Weakly $\tilde{\alpha}$-Continuous Functions

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Article Info
Page Number: 6654 - 6659
Publication Issue:
Vol 71 No. 4 (2022)

Article History
Article Received*: 15 September 2022*
Revised*: 25 October 2022*
Accepted*: 14 November 2022*
Publication*: 21 December 2022*

Abstract

In this paper, we introduce two new class of generalized closed sets called $\tilde{\alpha}$-closed and weakly $\tilde{\alpha}$-closed sets. Also, we investigate their relationships with others generalized closed sets.

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1. INTRODUCTION

Levine introduced generalized closed sets in general topology as a generalization ofclosed sets. Sheik John introduced a study on generalization of closed sets and continuous maps in topological space. Sundaram and Nagaveni introduced the concept of weakly generalized continuous maps, weakly generalized closed maps and weakly generalized irresolute maps in topological spaces.

In this paper, we introduce a new class of generalized continuous functions called weakly $\tilde{\alpha}$-closed continuous functions and study the properties of the class of functions.

Throughout this paper (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent topological spaces (briefly, **TPS**) on which no separation axioms are assumed unless otherwise mentioned. For a subset N of a space X, cl(N), int(N) and N^c or X| N or X − N denote the closure of N, the interior of N and the complement of N, respectively.

2. PRELIMINARIES

Definition 2.1

A subset *N* of a **TPS** is called

(i) $\tilde{\alpha}$-closed (briefly $\tilde{\alpha}$-cld) if cl(N) ⊆ **B** whenever N ⊆ **B** and **B** is sg-open in X.

(ii) weakly $\tilde{\alpha}$-closed (briefly w$\tilde{\alpha}$-cld) if cl(int(**N**)) $\subseteq$ **B** whenever

N $\subseteq$ **B** and **B** is sg-open in X.

The complements of the above mentioned closed sets are called their respective open sets.

Theorem 2.2

If a subset **N** of a **TPS** X is both open and w$\tilde{\alpha}$-cld, then it is closed.

Corollary 2.3

If a subset **N** of a **TPS** X is both open and w$\tilde{\alpha}$-cld, then it is both regular open and regular cld in X.

Definition 2.4

A subset **N** of a **TPS** X is called w$\tilde{\alpha}$-open if $\mathbf{N}^c$ is w$\tilde{\alpha}$-cld in X.

Theorem 2.5

Any open set is w$\tilde{\alpha}$-open.

Example 2.6

Let X = $\{w_1, w_2, w_3\}$ and $\tau = \{\phi, \{w_1\}, \{w_2\}, \{w_1, w_2\}, X\}$. The set $\{w_3\}$ is w$\tilde{\alpha}$-open set but it is not open in X.

Proposition 2.7

(i) Any w$\tilde{\alpha}$-open set is w$\tilde{\alpha}$-open but the reverse is not true.

(ii) Any regular open set is w$\tilde{\alpha}$-open but the reverse is not true.

(iii) Any g-open set is w$\tilde{\alpha}$-open but the reverse is not true.

(iv) Any w$\tilde{\alpha}$-open set is gsp-open but the reverse is not true.

It can be shown that the converse of (i), (ii), (iii) and (iv) need not be true.

3. WEAKLY $\tilde{\alpha}$-CONTINUOUS FUNCTIONS

Definition 3.1

Let X and Y be **TPS**. A function f : X $\rightarrow$ Y is called

(i) weakly $\tilde{\alpha}$-continuous (briefly w$\tilde{\alpha}$-continuous) if $f^{-1}(U)$ is a w$\tilde{\alpha}$-open in X, for each open U in Y.

(ii) $\tilde{\alpha}$-continuous (briefly $\tilde{\alpha}$-continuous) if $f^{-1}(U)$ is a $\tilde{\alpha}$-open in X, for each open U in Y.

Example 3.2

Let X = Y = $\{i_1, s_1, d_1\}$, $\tau = \{\phi, \{i_1\}, \{s_1, d_1\}, X\}$ and $\sigma = \{\phi, \{i_1\}, Y\}$. The identity function f :

$(X, \tau) \rightarrow (Y, \sigma)$ is w$\tilde{\alpha}$-continuous, because any subset of Y is w$\tilde{\alpha}$-cld in X.

Proposition 3.3
Any continuous function is w$\tilde{\alpha}$-continuous.

Proof
It follows from Theorem 2.5.

The converse of Proposition 3.3 need not be true as seen in the following example.

Example 3.4
Let $X = Y = \{i_1, s_1, d_1\}$, $\tau = \{\phi, \{i_1\}, \{s_1, d_1\}, X\}$ and $\sigma = \{\phi, \{s_1\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is w$\tilde{\alpha}$- continuous but $f^{-1}(\{s_1\}) = \{s_1\}$, which is not open in X. So f is not continuous.

Theorem 3.5
Any $\tilde{\alpha}$-continuous function is w$\tilde{\alpha}$-continuous.

Proof
It follows from Proposition 2.7 (i).
The converse of Theorem 3.5 need not be true as seen in the following example.

Example 3.6
In the Example 3.4, $\tilde{\alpha}$-cld in X is $\{\phi, \{i\}, \{s, d\}, X\}$ and the function f is w$\tilde{\alpha}$-continuous but $f^{-1}(s) = s$ which is not $\tilde{\alpha}$-open in X. So f is not $\tilde{\alpha}$-continuous.

Theorem 3.7
Any completely continuous function is w$\tilde{\alpha}$-continuous.

Proof
It follows from Proposition 2.7 (ii).

Remark 3.8
The converse of Theorem 3.7 need not be true in general. The function f in the Example 3.4 is w$\tilde{\alpha}$-continuous but not completely continuous.

Theorem 3.9
A function $f : X \rightarrow Y$ is w$\tilde{\alpha}$-continuous if and only if $f^{-1}(U)$ is a w$\tilde{\alpha}$-cld in X, for each closed set U in Y.

Proof
Let U be any closed in Y. According to the assumption $f^{-1}(U^c) = X \setminus f^{-1}(U)$ is w$\tilde{\alpha}$-open in X, so $f^{-1}(U)$ is w$\tilde{\alpha}$-cld in X.
The converse can be proved in a similar manner.

Proposition 3.10

If f : X $\rightarrow$ Y is perfectly continuous and w$\tilde{\alpha}$-continuous, then it is R-map.

Proof

Let V be any regular open subset of Y. According to the assumption, $f^{-1}(V)$ is both open and closed in X. Since $f^{-1}(V)$ is closed, it is w$\tilde{\alpha}$-cld. We have $f^{-1}(V)$ is both open and w$\tilde{\alpha}$-cld. Hence, by Corollary 2.3, it is regular open in X, so f is R-map.

Definition 3.11

A **TPS** X is called weakly $\tilde{\alpha}$-compact (briefly w$\tilde{\alpha}$-compact) if any w$\tilde{\alpha}$- open cover of X has a finite subcover.

Theorem 3.12

Let f : X $\rightarrow$ Y be surjective w$\tilde{\alpha}$-continuous function. If X is w$\tilde{\alpha}$- compact, then Y is compact.

Proof

Let $\{ N_i : i \in I\}$ be an open cover of Y. Then $\{f^{-1}(N_i) : i \in I\}$ is aw$\tilde{\alpha}$-open cover of X. Since X is w$\tilde{\alpha}$-compact, it has a finite subcover, say
$\{f^{-1}(N_1), f^{-1}(N_2),\ldots, f^{-1}(N_n)\}$. Since f is surjective $\{ N_1, N_2, \ldots, N_n\}$is a finite subcover of Y and hence Y is compact.

Definition 3.13

A **TPS** X is called weakly $\tilde{\alpha}$-connected (briefly w$\tilde{\alpha}$-connected) if X cannot be written as the disjoint union of two non-empty w$\tilde{\alpha}$-open sets.

Theorem 3.14

For a **TPS** X the following statements are equivalent:

(i) X is w$\tilde{\alpha}$-connected.

(ii) The empty set ϕ and X are only subsets which are both w$\tilde{\alpha}$-open and w$\tilde{\alpha}$-cld.

(iii) Each w$\tilde{\alpha}$-continuous function from X into a discrete space Y which has at least two points is a constant function.

Proof

(i) $\Rightarrow$ (ii). Let $N \subset X$ be any proper subset, which is both w$\tilde{\alpha}$-open and w$\tilde{\alpha}$-cld. Its complement $X \setminus N$ is also w$\tilde{\alpha}$-open and w$\tilde{\alpha}$-cld. Then $X = N \cup (X \setminus N)$ is a disjoint union of two non-empty w$\tilde{\alpha}$-open sets which is a contradiction with the fact that X is w$\tilde{\alpha}$-connected. Hence, $N = \phi$ or X.

(ii) $\Rightarrow$ (i). Let $X = A \cup B$ where $A \cap B = \phi$, $A \neq \phi$, $B \neq \phi$ and A, B arew$\tilde{\alpha}$-open. Since $A = X \setminus B$, A is w$\tilde{\alpha}$-cld. According to the assumption A =

ϕ, which is a contradiction.

(ii) $\Rightarrow$ (iii). Let $f : X \to Y$ be a $w\tilde{\alpha}$-continuous function where Y is a discrete space with at least two points. Then $f^{-1}(\{y\})$ is $w\tilde{\alpha}$-cld and $w\tilde{\alpha}$- open for each $y \in Y$ and $X = \cup \{f^{-1}(\{y\}) \mid y \in Y\}$. According to the assumption, $f^{-1}(\{y\}) = \phi$ or $f^{-1}(\{y\}) = X$. If $f^{-1}(\{y\}) = \phi$ for all $y \in Y$, f will not be a function. Also there is no exist more than one $y \in Y$ such that $f^{-1}(\{y\}) = X$. Hence, there exists only one $y \in Y$ such that $f^{-1}(\{y\}) = X$ and $f^{-1}(\{y_1\}) = \phi$ where $y \neq y_1 \in Y$. This shows that f is a constant function.

(iii) $\Rightarrow$ (ii). Let $N \neq \phi$ be both $w\tilde{\alpha}$-open and $w\tilde{\alpha}$-cld in X. Let $f : X \to Y$ be a $w\tilde{\alpha}$-continuous function defined by $f(N) = \{a\}$ and $f(X \setminus N) = \{b\}$ where $a \neq b$. Since f is constant function we get $N = X$.

Theorem 3.15

Let $f : X \to Y$ be a $w\tilde{\alpha}$-continuous surjective function. If X is $w\tilde{\alpha}$- connected, then Y is connected.

Proof

We suppose that Y is not connected. Then $Y = A \cup B$ where $A \cap B = \phi$, $A \neq \phi$, $B \neq \phi$ and A, B are open sets in Y. Since f is $w\tilde{\alpha}$-continuous surjective function, $X = f^{-1}(A) \cup f^{-1}(B)$ are disjoint union of two non- empty $w\tilde{\alpha}$-open subsets. This is contradiction with the fact that X is $w\tilde{\alpha}$-connected.

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