Mathematical Statistician and Engineering Applications
ISSN: 2094-0343
2326-9865

Generalized Pre-Semi Closed and Pre-Semi Open Sets in Intuitionistic Fuzzy Topological Spaces

R Revathy [1] And P. Thirunavukarasu [2]

[1]Part Time Research Scholar, PG & Research Department ofMathematics
Thanthai Periyar Government Arts & Science College,
Thiruchirapalli, Tamil Nadu , India
(Affiliated to Bharatidasan University)
e-mail : rrevathy085@gmail.com

[2]Assistant Professor, PG & Research Department of Mathematics
(Affiliated to Bharathidasan University)
Thanthai Periyar Government Arts & Science College
Thiruchirapalli, Tamil Nadu , India
email :ptavinash1967@gmail.com

Article Info
Page Number: 6948 - 6953
Publication Issue:
Vol 71 No. 4 (2022)

Abstract

The features of generalised pre-semi closed sets (gps -closed) in a topological space are defined in this chapter, and they are appropriately situated between pre-closed sets and generalised pre-closed sets.

Keywords: - Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, Intuitionistic fuzzy topologicalspaces, intuitionistic fuzzy ideal, generalized pre-semi closed sets

Article History
Article Received*: 25 March 2022*
Revised*: 30 April 2022*
Accepted*: 15 June 2022*
Publication *19 August 2022*

---

1. Introduction

Semi-open sets and semi-continuity in topological spaces were presented by Levine [3] in 1963.In 1970, he [4] proposed the concept of a generalised closed set ($g - closed\ set$). Maki et al. [5] established the concept of generalised preclosed sets and examined some of its basic features, whereas Sundaram and Sheik John [6] introduced the concept of weakly-closed sets. Dontchev [1] first proposedthe idea of generalised semi-pre open sets in topological spaces in 1995.

In 1997, Gnanambal [2] established the notion of $gpr - closed$ sets and investigated their features in topological spaces. The features of generalised pre-semi closed sets ($gps - closed$) in a topological space are defined in this chapter, and they are appropriately situated between pre-closed setsand generalised pre-closed sets.

Definition 1. Consider the subset A of the topological space (X, r) is generalized presemi closed ($gps - closed$) if $pcl(A) \subseteq U$ whenever $A \subseteq$ U and U is semi-open.

Theorem 2. Each closed set is $gps - closed$.

ISSN: 2094-0343
2326-9865

Theorem 3. Each $gps-closed$ set is $gpr-closed$.

Theorem 4. Each $gps-closed$ set is $gp-closed$.

Theorem 5. Each $w-closed$ set is $gps-closed$.

Theorem 6. Each $gps-closed$ set is $gprw-closed$.

Proof:

If A is a $gps-closed$ set and A □ U here U is the regular semi-open is. Each regular semi-open set will be semi-open.

Hence $pcl(A)$□ U.

Thus A is $gprw-closed$.

.Theorem 7. Each $gps-closed$ is $gsp-closed$.

Proof:

If A is a $gps-closed$ also A □ U here U is open.

Each open set is said to be semi-open and $spcl(A)$□ pcl(A), spcl(A)□ U. Therefore, A is $gsp-closed$.

Theorem 8. If A be $gsp-closed$ in (X, r). A is pre-closed only if $pcl(A)$□ A is always semi closed.

Proof:

Necessity : Let A be $gps-closed$.Assume that A is pre-closed.
Then $pcl(A) = A$ and so $pcl(A) - A =$ □.

Hence $pcl(A) - A$ is semi closed.

Sufficiency : Assume that $pcl(A) - A$ is semi closed. Since A is $gps-closed$, $pcl(A) - A =$ □ by Theorem 7.That is $pcl(A) = A$.

Therefore A is pre-closed.

Theorem 9. A is a $gps-closed$ subset of X so that A □ B □ pcl(A) then B is also a $gps-closed$ set in X.

Proof:

If A be $gps-closed$ set of X so that A □ B □ pcl(A) .

If U is a semi open set of X so that B □ U.

Then A □ U.

Since A is $gps-closed$, we have $pcl(A)$□ U.

Now (B)□pcl(pcl(A)) = $pcl(A)$□U. .Therefore B is $gps-closed$ set in .

2. Intuitionistic fuzzy generalized pre-semi closed set

Definition 10. An IFS A in an $IFTS$ $(X,τ)$ is said to be an intuitionistic fuzzy generalized pre-semi closed set ($IFGPSCS$) if $pcl(A)$□ U whenever A □ U and U is an $IFSOS$ in $(X,τ)$. The family of all $IFGPSCSs$ of an $IFTS$ $(X,τ)$ is denoted by $IFGPSC$ (X).

Theorem 11. Each $IFCS$ in (X, □) is an $IFGPSCS$ in $(X,τ)$.

Proof:

Let A be an $IFCS$.

Let A □ U and U be an $IFSOS$ in (X, □).

Then $pcl(A)$ □ $cl(A)$ = A □ U, by hypothesis.Hence A is an $IFGPSCS$ in $(X,τ)$

Theorem 12. Each $IFGPSCS$ in $(X,τ)$ is an $IFGPCS$ in $(X,τ)$.

Proof:

Let A be an $IFGPSCS$ in (X, □) and let A □ U, U is an $IFOS$ in (X, □).

Subsequently each $IFOS$ in (X, □) is an $IFSOS$ in (X, □) andby hypothesis $pcl(A)$ □ U.

Hence A is an $IFGPCS$ in (X, □).

Theorem 13. Each $IFGPSCS$ in (X, □) is an $IFGPRCS$ in (X, □) .

Proof:

If A is an $IFGPSCS$ in (X, □) and let A □ U, U is an $IFROS$ in (X, □).

Subsequently each $IFROS$ in (X, □) is an $IFSOS$ in (X, □) andby hypothesis $pcl(A)$ □ U.
Hence A is an $IFGPRCS$ in (X, □).

Theorem 14. Each $IFGPSCS$ in (X, □) is an $IFGSPCS$ in (X, □) .

Proof:

Let A be an $IFGPSCS$ in (X, □) and let A □ U, U is an $IFOS$ in (X, □).

Since every $IFOS$ in (X, □) is an $IFSOS$ in (X, □) andby hypothesis $spcl(A)$ □ $pcl(A)$ □ U.
Hence A is an $IFGSPCS$ in (X, □).

Remark 15. The combination of any two $IFGPSCSs$ in (X, □) will not be an $IFGPSCS$ in (X, □) in universalas realized from the subsequent instance.

Example 16. Let $X = \{a, b\}$ and let □ = {0~, $H1$, 1~ } where
$H1$ = □x, (0.3, 0.2), (0.7, 0.8)□.
Then the $IFSs$ A = □x, (0.2, 0.4), (0.8, 0.6)□ and
B = □x, (0.5, 0.1), (0.5, 0.9)□ are $IFGPSCSs$ in (X, □) but
A □ B = □x, (0.5, 0.4), (0.5, 0.6)□ is not an $IFGPSCS$ in (X, □).Let U = □x, (0.5, 0.4), (0.5, 0.6)□ be an $IFSOS$ in (X, □).
Since A □ B □U but $pcl(A$ □ $B)$ = □x, (0.7, 0.8), (0.3, 0.2)□ □ U.

Remark 17. The combination of any two $IFGPSCSs$ in (X, □) will not be an $IFGPSCS$ in (X, □) in universal as realized from the subsequent instance.

Example 18. Let $X = \{a, b\}$ and let □ = {0~, $H1$, 1~ } where
$H1$ = □x, (0.3, 0.2), (0.7, 0.8)□ .
Then the $IFSs$ A = □x, (0.5, 0.9), (0.5, 0.1)□ and
B = □x, (0.8, 0.4), (0.2, 0.6)□ are $IFGPSCSs$ in (X, □) but
A □ B = □x, (0.5, 0.4), (0.5, 0.6)□ is not an $IFGPSCS$ in (X, □).Let U = □x, (0.5, 0.4), (0.5, 0.6)□ be an $IFSOS$ in (X, □).
Since A □ B □ U but $pcl(A$ □ $B)$ = □x, (0.7, 0.8), (0.3, 0.2)□ □ U.

Theorem 19. Let (X, □) be an $IFTS$. Then for every A □ $IFGPSC(X)$ andfor every B □ $IFS(X)$, A □ B □ $pcl(A)$ implies B □ $IFGPSC(X)$.

Proof:

Let B □ U and U be an $IFSOS$ in (X, □). Then since A □ B, A □ U.

Since A is an $IFGPSCS$, it follows that $pcl(A)$ □ U. Now B □ $pcl(A)$ implies $pcl(B)$ □ $pcl(pcl(A)) = pcl(A)$[4]. Thus, $pcl(B)$ □ U. This proves that B □ $IFGPSC(X)$.

Theorem 20. Let A be an $IFSOS$ and an $IFGPSCS$ in (X, □), then A is an $IFPCS$ in (X, □).

Proof:

Since A □ A and A is an $IFSOS$ in (X, □), by hypothesis, $pcl(A)$ □ A. Always A □ $pcl(A)$. Therefore $pcl(A) = A$. Hence A is an $IFPCS$ in (X, □).

Theorem 21. Let A be an $IFGPSCS$ in (X, □) then $pcl(A) - A$ does not contain any non empty $IFSCS$.

Proof:

Let F be an $IFSCS$ such that F □ $pcl(A) - A$. Then F □ $X - A$ implies

A □ $X - F$. Since A is an $IFGPSCS$ and $X - F$ is an $IFSOS$, $pcl(A)$ □ $X - F$ .That is F □ $X - pcl(A)$. Hence F □ $pcl(A)$ □ ($X - pcl(A)$) = □ . This shows F = □.

3. Intuitionistic Fuzzy Generalized Pre-Semi Open Sets

In this section, the concept of an intuitionistic fuzzy generalized pre-semi opensets is introduced and some of their properties are studied.

Definition 22. An IFS A is said to be an intuitionistic fuzzy generalized presemi open set ($IFGPSOS$) in (X, □) if the complement A^c is an $IFGPSCS$ in X.

The family of all $IFGPSOSs$ of an $IFTS$ (X, □) is denoted by $IFGPSO(X)$.

Definition 23. Let A be an IFS in an $IFTS$ (X, □) . Then intuitionistic fuzzy generalized pre-semi interior of $A(gpsint(A))$ and intuitionistic fuzzy generalized pre-semi closure of $A(gpscl(A))$ are defined by

(i) $gpsint(A)$ =□{G/G is an $IFGPSOS$ in X and G _ A} .

(ii) $gpscl(A)$ =□{K/K is an $IFGPSCS$ in X and A □ K} .

Note that for any IFS

A in $(X, □)$, we have $gpscl(A^{c}) = (gpsint(A))^{c}$ and $gpsint(A^{c}) = (gpscl(A))^{c}$.

Theorem 24. Let $(X, □)$ be a $IFTS$. Then for every $A □ IFGPSO(X)$ and for every $B □ IFS(X)$, $pint(A)$ $□ B □ A$ implies $B □ IFGPSO(X)$.

Proof:

Let A be any $IFGPSOS$ of X and B be any IFS of X, such that

$pint(A) □ B □ A$.

Then A^{c} is an $IFGPSCS$ in X and $A^{c} □ B^{c} □ pcl(A^{c})$.By Theorem, B^{c} is an $IFGPSCS$ in $(X, □)$. Therefore B is an $IFGPSOS$ in $(X, □)$.Hence $B □ IFGPSO(X)$.

Theorem 25. An IFS A of an $IFTS$ $(X, □)$ is an $IFGPSOS$ in $(X, □)$ if and only if $F □ pint(A)$ whenever F is an $IFSCS$ in $(X, □)$ and $F □ A$.

Proof:

Necessity: Suppose A is an $IFGPSOS$ in $(X, □)$.

Let F be an $IFSCS$ in $(X, □)$ such that $F □ A$.Then F^{c} is an $IFSOS$ and $A^{c} □ F^{c}$.
By hypothesis A^{c} is an $IFGPSCS$ in $(X, □)$, we have $pcl(A^{c}) □ F^{c}$.Therefore $F □ pint(A)$, since $(pcl(A))^{c} = pcl(A)^{c}$[4].
Sufficiency: Let U be an $IFSOS$ in $(X, □)$ such that $A^{c} □ U$.By hypothesis, $U^{c} □ pint(A)$.
Therefore $pcl(A^{c}) □ U$ and A^{c} is an $IFGPSCS$ in $(X, □)$.Hence A is an $IFGPSOS$ in $(X, □)$.

References

[1]J.Dontchev, On generalizing semi-pre open sets, Mem. Fac. Sci. Kochi. Univ. Ser. A, Math., 16, (1995), 35-48.
[2]Y.Gnanambal, On generalized pre-regular closed sets in topological spaces,Indian J. Pure App.Math., 28, (1997), 351-360.
[3]N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70, (1963), 36-41.zN.Levine, Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19, (1970), 89-96.
[4]H. Maki, J. Umehara and T. Noiri, Every topological space is pre- $T_{1/2}$ space,Mem. Fac. Sci. Kochi Univ. Ser. A. Math. 17, (1996), 33-42.
[5]P.Sundaram and M.ShiekJohn, On ! -closed sets in topology, Acta SinciaIndica., 4, (2000), 389-392.