. In the case of heteroscedasticity, GLS is an appropriate method of estimation as given in

$$\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y. \qquad (33)$$

Further, when there is multicollinearity, often used method is the ridge regression as mentioned in (25).

$$\hat{\beta}_k^{ridge} = \left(X^TX + kI_p\right)^{-1}X^Ty$$

Combining these two methods, we propose for the autocorrelated model with multicollinearity a generalized ridge type estimator represented as

$$\hat{\beta}_{RGLS} = (X'\Omega^{-1}X + kI)^{-1}X'\Omega^{-1}Y. \quad (34)$$

Hence the model under consideration contains the unknown parameters k,ρ, σ^2 and β.

4.0 Results and Discussions

This section presents the result of the analysis carried out. Data on Gross Domestic Product (GDP), inflation rate (INF), exchange rate (EXG), and money supply (MS) were obtained from the Central Bank of Nigeria's (CBN) Statistical Bulletin and used for analysis from 1981 to 2020.

Table 1 Correlation Coefficients between Variables

	INF	EXG	MS	GDP
INF	1.0000	-0.1708	0.9962	0.0472
EXG	-0.1708	1.0000	-0.1682	0.2831
MS	0.9962	-0.1682	1.0000	0.0432
GDP	0.0472	0.2831	0.0432	1.0000

Table 2: Test for Normality

Test Name	Test Statistic to Test H0: Normal	Prob Level	Reject H0 at 20%?
Shapiro Wilk	0.974	0.4842	No
Anderson Darling	0.306	0.5657	No
D'Agostino Skewness	0.336	0.7372	No
D'Agostino Kurtosis	-0.989	0.3229	No
D'Agostino Omnibus	1.090	0.5799	No

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Table 3: Least Squares, Ridge, Lasso Regression and Hybrid Regression Coefficients

	Least Squares		Ridge Regression		Lasso Regression		Proposed Regression	
	$\bar{\beta}_i$	S_{β_i}	$\bar{\beta}_i$	S_{β_i}	$\bar{\beta}_i$	S_{β_i}	$\bar{\beta}_i$	S_{β_i}
Intercept	0.4194	0.4916	0.3783		0.3411		0.3681	0.5588
$\boldsymbol{INF}$	2.0459	5.5002	0.9585	2.3723	0.1429	0.2420	0.9348	0.2315
$\boldsymbol{EXC}$	1.0041	0.5375	0.9956	0.5352	0.9984	0.5382	0.9556	0.5110
$\boldsymbol{MS}$	-0.1752	0.5483	-0.0669	0.2365	0.0145	0.0241	-0.0607	0.02327
MSE	**1.7694**		**1.0480**		**0.2681**		**0.2311**	
AIC	**0.7493**		**0.6699**		**0.1874**		**0.1751**	

5.0 Conclusion

It has been shown that in the presence of multicollinearity with sufficient high degrees of autocorrelation. The OLS estimates of regression coeffcients can be highly inaccurate. Improving the estimation procedure is obviously necessary. Combining GLS and Ridge regression, we derived an estimator.

$$\tilde{\beta}_{GR}(k) = (X'\Omega^{-1}X + kI)^{-1}X'\Omega^{-1}Y$$

where $0 \leq k \leq 1$. $\tilde{\beta}_{GR}(k)$, though biased, is expected to perform well in the joint presence of multicollinearity and autocorrelation. However, since Ω is unknown, parameter estimates based on the biased estimator $\tilde{\beta}_{GR}(k)$ cannot be obtained in practice. Therefore, we combined Durbin's two-step method with ordinary Ridge regression to approximate those parametrs. The effectiveness of our approximation can then best be examined by the Monte Carlo simulation.

The study has revealed that the hybrid regression technique provides the preferred estimator in estimating all the parameters of the model based on the criteria used namely; Mean Square Error and Akaike Information Criterion (AIC).

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