On Nano Ideal Generalized Scattered Spaces

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Article Info

Page Number: 7047 - 7057

Publication Issue:

Vol 71 No. 4 (2022)

Article History

Article Received*: 25 March 2022*

Revised*: 30 April 2022*

Accepted*: 15 June 2022*

Publication*: 19 August 2022*

Abstract

The purpose of this paper is to investigate scatteredness on nano ideal topological spaces. We introduce the notion of $\mathrm{nIs_\alpha g}-$ scattered space and investigated certain properties. Also, we have introduced $\mathcal{N}^{s_\alpha g^\star}-$ isolated points, $\mathcal{N}^{s_\alpha g^\star}-$ derived sets, $\mathcal{N}^{s_\alpha g^\star}-$ dense sets, $\mathrm{nIs_\alpha g}-$ dense sets and discuused their characteristics. Further, we have studied the equivalent conditions of $\mathrm{nIs_\alpha g}-$ scattered space.

Keywords:$\mathrm{nIs_\alpha g}-$ scattered space, $\mathcal{N}^{s_\alpha g^\star}-$ isolated points, $\mathcal{N}^{s_\alpha g^\star}-$ derived sets, $\mathcal{N}^{s_\alpha g^\star}-$ dense sets, $\mathrm{nIs_\alpha g}-$ dense sets

1. Introduction

Parimal et.al[2] introduce the notion of nano ideal topological spaces as follows: Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a nano ideal topological space with an ideal $\mathcal{I}$ on Γ, where $\mathcal{N} = \tau_{\mathcal{R}}(\mathcal{X})$ and if 2^Γ is the set of all subsets of Γ, a set operator $(.)_n^*: 2^\Gamma \to 2^\Gamma$, called a nano local function (briefly, $\mathrm{n}-$ local function of $\mathcal{H}$ with respect to $\mathcal{N}$ and $\mathcal{I}$ is defined as follows: for $\mathcal{H} \subset \mathcal{X}, \mathcal{H}_n^*(\mathcal{I}, \mathcal{N}) = \{\gamma \in \Gamma : \mathcal{G}_n \cap \mathcal{H} \notin \mathcal{I}$ for every $\mathcal{G}_n \in \mathcal{G}_n(\gamma)\}$, where $\mathcal{G}_n = \{\mathcal{G}_n : \gamma \in \mathcal{G}_n, \mathcal{G}_n \in \mathcal{N}\}$. We will simply write $\mathcal{H}_n^*$ for $\mathcal{H}_n^*(\mathcal{I}, \mathcal{N})$.

Pasunkilipandian et.al[4] introduce a new class of generalized closed sets in nano ideal topological space namely, $n\mathcal{J}s_{\alpha}g$ closed sets.

2.Preliminaries

Definition 2.1[1] Let Γ be a nonempty finite set of objects called the universe and $\mathcal{R}$ be an equivalence relation on Γ named as indiscernibility relation. Then Γ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\Gamma, \mathcal{R})$ is said to be an approximation space. Let $\mathcal{X} \subseteq \Gamma$. Then,

(i)The lower approximation of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all objects which can be for certain classified as $\mathcal{X}$ with respect to $\mathcal{R}$ and is denoted by $L_{\mathcal{R}}(\mathcal{X})$. That is, $L_{\mathcal{R}}(\mathcal{X}) = \cup_{x\in\Gamma} \{\mathcal{R}(\mathcal{X}) \subseteq X: x \in \Gamma\}$ where $\mathcal{R}(\mathcal{X})$ denotes the equivalence class determined by $x \in \Gamma$.

(ii)The upper approximation of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all objects which can be possibly classified as $\mathcal{X}$ with respect to $\mathcal{R}$ and is denoted by $U_{\mathcal{R}}(\mathcal{X})$. That is, $U_{\mathcal{R}}(\mathcal{X}) = \cup_{x\in\Gamma} \{\mathcal{R}(\mathcal{X}): \mathcal{R}(\mathcal{X}) \cap X \neq \emptyset, x \in \Gamma\}$ where $\mathcal{R}(\mathcal{X})$ denotes the equivalence class determined by $x \in \Gamma$.

(iii) The boundary region of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all objects which can be classified neither as $\mathcal{X}$ nor as not $-\mathcal{X}$ with respect to $\mathcal{R}$ and is denoted by $B_{\mathcal{R}}(\mathcal{X})$. That is, $B_{\mathcal{R}}(\mathcal{X}) = U_{\mathcal{R}}(\mathcal{X}) - L_{\mathcal{R}}(\mathcal{X})$.

Definition 2.2 [1] Let Γ be a universe, $\mathcal{R}$ be an equivalence relation on Γ and $\mathcal{N}_{\mathcal{R}}(\mathcal{X}) = \{\mathcal{U}, \emptyset, L_{\mathcal{R}}(\mathcal{X}), U_{\mathcal{R}}(\mathcal{X}), B_{\mathcal{R}}(\mathcal{X})\}$, where $X \subseteq \Gamma$, satisfies the following axioms:

(i) $\mathcal{U}, \emptyset \in \mathcal{N}_{\mathcal{R}}(\mathcal{X})$.

(ii) The union of the elements of any sub-collection of $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$ is in $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$.

(iii) The intersection of the elements of any finite subcollection of $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$ is in $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$.

Therefore, $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$ is a topology on Γ called the nano topology on Γ with respect to $\mathcal{X}$. We call $\left(\Gamma, \mathcal{N}_{\mathcal{R}}(\mathcal{X})\right)$ as the nano topological space. The elements of $\mathcal{N}_{\mathcal{R}}(\mathcal{X})$ are called nano open sets (briefly, n- open sets). The complement of a nano open set is called a nano closed set (briefly, $n-$ closed set).

Definition 2.3 [3] A subset $\mathcal{C}$ of a nano topological space $(\Gamma, \mathcal{N})$ is said to be nano semi $\alpha-$ open set (briefly, $NS_{\alpha}-$ O.S) if there exists a $n\alpha-$open set $\mathcal{P}$ in Γ such that $\mathcal{P} \subseteq \mathcal{C} \subseteq n - cl(\mathcal{P})$ or equivalently if $\mathcal{C} \subseteq n - cl\left(n\alpha - int(\mathcal{P})\right)$.The family of all $NS_{\alpha}-$ O.S of $\mathcal{U}$ is denoted by $NS_{\alpha}O(\mathcal{U}, \mathcal{M})$.

Definition 2.4 [4] A subset $\mathcal{H}$ of a nano ideal topological space $(\Gamma, \mathcal{N}, \mathcal{I})$ is said to be nano ideal semi $\alpha -$ generalized closed set (briefly, $\mathrm{n}\mathcal{I}\mathcal{s}_{\alpha}\mathrm{g} -$ closed set) if $\mathcal{H}_n^* \subseteq \mathcal{K}$ whenever $\mathcal{H} \subseteq \mathcal{K}$ and $\mathcal{K}$ is nano semi $\alpha -$ open.

Definition 2.5 [4] A subset $\mathcal{H}$ of a nano topological space $(\Gamma, \mathcal{N})$ is said to be nano semi $\alpha -$ generalized closed set (briefly, $\mathrm{n}\mathcal{s}_{\alpha}\mathrm{g} -$ closed set) if $\mathrm{n} - \mathrm{cl}(\mathcal{H}) \subseteq \mathcal{K}$ whenever $\mathcal{H} \subseteq \mathcal{K}$ and $\mathcal{K}$ is nano semi $\alpha -$ open.

Definition 2.6[8]Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $\mathrm{n}\mathcal{I} -$ topological space and $(.)^{s_{\alpha}g^*}$ be a set operator from 2^{Γ} to 2^{Γ}, where 2^{Γ} is the set of all subsets of Γ. For a subset $\mathcal{H} \subset \Gamma$, $\mathcal{H}^*_{n\mathcal{s}_{\alpha}\mathcal{g}}(\mathcal{I}, \mathcal{N}) = \{x \in \Gamma : \mathcal{G}_n \cap \mathcal{H} \notin \mathcal{I}$, for every $\mathcal{G}_n \in \mathcal{N}^{\mathcal{s}_{\alpha}\mathcal{g}}(\gamma)\}$ is called the nano semi α generalized local function (briefly, $\mathrm{ns}_{\alpha}\mathrm{g} -$ local function) of $\mathcal{H}$ with respect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write $\mathcal{H}^*_{n\mathcal{s}_{\alpha}\mathcal{g}}$ instead of $\mathcal{H}^*_{n\mathcal{s}_{\alpha}\mathcal{g}}(\mathcal{I}, \mathcal{N})$.

Definition 2.6[9]Let (X, τ, I) be an ideal space and let $h \in A \subset X$.

(1) h is called a $* -$ isolated point of A in h if there exists $U \in \tau^*(h)$ such that $U \cap A = \{h\}$.

(2) h is called a $* -$ accumulation point of A in h if $U \cap (A - \{h\}) \neq \emptyset$ for any $U \in \tau^*(h)$.

The set of all $* -$ isolated points of A in h is denoted by $I^*(A)(I, \tau)$ or $I^*(A)$. The set of all $* -$ accumulation points of A in h is denoted by $d^*(A)(I, \tau)$ or $d^*(A)$, which is called the $* -$ derived set of A in h.

Definition 2.7[7] Let (X, τ, I) be an ideal space.

1. $A \subset X$ is called $* -$ dense in h if $cl^*(A) = X$.

2. $A \subset X$ is called $I -$ dense in h if $A^* = X$.

Definition2.8[9] Let (X, τ, I) be an ideal space. X is called $I -$ scattered if $I^*(A) \neq \emptyset$ for any $A \in 2^X - \{\emptyset\}$.

Definition 2.9[6] An ideal space (X, τ, I) is called $I -$ resolvable if X has two disjoint $I -$ dense subsets. Otherwise, X is $I -$ irresolavble.

3. $\mathcal{N}^{\mathcal{s}_{\alpha}\mathcal{g}^*} -$ isolated points and $\mathcal{N}^{\mathcal{s}_{\alpha}\mathcal{g}^*} -$ derived sets

Definition 3.1 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $\mathrm{n}\mathcal{I} -$ topological space and let $\gamma \in \mathcal{H} \subset \Gamma$. Then γ is called a

(i) $\mathcal{N}^* -$ isolated point of $\mathcal{H}$ in Γ if there exists $\mathcal{G} \in \mathcal{N}^*(\gamma)$ such that $\mathcal{G} \cap \mathcal{H} = \{\gamma\}$.

(ii) $\mathcal{N}^* -$ accumulation point of $\mathcal{H}$ in Γ if $\mathcal{G} \cap (\mathcal{H} - \{\gamma\}) \neq \emptyset$ for any $\mathcal{G} \in \mathcal{N}^*(\gamma)$.

(iii) $\mathcal{N}^{s_\alpha g^*}-$ isolated point of $\mathcal{H}$ in Γ if there exists $\mathcal{G} \in \mathcal{N}^{s_\alpha g^*}(\gamma)$ such that $\mathcal{G} \cap \mathcal{H} = \{\gamma\}$.

(iv) $\mathcal{N}^{s_\alpha g^*}-$ accumulation point of $\mathcal{H}$ in Γ if $\mathcal{G} \cap (\mathcal{H} - \{\gamma\}) \neq$ for any $\mathcal{G} \in \mathcal{N}^{s_\alpha g^*}(\gamma)$.

The set of all $\mathcal{N}^*-$ isolated points (resp.$\mathcal{N}^{s_\alpha g^*}-$ isolated points) of $\mathcal{H}$ in Γ is denoted by $\mathrm{nI}^*(\mathcal{H})(\mathcal{I},\mathcal{N})$ or $\mathrm{nI}^*(\mathcal{H})$ (resp. $\mathrm{ns}_\alpha\mathrm{gI}^*(\mathcal{H})(\mathcal{I},\mathcal{N})$ or $\mathrm{ns}_\alpha\mathrm{gI}^*(\mathcal{H})$. The set of all $\mathcal{N}^*-$ accumulation points (resp.$\mathcal{N}^{s_\alpha g^*}-$ accumulation points) of $\mathcal{H}$ in Γ is denoted by $\mathrm{nd}^*(\mathcal{H})(\mathcal{I},\mathcal{N})$ or $\mathrm{nd}^*(\mathcal{H})$, (resp. $\mathrm{ns}_\alpha\mathrm{gd}^*(\mathcal{H})(\mathcal{I},\mathcal{N})$ or $\mathrm{ns}_\alpha\mathrm{gd}^*(\mathcal{H})$) which is called the n^*- derived set (resp. $\mathrm{ns}_\alpha\mathrm{g}^*-$ derived set) of $\mathcal{H}$ in Γ.

Example 3.2 Consider the $\mathrm{n}\mathcal{I}-$ topological spaces $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ and $(\Delta_1, \mathcal{N}_1, \mathcal{I}_2)$ as follows:

$\Delta_1 = \{\delta_1, \delta_2, \delta_3, \delta_4\}$; $\Delta_1/\mathcal{R} = \{\{\delta_1\}, \{\delta_2, \delta_4\}, \{\delta_3\}\}$; $\mathcal{X} = \{\delta_2, \delta_4\}$; $\mathcal{N}_1 = \{\emptyset, \Delta_1, \{\delta_1\}, \{\delta_1, \delta_2, \delta_4\}, \{\delta_2, \delta_4\}\}$; $\mathcal{I}_1 = \{\emptyset, \{\delta_2\}\}$ and

$\Delta_1 = \{\delta_1, \delta_2, \delta_3, \delta_4\}$; $\Delta_1/\mathcal{R} = \{\{\delta_1\}, \{\delta_2, \delta_4\}, \{\delta_3\}\}$; $\mathcal{X} = \{\delta_2, \delta_4\}$; $\mathcal{N}_1 = \{\emptyset, \Delta_1, \{\delta_1\}, \{\delta_1, \delta_2, \delta_4\}, \{\delta_2, \delta_4\}\}$; $\mathcal{I}_2 = \{\emptyset, \{\delta_2\}\}$.

Refer the following table for $\mathcal{N}^*$ isolated points (resp.$\mathcal{N}^{s_\alpha g^*}-$ isolated points) and $\mathcal{N}^*$ accumulation points (resp.$\mathcal{N}^{s_\alpha g^*}-$ accumulation points).

$\mathcal{H}$	$\mathcal{I}_1 = \{\emptyset, \{\delta_2\}\}$				$\mathcal{I}_2 = \{\emptyset, \{\delta_2\}, \{\delta_3\}, \{\delta_2, \delta_3\}\}$	
	$nI^*(\mathcal{H})$	$nd^*(\mathcal{H})$	$ns_\alpha gI^*(\mathcal{H})$	$ns_\alpha gd^*(\mathcal{H})$	$ns_\alpha gI^*(\mathcal{H})$	$ns_\alpha gd^*(\mathcal{H})$
Δ_1	$\{\delta_1, \delta_4\}$	$\{\delta_2, \delta_3\}$	$\{\delta_1, \delta_2, \delta_4\}$	$\{\delta_3\}$	$\{\delta_1, \delta_2, \delta_4\}$	$\{\delta_3\}$
$\{\delta_1\}$	$\{\delta_1\}$	$\{\delta_3\}$	$\{\delta_1\}$	$\{\delta_3\}$	$\{\delta_1\}$	$\{\delta_3\}$
$\{\delta_2\}$	$\{\delta_2\}$	$\emptyset$	$\{\delta_2\}$	$\emptyset$	$\{\delta_2\}$	$\emptyset$
$\{\delta_3\}$	$\{\delta_3\}$	$\emptyset$	$\{\delta_3\}$	$\emptyset$	$\{\delta_3\}$	$\emptyset$
$\{\delta_4\}$	$\{\delta_4\}$	$\{\delta_3\}$	$\{\delta_4\}$	$\{\delta_3\}$	$\{\delta_4\}$	$\{\delta_3\}$
$\{\delta_1, \delta_2\}$	$\{\delta_1, \delta_2\}$	$\{\delta_3\}$	$\{\delta_1, \delta_2\}$	$\{\delta_3\}$	$\{\delta_1, \delta_3\}$	$\{\delta_3\}$
$\{\delta_1, \delta_3\}$	$\{\delta_1\}$	$\{\delta_3\}$	$\{\delta_1\}$	$\{\delta_3\}$	$\{\delta_1\}$	$\{\delta_3\}$
$\{\delta_1, \delta_4\}$	$\{\delta_1, \delta_4\}$	$\{\delta_2, \delta_3\}$	$\{\delta_1, \delta_4\}$	$\{\delta_3\}$	$\{\delta_1, \delta_4\}$	$\{\delta_3\}$
$\{\delta_2, \delta_3\}$	$\{\delta_2, \delta_3\}$	$\emptyset$	$\{\delta_2, \delta_3\}$	$\emptyset$	$\{\delta_2, \delta_3\}$	$\emptyset$
$\{\delta_2, \delta_4\}$	$\{\delta_4\}$	$\{\delta_2, \delta_3\}$	$\{\delta_2, \delta_4\}$	$\{\delta_3\}$	$\{\delta_2, \delta_4\}$	$\{\delta_3\}$
$\{\delta_3, \delta_4\}$	$\{\delta_4\}$	$\{\delta_2, \delta_3\}$	$\{\delta_4\}$	$\{\delta_3\}$	$\{\delta_4\}$	$\{\delta_3\}$
$\{\delta_1, \delta_2, \delta_3\}$	$\{\delta_1, \delta_2\}$	$\{\delta_3\}$	$\{\delta_1, \delta_2\}$	$\{\delta_3\}$	$\{\delta_1, \delta_2\}$	$\{\delta_3\}$
$\{\delta_1, \delta_2, \delta_4\}$	$\{\delta_1, \delta_4\}$	$\{\delta_2, \delta_3\}$	$\{\delta_1, \delta_2, \delta_4\}$	$\{\delta_3\}$	$\{\delta_1, \delta_2, \delta_4\}$	$\{\delta_3\}$

$\{\delta_1,\delta_3,\delta_4\}$	$\{\delta_1,\delta_4\}$	$\{\delta_2,\delta_3\}$	$\{\delta_1,\delta_4\}$	$\{\delta_3\}$	$\{\delta_1,\delta_4\}$	$\{\delta_3\}$
$\{\delta_2,\delta_3,\delta_4\}$	$\{\delta_4\}$	$\{\delta_2,\delta_3\}$	$\{\delta_2,\delta_4\}$	$\{\delta_3\}$	$\{\delta_2,\delta_4\}$	$\{\delta_3\}$

Table 3.1. $\mathcal{N}^{s_\alpha g^*}-$ isolated points and $\mathcal{N}^{s_\alpha g^*}-$ accumulation points

Theorem 3.3 Let $(\Gamma,\mathcal{N},\mathcal{I})$ be a $n\mathcal{I}-$ topological space. Then for $\mathcal{H},\mathcal{K}\subset\Gamma$,

(i) $ns_\alpha gI^*(\mathcal{H})=\mathcal{H}-\left(ns_\alpha gd^*(\mathcal{H})\right)$.

(ii) $nI^*(\mathcal{H})\subset ns_\alpha gI^*(\mathcal{H})\subset\mathcal{H}$.

(iii) (a) $\mathcal{H}=\left(ns_\alpha gI^*(\mathcal{H})\right)\cup\left(\left(ns_\alpha gd^*(\mathcal{H})\right)\cap\mathcal{H}\right)$;

(b) $\left(ns_\alpha gd^*(\mathcal{H})\right)\cap\mathcal{H}=\mathcal{H}\setminus\left(ns_\alpha gI^*(\mathcal{H})\right)$.

(iv) If $\mathcal{H}\in\mathcal{N}^{s_\alpha g^*}-\{\emptyset\}$ and $\mathcal{H}\subset\mathcal{K}$ then $ns_\alpha gI^*(\mathcal{H})\subset ns_\alpha gI^*(\mathcal{K})$.

(v) (a) $\left(ns_\alpha gI^*(\mathcal{H})\right)\cap\left(ns_\alpha gI^*(\mathcal{K})\right)\subset ns_\alpha gI^*(\mathcal{H}\cap\mathcal{K})$;

(b) $ns_\alpha gI^*(\mathcal{H}\cup\mathcal{K})\subset\left(ns_\alpha gI^*(\mathcal{H})\right)\cup\left(ns_\alpha gI^*(\mathcal{K})\right)$.

Proof: (i) Let $\gamma\in ns_\alpha gI^*(\mathcal{H})$. Then $\mathcal{G}\cap\mathcal{H}=\{\gamma\}$ for some $\mathcal{G}\in\mathcal{N}^{s_\alpha g^*}(\gamma)$. This implies $\mathcal{G}\cap(\mathcal{H}-\{\gamma\})=\emptyset$. Then $\gamma\notin ns_\alpha gd^*(\mathcal{H})$. Thus, $\gamma\in\mathcal{H}-ns_\alpha gd^*(\mathcal{H})$ and so $ns_\alpha gI^*(\mathcal{H})\subset\mathcal{H}-ns_\alpha gd^*(\mathcal{H})$. Conversely, let $\gamma\in\mathcal{H}-ns_\alpha gd^*(\mathcal{H})$. Since $\gamma\notin ns_\alpha gd^*(\mathcal{H})$, we have $\mathcal{G}\cap(\mathcal{H}-\{\gamma\})=\emptyset$ for some $\mathcal{G}\in\mathcal{N}^{s_\alpha g^*}(\gamma)$. Note that $\mathcal{G}\cap\mathcal{H}=\{\gamma\}$. Then $\gamma\in ns_\alpha gI^*(\mathcal{H})$ and so $ns_\alpha gI^*(\mathcal{H})\supset\mathcal{H}-ns_\alpha gd^*(\mathcal{H})$. Hence, $ns_\alpha gI^*(\mathcal{H})=\mathcal{H}-ns_\alpha gd^*(\mathcal{H})$.

(ii) Since every $\mathcal{N}^*-$ closed set is $\mathcal{N}^{s_\alpha g^*}-$ closed, the result follows.

(iii) (a) For any $\gamma\in\mathcal{H}$ and $\mathcal{G}\in\mathcal{N}^{s_\alpha g^*}(\gamma)$, $\mathcal{G}\cap\mathcal{H}=\{\gamma\}$ or $\mathcal{G}\cap\{\mathcal{H}-\{\gamma\}\}\neq\emptyset$, then $\gamma\in\left(ns_\alpha gI^*(\mathcal{H})\cup ns_\alpha gd^*(\mathcal{H})\right)$ and $\mathcal{H}\subset\left(ns_\alpha gI^*(\mathcal{H})\right)\cup\left(ns_\alpha gd^*(\mathcal{H})\right)$. Thus, $\mathcal{H}\subset\left(ns_\alpha gI^*(\mathcal{H})\cup\right.$ $ns\alpha gd*\mathcal{H}\cap\mathcal{H}=ns\alpha gI*\mathcal{H}\cup ns\alpha gd*\mathcal{H}\cap\mathcal{H}$ and $\mathcal{H}\supset ns\alpha gI*\mathcal{H}\cup ns\alpha gd*\mathcal{H}\cap\mathcal{H}$. Hence, $\mathcal{H}=ns_\alpha gI^*(\mathcal{H})\cup(ns_\alpha gd^*(\mathcal{H})\cap\mathcal{H})$.

(b) The result follows from (a).

(iv) Let $\gamma\in ns_\alpha gI^*(\mathcal{H})$. Then $\mathcal{G}\cap\mathcal{H}=\{\gamma\}$ for some $\mathcal{G}\in\mathcal{N}^{s_\alpha g^*}(\gamma)$. Since $\mathcal{H}\in\mathcal{N}^{s_\alpha g^*}-\{\emptyset\}$, $\mathcal{G}\cap\mathcal{H}\in\mathcal{N}^{s_\alpha g^*}-\{\emptyset\}$. Note that $(\mathcal{G}\cap\mathcal{H})\cap\mathcal{K}=\{\gamma\}$. Then $\gamma\in ns_\alpha gI^*(\mathcal{K})$. Thus $ns_\alpha gI^*(\mathcal{H})\subset ns_\alpha gI^*(\mathcal{K})$.

(v) The result is trivial.

Lemma 3.4 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ and $(\Gamma, \mathcal{N}, \mathcal{I}')$ be two $n\mathcal{I}-$ topological spaces with $\mathcal{I} \subset \mathcal{I}^{'}$. Then for $\mathcal{H} \subset \Gamma$, $ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}, \mathcal{N}) \subset ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}^{'}, \mathcal{N})$.

Proof: Let $\gamma \in ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}, \mathcal{N})$. Then $\mathcal{G} \cap \mathcal{H} = \{\gamma\}$ for some $\mathcal{G} \in \mathcal{N}^{s_{\alpha}g*}(\gamma)(\mathcal{I}, \mathcal{N})$. Since $\mathcal{I} \subset \mathcal{I}^{'}$, it is clear that $\mathcal{N}^{s_{\alpha}g*}(\gamma)(\mathcal{I}, \mathcal{N}) \subset \mathcal{N}^{s_{\alpha}g*}(\gamma)(\mathcal{I}^{'}, \mathcal{N})$ which implies $\mathcal{G} \in \mathcal{N}^{s_{\alpha}g*}(\gamma)(\mathcal{I}^{'}, \mathcal{N})$ so that $\gamma \in ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}^{'}, \mathcal{N})$. Hence, $ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}, \mathcal{N}) \subset ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}^{'}, \mathcal{N})$.

Remark 3.5 If $(\Gamma, \mathcal{N}, \mathcal{I})$ and $(\Gamma, \mathcal{N}, \mathcal{I}^{'})$ are two $n\mathcal{I}-$ topological spaces, $ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}, \mathcal{N}) \subset ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}^{'}, \mathcal{N})$ does not imply $\mathcal{I} \subset \mathcal{I}^{'}$.

Example 3.6 Consider the $n\mathcal{I}-$ topological spaces of Example 3.2.

Here, $ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}^{'}, \mathcal{N}) \subseteq ns_{\alpha}gI^{*}(\mathcal{H})(\mathcal{I}, \mathcal{N})$ for every subset $\mathcal{H}$ of Γ. But $\mathcal{I}^{'} \nsubseteq \mathcal{I}$. Refer Table 3.1.

4. $n\mathcal{I}s_{\alpha}g-$ Scattered Spaces

Definition 4.1 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space.

(i) Γ is called $n\mathcal{I}-$ scattered if n$I^{*}(\mathcal{H}) \neq$ for any $\mathcal{H} \in 2^{\Gamma} \setminus \{\emptyset\}$.

(ii) Γ is called $n\mathcal{I}s_{\alpha}g-$ scattered if $ns_{\alpha}gI^{*}(\mathcal{H}) \neq \emptyset$ for any $\mathcal{H} \in 2^{\Gamma} \setminus \{\emptyset\}$.

Example 4.2 (i) Consider the $n\mathcal{I}-$ topological space $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ of Example 3.2. Since $nI^{*}(\mathcal{H}) \neq \emptyset$ (resp.$ns_{\alpha}gI^{*}(\mathcal{H}) \neq \emptyset$) for any non empty subset $\mathcal{H}$ of Δ_1, the space $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ is $n\mathcal{I}-$ scattered (resp. $n\mathcal{I}s_{\alpha}g-$ scattered) space.

Remark 4.3 A $n\mathcal{I}s_{\alpha}g-$ scattered space need not be $n\mathcal{I}-$ scattered space.

Example 4.4 Consider the $n\mathcal{I}-$ topological space $(\Delta, \mathcal{N}, \mathcal{I})$ as follows: $= \{\delta_1, \delta_2, \delta_3, \delta_4\}$; $\Delta/\mathcal{R} = \{\{\delta_1\}, \{\delta_2, \delta_3\}, \{\delta_4\}\}$; $\mathcal{X} = \{\delta_1, \delta_2, \delta_3\}$; $\mathcal{N} = \{\emptyset, \Delta_4, \{\delta_1, \delta_2, \delta_3\}\}$; $\mathcal{I} = \{\emptyset, \{\delta_4\}\}$. Since $ns_{\alpha}gI^{*}(\mathcal{H}) \neq \emptyset$ for any non empty subset $\mathcal{H}$ of Δ, $(\Delta, \mathcal{N}, \mathcal{I})$ is $n\mathcal{I}s_{\alpha}g-$ scattered. But the space is not $n\mathcal{I}-$ scattered, since for the set $\mathcal{H} = \{\delta_1, \delta_2\}$, $nI^{*}(\mathcal{H}) = \emptyset$.

Theorem 4.5 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ and $(\Gamma, \mathcal{N}, \mathcal{I}^{'})$ be two $n\mathcal{I}-$ topological spaces. If $\mathcal{I} \subset \mathcal{I}^{'}$ and $(\Gamma, \mathcal{N}, \mathcal{I})$ is $n\mathcal{I}s_{\alpha}g-$ scattered with respect to ideal $\mathcal{I}$, then $(\Gamma, \mathcal{N}, \mathcal{I}^{'})$ is $nIs_{\alpha}g-$ scattered with respect to ideal $\mathcal{I}^{'}$.

Proof: The proof is trivial from Lemma 3.4.

Remark 4.6 Both $(\Gamma, \mathcal{N}, \mathcal{I})$ and $(\Gamma, \mathcal{N}, \mathcal{I}^{'})$ are $n\mathcal{I}s_{\alpha}g-$ scattered spaces does not imply $\mathcal{I} \subset \mathcal{I}^{'}$.

Example 4.7 Consider the $n\mathcal{I}-$ topological spaces of Example 3.2. Both $(\Delta_1, \mathcal{N}_1, \mathcal{I}_2)$and $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ are $n\mathcal{I}s_\alpha g-$ scattered but $\mathcal{I}_2 \nsubseteq \mathcal{I}_1$.

Definition 4.8 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space. Then,

(i) $\mathcal{H} \subset \Gamma$ is called $\mathcal{N}^{s_\alpha g *}-$ dense in Γ if $ns_\alpha g - cl^*(\mathcal{H}) = \Gamma$.

(ii) $\mathcal{H} \subset \Gamma$ is called $n\mathcal{I}s_\alpha g -$ dense in Γ if $\mathcal{H}^*_{ns_\alpha g} = \Gamma$.

Example 4.9 Consider the $n\mathcal{I}-$ topological space $(\Delta, \mathcal{N}, \mathcal{I})$ as follows:

$\Delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$; $\Delta/\mathcal{R} = \{\{\delta_1\}, \{\delta_2, \delta_3\}, \{\delta_4\}\}$; $\mathcal{X} = \{\delta_1, \delta_2, \delta_3\}$; $\mathcal{N} = \{\emptyset, \Delta_4, \{\delta_1, \delta_2, \delta_3\}\}$; $\mathcal{I} = \{\emptyset, \{\delta_4\}\}$

Here, (i) None of the set is $\mathcal{N}^{s_\alpha g *}-$ dense set except Γ.

(ii) None of the set is $n\mathcal{I}s_\alpha g -$ dense set.

Example 4.10 Consider the $n\mathcal{I}-$ topological space $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ as follows:

$\Delta_1 = \{\delta_1, \delta_2, \delta_3, \delta_4\}$; $\Delta_1/\mathcal{R} = \{\delta_1\}, \{\delta_2, \delta_3\}, \{\delta_4\}\}$; $\mathcal{X} = \{\delta_1, \delta_4\}$; $\mathcal{N}_1 = \{\emptyset, \Delta_5, \{\delta_1, \delta_4\}\}$; $\mathcal{I}_1 = \{\emptyset, \{\delta_1\}\}$. Here, the sets $\Delta_1, \{\delta_1, \delta_2\}$ are both $\mathcal{N}^{s_\alpha g *}-$ dense and $n\mathcal{I}s_\alpha g -$ dense.

Proposition 4.11 Every $n\mathscr{I}s_\alpha g-$ dense set is $\mathscr{N}^{s_\alpha g\ast}-$ dense.

Proof: A subset $\mathcal{H}$ of Γ is said to be $n\mathcal{I}s_\alpha g -$ dense in Γ if $\mathcal{H}^*_{ns_\alpha g} =. Since ns_\alpha g - cl^*(\mathcal{H}) = \mathcal{H} \cup \mathcal{H}^*_{ns_\alpha g}, ns_\alpha g - cl^*(\mathcal{H}) = \Gamma$. Hence, $\mathcal{H}$ is $\mathcal{N}^{s_\alpha g *}-$ dense set.

Remark 4.12 A $\mathscr{N}^{s_\alpha g\ast}-$ dense set need not be $n\mathscr{I}s_\alpha g-$ dense set.

For instance, consider the $n\mathcal{I}-$ topological space of Example 4.9. Here, the set Δ is $\mathcal{N}^{s_\alpha g *}-$ dense set but not $n\mathcal{I}s_\alpha g -$ dense.

Remark 4.13 (i) Γ need not always be $n\mathcal{I}s_\alpha g -$ dense.

(ii) $\emptyset$ is not $\mathcal{N}^{s_\alpha g *}-$ dense and $n\mathcal{I}s_\alpha g -$ dense.

Theorem 4.14 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space. Then $\mathcal{H} \subset \Gamma$ is $\mathcal{N}^{s_\alpha g *}-$ dense in Γ if and only if $\mathcal{G} \cap \mathcal{H} \neq \emptyset$ for any $\mathcal{G} \in \mathcal{N}^{s_\alpha g *} - \{\emptyset\}$.

Proof: Necessity: Let $\mathcal{H}$ be a $\mathcal{N}^{s_\alpha g*}-$ dense in Γ and let $\mathcal{G} \in \mathcal{N}^{s_\alpha g*} - \{\emptyset\}$. Pick $\gamma \in \mathcal{G}$. Then $\gamma \in \Gamma = ns_\alpha g - cl^*(\mathcal{H}) = \mathcal{H} \cup \mathcal{H}^*_{ns_\alpha g}$.

Case 1. $\gamma \in \mathcal{H}$.

Then $\gamma \in \mathcal{G} \cap \mathcal{H}$, so that $\mathcal{G} \cap \mathcal{H} \neq \emptyset$.

Case 2. $\gamma \in \mathcal{H}^*_{ns_\alpha g}$.

Suppose $\mathcal{G} \cap \mathcal{H} = \emptyset$. Since $\mathcal{G}^c$ is $\mathcal{N}^{s_\alpha g*}-$ closed in Γ, $(\mathcal{G}^c)^*_{ns_\alpha g} \subset \mathcal{G}^c$. Then $\mathcal{G} \subset \left((\mathcal{G}^c)^*_{ns_\alpha g}\right)^c$. Since $\gamma \in \mathcal{G}$, $\gamma / \in (\mathcal{G}^c)^*_{ns_\alpha g}$. It follows that $\mathcal{F} \cap (\mathcal{G}^c) \in \mathcal{I}$ for some $\mathcal{F} \in \mathcal{N}^{s_\alpha g}(\gamma)$. By $\mathcal{G} \cap \mathcal{H} = \emptyset$, $\mathcal{H} \subset \mathcal{G}^c$. This implies that $\mathcal{F} \cap \mathcal{H} \subset \mathcal{F} \cap (\mathcal{G}^c)$. Then $\mathcal{F} \cap \mathcal{H} \in \mathcal{I}$. So $\gamma \notin \mathcal{H}^*_{ns_\alpha g}$, a contradiction. Thus, $\mathcal{G} \cap \mathcal{H} \neq \emptyset$.

Sufficiency: Suppose that $ns_\alpha g - cl^*(\mathcal{H}) \neq \Gamma$. Put $\mathcal{G} = \left(ns_\alpha g - cl^*(\mathcal{H})\right)^c$. Then $\mathcal{G} \in \mathcal{N}^{s_\alpha g*} - \{\emptyset\}$. But $\mathcal{G} \cap \mathcal{H} = \left(ns_\alpha g - cl^*(\mathcal{H})\right)^c \cap \mathcal{H} = \emptyset$, which is a contradiction. Hence, the result.

Definition 4.15 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space. The family of all $\mathcal{N}^{s_\alpha g*}-$ dense subsets of Γ is denoted by $\mathcal{N}^{s_\alpha g\mathcal{D}^*}$. For the subspace $(\mathcal{V}, \mathcal{N}_\mathcal{V}, \mathcal{I}_\mathcal{V})$,the family of all $\mathcal{N}^{s_\alpha g*}-$ dense subsets of $\mathcal{V}$ is denoted by $\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V})$. (i.e.,) $\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V}) = \{\mathcal{H} \subset \mathcal{V}: ns_\alpha g - cl_{\mathcal{V}^*}(\mathcal{H}) = \mathcal{V}\}$ where $\mathcal{N}_\mathcal{V} = \{\mathcal{G} \cap \mathcal{V}: \mathcal{G} \in \mathcal{N}\}$ and $\mathcal{I}_\mathcal{V} = \{\mathcal{I} \cap \mathcal{V}: \mathcal{I} \in \mathcal{I}\}$. Obviously, $\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\Gamma) = \mathcal{N}^{s_\alpha g\mathcal{D}^*}$.

Example 4.16 Consider the $n\mathcal{I}-$ topological space $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ of Example 3.2. Refer the following table for the subspace $(\mathcal{V}, \mathcal{N}_\mathcal{V}, \mathcal{I}_\mathcal{V})$, the family of all $\mathcal{N}^{s_\alpha g*}-$ dense subsets of $\mathcal{V} \subseteq \Gamma$.

$\mathcal{V}$	$\mathcal{N}_\mathcal{V}$	$\mathcal{I}_\mathcal{V}$	$\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V})$
Δ_1	$\emptyset, \Delta_1, \{\delta_1\}, \{\delta_1, \delta_2, \delta_3\}, \{\delta_2, \delta_3\}$	$\emptyset, \{\delta_2\}$	$\{\delta_1, \delta_2, \delta_4\}, \Delta_1$
$\{\delta_1\}$	$\emptyset, \mathcal{V}$	$\emptyset$	$\mathcal{V}$
$\{\delta_2\}$	$\emptyset, \mathcal{V}$	$\emptyset, \{\delta_2\}$	$\mathcal{V}$
$\{\delta_3\}$	$\emptyset$	$\emptyset$	$\mathcal{V}$
$\{\delta_4\}$	$\emptyset, \mathcal{V}$	$\emptyset$	$\mathcal{V}$
$\{\delta_1, \delta_2\}$	$\emptyset, \mathcal{V}, \{\delta_1\}, \{\delta_2\}$	$\emptyset, \{\delta_2\}$	$\mathcal{V}$
$\{\delta_1, \delta_3\}$	$\emptyset, \mathcal{V}, \{\delta_1\}$	$\emptyset$	$\mathcal{V}$
$\{\delta_1, \delta_4\}$	$\emptyset, \mathcal{V}, \{\delta_1\}, \{\delta_4\}$	$\emptyset$	$\mathcal{V}$
$\{\delta_2, \delta_3\}$	$\emptyset, \mathcal{V}, \{\delta_2\}$	$\emptyset, \{\delta_2\}$	$\mathcal{V}$
$\{\delta_2, \delta_4\}$	$\emptyset, \mathcal{V}$	$\emptyset, \{\delta_2\}$	$\mathcal{V}$
$\{\delta_3, \delta_4\}$	$\emptyset, \mathcal{V}, \{\delta_4\}$	$\emptyset$	$\{\delta_4\}, \mathcal{V}$
$\{\delta_1, \delta_2, \delta_3\}$	$\emptyset, \mathcal{V}, \{\delta_1\}, \{\delta_2\}, \{\delta_1, \delta_2\}$	$\emptyset, \{\delta_2\}$	$\{\delta_1, \delta_2\}, \mathcal{V}$
$\{\delta_1, \delta_2, \delta_4\}$	$\emptyset, \mathcal{V}, \{\delta_1\}, \{\delta_2, \delta_4\}$	$\emptyset, \{\delta_2\}$	$\mathcal{V}$
$\{\delta_1, \delta_3, \delta_4\}$	$\emptyset, \mathcal{V}, \{\delta_1\}, \{\delta_4\}, \{\delta_1, \delta_4\}$	$\emptyset$	$\{\delta_1, \delta_4\}, \mathcal{V}$

$\{\delta_2,\delta_3,\delta_4\}$	$\emptyset,\mathcal{V},\{\delta_2,\delta_4\}$	$\emptyset,\{\delta_2\}$	$\{\delta_2,\delta_4\},\mathcal{V}$

Theorem 4.17 Let $(\Gamma,\mathcal{N},\mathcal{J})$ be a $n\mathcal{J}-$ topological space. Then the underneath affirmations are analogous.

(i) $(\Gamma,\mathcal{N},\mathcal{J})$ is $n\mathcal{J}s_\alpha g-$ scattered.

(ii) $ns_\alpha gI^*(\mathcal{V})\in\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V})$ for any $\mathcal{V}\in 2^\Gamma\setminus\{\emptyset\}$.

(iii) For any $\mathcal{V}\in 2^\Gamma\setminus\{\emptyset\}$, $\mathcal{D}\in\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V})$ if and only if $\mathcal{D}\supset ns_\alpha gI^*(\mathcal{V})$.

(iv) $ns_\alpha gd^*(\mathcal{V})=ns_\alpha gd^*\big(ns_\alpha gI^*(\mathcal{V})\big)$ for any $\mathcal{V}\in 2^\Gamma\setminus\{\emptyset\}$.

(v) If $\mathcal{V}\in\mathcal{N}^{s_\alpha g*}-\{\emptyset\}$, then $ns_\alpha gI^*(\mathcal{V})\neq\emptyset$.

Proof: (i) $\Longrightarrow$ (ii): Let $\mathcal{F}\in\mathcal{N}_\mathcal{V}^{s_\alpha g*}-\{\emptyset\}$. Then $\mathcal{F}=\mathcal{W}\cap\mathcal{V}$ for some $\mathcal{W}\in\mathcal{N}^{s_\alpha g*}$.Since $(\Gamma,\mathcal{N},\mathcal{J})$ is $n\mathcal{J}s_\alpha g-$ scattered, $ns_\alpha gI^*(F)\neq\emptyset$. Pick $\gamma\in ns_\alpha gI^*(F)$. Then $\mathcal{G}\cap F=\{\gamma\}$ for some $\mathcal{G}\in\mathcal{N}^{s_\alpha g*}(\gamma)$ so that $(\mathcal{G}\cap\mathcal{W})\cap\mathcal{V}=\mathcal{G}\cap(\mathcal{W}\cap\mathcal{V})=\mathcal{G}\cap F=\{\gamma\}$. Note that $\mathcal{G}\cap\mathcal{W}\in\mathcal{N}^{s_\alpha g*}(\gamma)$. This implies $\gamma\in ns_\alpha gI^*(\mathcal{V})$. Then $\gamma\in F\cap ns_\alpha gI^*(\mathcal{V})$ and so $\mathcal{F}\cap ns_\alpha gI^*(\mathcal{V})\neq\emptyset$. By Theorem 4.14, $ns_\alpha g-cl_\mathcal{V}^*\big(ns_\alpha gI^*(\mathcal{V})\big)=\mathcal{V}$.Thus, $ns_\alpha gI^*(\mathcal{V})\in\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V})$.

(ii) $\Longrightarrow$ (iii) : Let $D\supset ns_\alpha gI^*(\mathcal{V})$. By hypothesis, $\mathcal{V}=ns_\alpha g-cl_\mathcal{V}^*\big(ns_\alpha gI^*(\mathcal{V})\big)\subset ns_\alpha g-cl_\mathcal{V}^*(D)$. Thus $D\in\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V})$. Conversely, suppose $ns_\alpha gI^*(\mathcal{V})\not\subseteq D$ for some $D\in\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V})$. Then $ns_\alpha gI^*(\mathcal{V})\setminus D\neq\emptyset$. Pick $\gamma\in ns_\alpha gI^*(\mathcal{V})\setminus D$. Then $\mathcal{G}\cap\mathcal{V}=\{\gamma\}$ for some $\mathcal{G}\in\mathcal{N}^{s_\alpha g*}(\gamma)$. Note that $\mathcal{G}\cap\mathcal{V}\in\mathcal{N}_\mathcal{V}^{s_\alpha g*}(\gamma)$ and $D\in\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V})$. By Theorem 4.14, $D\cap(\mathcal{G}\cap\mathcal{V})\neq\emptyset$. But $D\cap(\mathcal{G}\cap\mathcal{V})=D\cap\{\gamma\}=$, a contradiction.

(iii) $\Longrightarrow$ (ii):The result is trivial.

(iii) $\Longrightarrow$ (iv) : Since $\mathcal{V}\supset ns_\alpha gI^*(\mathcal{V})$, we have $ns_\alpha gd^*(\mathcal{V})\supset ns_\alpha gd^*\big(ns_\alpha gI^*(\mathcal{V})\big)$.It suffices to show that $ns_\alpha gd^*(\mathcal{V})\subset ns_\alpha gd^*\big(ns_\alpha gI^*(\mathcal{V})\big)$. Suppose $ns_\alpha gd^*(\mathcal{V})\not\subseteq ns_\alpha gd^*\big(ns_\alpha gI^*(\mathcal{V})\big)$. Then $ns_\alpha gd^*(\mathcal{V})\setminus ns_\alpha gd^*\big(ns_\alpha gI^*(\mathcal{V})\big)\neq\emptyset$. Pick $\gamma\in ns_\alpha gd^*(\mathcal{V})\setminus ns_\alpha gd^*\big(ns_\alpha gI^*(\mathcal{V})\big)$.By Theorem 3.3 (i), $ns_\alpha gI^*(\mathcal{V})=\mathcal{V}\setminus ns_\alpha gd^*(\mathcal{V})$. Then $\gamma\notin ns_\alpha gI^*(\mathcal{V})$. $\gamma\notin ns_\alpha gd^*\big(ns_\alpha gI^*(\mathcal{V})\big)$ implies that $\mathcal{G}\cap(ns_\alpha gI^*(\mathcal{V})\setminus\{\gamma\})=\emptyset$ for some $\mathcal{G}\in\mathcal{N}^{s_\alpha g*}(\gamma)$.Note that $\gamma\notin ns_\alpha gI^*(\mathcal{V})$. Then $(\mathcal{G}\cap\mathcal{V})\cap ns_\alpha gI^*(\mathcal{V})=\mathcal{G}\cap ns_\alpha gI^*(\mathcal{V})=\emptyset$ with $\mathcal{G}\cap\mathcal{V}\in\mathcal{N}_\mathcal{V}^{s_\alpha g*}$. By hypothesis, $ns_\alpha gI^*(\mathcal{V})\in\mathcal{N}^{s_\alpha g\mathcal{D}^*}(\mathcal{V})$. Then $\mathcal{F}\cap ns_\alpha gI^*(\mathcal{V})\neq\emptyset$ for any $\mathcal{F}\in\mathcal{N}_\mathcal{V}^{s_\alpha g*}$, which is a contradiction. Hence, $ns_\alpha gd^*(\mathcal{V})=ns_\alpha gd^*\big(\mathcal{V}-ns_\alpha gd^*(\mathcal{V})\big)=ns_\alpha gd^*\big(ns_\alpha gI^*(\mathcal{V})\big)$.\\

(iv) $\Longrightarrow$ (i): Suppose $ns_\alpha gI^*(\mathcal{V}) = \emptyset$ for some $\mathcal{V} \in 2^\Gamma \setminus \{\emptyset\}$. By hypothesis, $ns_\alpha gd^*(\mathcal{V}) = ns_\alpha gd^*\big(ns\alpha gI^*(\mathcal{V})\big) = ns_\alpha gd^*(\emptyset) = \emptyset$. By Theorem 3.3 (iii), $\mathcal{V} = ns_\alpha gI^*(\mathcal{V}) \cup (ns_\alpha gd^*(\mathcal{V}) \cap \mathcal{V}=$, which is a contradiction.

(i) $\Longrightarrow$ (v): Since $(\Gamma, \mathcal{N}, \mathcal{I})$ is $n\mathcal{I}s_\alpha g -$ scattered space, the result is trivial.

(v) $\Longrightarrow$ (i): Let $\mathcal{V} \in 2^\Gamma \setminus \{\emptyset\}$. Since $ns_\alpha g - cl^*(\mathcal{V}) \in \mathcal{N}^{s_\alpha g*} \setminus \{\emptyset\}$, by hypothesis, $ns_\alpha gI^*\big(ns_\alpha g - cl*\mathcal{V} \neq \emptyset$. Pick $\gamma \in ns\alpha gI* ns\alpha g - cl*\mathcal{V}$. Then $\mathcal{F} \cap ns\alpha g - cl*\mathcal{V} = \{\gamma\}$ for some $\gamma \in \mathcal{N} s\alpha g*\gamma$. Suppose $\mathcal{F} \cap \mathcal{V} = \emptyset$. Then $\mathcal{F}^c \supset \mathcal{V}$ implies $\mathcal{F}^c \supset ns_\alpha g - cl^*(\mathcal{V})$. So $\mathcal{F} \cap ns_\alpha g - cl^*(\mathcal{V}) = \emptyset$, which is a contradiction. Thus, $\mathcal{F} \cap \mathcal{V} \neq \emptyset$. Since $\mathcal{F} \cap \mathcal{V} \subset \mathcal{F} \cap ns_\alpha g - cl^*(\mathcal{V}) = \{\gamma\}$, we have $\mathcal{F} \cap \mathcal{V} = \{\gamma\}$. So, $\gamma \in ns_\alpha gI^*(\mathcal{V})$ implies $ns_\alpha gI^*(\mathcal{V}) \neq \emptyset$. Hence, $(\Gamma, \mathcal{N}, \mathcal{I})$ is $n\mathcal{I}s_\alpha g -$ scattered space.

Theorem 4.18 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a n$\mathcal{I} -$ topological space and let $\mathcal{V} \in 2^\Gamma \setminus \{\emptyset\}$. If Γ is $n\mathcal{I}s_\alpha g -$ scattered, then $(\mathcal{V}, \mathcal{N}_\mathcal{V}, \mathcal{I}_\mathcal{V})$ is $n\mathcal{I}s_\alpha g -$ scattered with respect to $\mathcal{V}$.

Proof: Let $\mathcal{H} \in 2^\Gamma \setminus \{\emptyset\}$. Since Γ is $n\mathcal{I}s_\alpha g -$ scattered, $ns_\alpha gI^*(\mathcal{H}) \neq \emptyset$. Pick $\gamma \in ns_\alpha gI^*(\mathcal{H})$. Then $\mathcal{G} \cap \mathcal{H} = \{\gamma\}$ for some $\mathcal{G} \in \mathcal{N}^{s_\alpha g*}(\gamma)$. Note that $\mathcal{G} \cap \mathcal{V} \in \mathcal{N}_\mathcal{V}^{s_\alpha g*}(\gamma)$ and $(\mathcal{G} \cap \mathcal{V}) \cap \mathcal{H} = (\mathcal{G} \cap \mathcal{H}) \cap \mathcal{V} = \{\gamma\}$. Then $\gamma \in ns_\alpha gI_\mathcal{V}^*(\mathcal{H})$ so that $ns_\alpha gI_\mathcal{V}^*(\mathcal{H}) \neq \emptyset$. Hence $(\mathcal{V}, \mathcal{N}_\mathcal{V}, \mathcal{I}_\mathcal{V})$ is $n\mathcal{I}s_\alpha g -$ scattered with respect to $\mathcal{V}$.

Definition 4.19 A $n\mathcal{I} -$ topological space $(\Gamma, \mathcal{N}, \mathcal{I})$ is said to be:

(i) $n\mathcal{I} -$ resolvable if Γ has two disjoint $n\mathcal{I} -$ dense subsets. Otherwise, Γ is called $n\mathcal{I} -$ irresolvable.

(ii) $n\mathcal{I}s_\alpha g -$ resolvable if Γ has two disjoint $n\mathcal{I}s_\alpha g -$ dense subsets. Otherwise, Γ is called $n\mathcal{I}s_\alpha g -$ irresolvable.

Example 4.20 Consider the $n\mathcal{I} -$ topological space $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ of Example 3.2.

(i) $n\mathcal{I} -$ dense sets are $\Delta_1, \{\delta_1\}, \{\delta_2\}, \{\delta_3\}, \{\delta_1, \delta_2\}, \{\delta_1, \delta_3\}, \{\delta_1, \delta_4\}, \{\delta_2, \delta_3\}, \{\delta_2, \delta_4\}, \{\delta_3, \delta_4\}, \{\delta_1, \delta_2, \delta_3\}, \{\delta_1, \delta_2, \delta_4\}, \{\delta_1, \delta_3, \delta_4\}, \{\delta_2, \delta_3, \delta_4\}$. Therefore, $\{\delta_1\}, \{\delta_2, \delta_3, \delta_4\}$ (resp.$\{\delta_2\}, \{\delta_1, \delta_3, \delta_4\}$ and $\{\delta_3\}, \{\delta_1, \delta_2, \delta_4\}$) are pair of disjoint $n\mathcal{I} -$ dense sets. Hence $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ is $n\mathcal{I} -$ resolvable.

(ii) $n\mathcal{I}s_\alpha g -$ dense sets are $\Delta_1, \{\delta_1, \delta_2, \delta_3$. Therefore, $(\Delta_1, \mathcal{N}_1, \mathcal{I}_1)$ is $n\mathcal{I}s_\alpha g -$ irresolvable.

Remark 4.21 A $n\mathcal{I} -$ resolvable space need not be $n\mathcal{I}s_\alpha g -$ resolvable.

Theorem 4.22 Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space. If Γ is $n\mathcal{I}s_\alpha g-$ scattered, then Γ is $n\mathcal{I}s_\alpha g-$ irresolvable.

Proof: For any $\mathcal{H}, \mathcal{K} \in 2^\Gamma \setminus \{\emptyset\}$ with $\mathcal{H}^*_{ns_\alpha g} = \mathcal{K}^*_{ns_\alpha g} = \Gamma$ and $\Gamma = \mathcal{H} \cup \mathcal{K}$, we have $\mathcal{H}, \mathcal{K} \in \mathcal{N}^{s_\alpha g\mathcal{D}^*}(\Gamma)$. By Theorem 4.17, $\mathcal{H}, \mathcal{K} \supset ns_\alpha gI^*(\Gamma)$. Then $\mathcal{H} \cap \mathcal{K} \supset ns_\alpha gI^*(\Gamma)$. Since Γ is $n\mathcal{I}s_\alpha g-$ scattered, $ns_\alpha gI^*(\Gamma) \neq \emptyset$. So, $\mathcal{H} \cap \mathcal{K} \neq \emptyset$. Thus, Γ is $n\mathcal{I}s_\alpha g-$ irresolvable.

Remark 4.23 A $n\mathcal{I}s_\alpha g-$ irresolvable space need not be $n\mathcal{I}s_\alpha g-$ scattered.

For instance, consider the $n\mathcal{I}-$ topological space $(\Delta, \mathcal{N}, \mathcal{I})$ as follows:

$\Delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}$; $\Delta/\mathcal{R} = \{\delta_1, \{\delta_2, \delta_3\}, \{\delta_4\}$; $\mathcal{X} = \{\delta_1, \delta_4\}$; $\mathcal{N} = \{\emptyset, \Delta, \{\delta_1, \delta_4\}\}$; $\mathcal{I} = \{\emptyset, \{\delta_1\}\}$. The space is $n\mathcal{I}s_\alpha g-$ irresolvable but it is not $n\mathcal{I}s_\alpha g-$ scattered, since for the set $\mathcal{H} = \{\delta_2, \delta_3\}$, $ns_\alpha gI^*(\mathcal{H}) = \emptyset$.

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