Mathematical Statistician and Engineering Applications
ISSN: 2094-0343
2326-9865

Probability Distributions of Hidden Markov Models for Stock Closing Prices

Tirupathi Rao Padi[1], *Sarode Rekha[2], Gulbadin Farooq Dar[3]

[1,2,3]Department of Statistics, Pondicherry University, Puducherry, India
[1]drtrpadi@gmail.com
[2]statisticsrekha@gmail.com
[3]gulbadinstst.pu@gmail.com

Article Info
Page Number: 7174-7198
Publication Issue:
Vol. 71 No. 4 (2022)

Article History
Article Received: 25 March 2022
Revised: 30 April 2022
Accepted: 15 June 2022
Publication: 19 August 2022

Abstract

This study is on developing a Hidden Markov Model (HMM) for a daily closing price of the National Stock Exchange (NSE) and Industrial Credit and Investment Corporation of India (ICICI) bank. The changing states of NSE are considered hidden and the influencing states of change in the visible states of ICICI bank. Obtained the parameters of HMM namely Initial Probability Vector (IPV), Transition Probability Matrix (TPM) and Observed Probability Matrix (OPM) by assuming the discrete Markov chains among (i) within hidden states (*Gain* & *Loss* in NSE) and (ii) between hidden and observed states (*Rise* & *Fall*) respectively. Separate probability distributions for a sequence of n days' transits on *Rise* & *Fall* are formulated. The behaviours of *Rise* and *Fall* states in the closing prices of ICICI bank are explored through explicit mathematical relations of different statistical measures and the related Pearson's coefficients. Numerical illustrations are considered for a proper understanding of the developed models. These models will be useful for short-term business investors for getting the indicators on when to sell and when to purchase by observing the chances of emission states. Computer automation will make this tool more user-friendly in understanding the day-to-day share market contexts.

Keywords: - Hidden Markov Model; Transition Probability Matrix; Stock Market; Share Price; Probability distribution; Observed Probability Matrix; Initial Probability Vector.

1. Introduction

To keep the resources, evergreen without expiry they have to be maintained in a reproductive cycle. Management of resources like Money, Water, Mines, Seeds, Time, etc. requires suitable prototyping for their continuous and never-ending utilization. Agriculture is the process of keeping seeds alive and making the seeds a long-time living organism. Similarly, investment is the process to keep the money/ resources/ assets for never lasting life. Hence, needless to say, for keeping the resources in continuous and

* Corresponding Author
Email ID: statisticsrekha@gmail.com

uninterrupted usage. They have to be in a cycle of initial investment, finance culturing, asset transformation, asset growth, money cropping, investment yielding, reinvestment, etc. For instance, if the water is allowed to stagnate without flow, it will become silty and useless. Hence, in order to keep the water in usable conditions the system has to make the water in a cycle of continuous flow. The formats of the assets are classified in two ways namely movable and fixed. Immovable assets like lands, buildings and other fixed assets are rigid in transforming them from one state to another. Liquidation is one the most flexible state where the money can have an easy flow to any transition state. Investment is the process of growing the current levels of assets/ money/ resources into improving states. Finance institutions are the entities where the trade of investment will be taken place. They are the industries for making and growing the products of monetary funds. Stock exchanges and banks are similar agencies that will handle the task of money growth as productivity. Banking is one of the most important industries to influence national Inco me. The banking industries have multiple investments as inputs for extracting several revenue outputs.

The stock exchange plays a vital role in the progress of the economy. It is a place where securities, shares, bonds and other financial products are available for transactions of buy and sale. The business of stock exchange products is mostly dealt with by traders and brokers. Stock exchanges help companies to Rise funds. Therefore, the company needs to list itself on the stock exchange. Shares listed on the stock exchange are known as equity and these shareholders are known as Equity Shareholders. It acts as a continuous market for securities by taking the responsibilities of evaluation, mobilization, savings, speculation, protecting the investors and ensuring liquidity. It will exercise vigilance, monitoring, abstracting, etc. in any financial portfolio. It ensures the safety of capital with fair dealing and regulating company management.

The flow of investments from different sources either internal or external mode needs standard operating protocols. The portfolio manager has to plan the investment policies of the organization by allocating the funds in risky and non-risky assets on the basis of both long-term and short-term periods. He shall make use of all possible alternatives regarding the diversification of investments more specific to risky assets. The usual market behaviour suggests that investing in risky assets has the potential scope for getting more returns compared to investing in non-risky assets. The reason is quite obvious the non-risky assets have a slower rate of returns mostly linked to the product under the transaction. It is interesting to observe that the incomings and outgoings of the investment market are more random and influenced by many factors of uncertainty. The model developers shall formulate the stochastic models as they are non-deterministic [6].

By using the Markov chain, the performance of the stock exchange market of Nigeria has been studied to decide the moment between the states like rising and falling. Expected long-run and short-run returns were calculated, and the outcome were compared to the expected profit of the CAPM. It was found that, regardless of the current situation, both the share and the expected rate of return of a CAPM will be attained in the long run [1]. With the help of Markov-switching Bayesian VAR models, the non-linear relationship between the price of gold and the stock market index is examined and assessed. The number regime is investigated with the LR test, MCMC algorithm, and by employing the Sims and Zan 1989 prior distribution to estimate the model [2]. The behaviour of the stock price of the State Bank of India (SBI) was analyzed by adopting a Markov chain model [3]. The Markov chain has been used to investigate the Nepal Stock Exchange Index, in order to find the anticipated number of visits to a particular state and to calculate the anticipated initial return time of distinct states [4]. A two-state (Low and High volatility business) Capital Asset Price Model (CAPM) is proposed by assuming excess returns of the market and specific security are bivariate and normally distributed. MLEs are obtained with the Baum-Welch algorithm for the Markov regime model (MRM) or hidden Markov model (HMM) [5]. The forecasting model used a Markov chain to predict the stock price by combining two Markov chain models. The first is a usual Markov chain model that illustrates trends in stabilizing states over time distribution, and the second is a absorbing Markov chain that gives details on the interval between increases before reaching the decreasing state [7]. For predicting the customer behaviour in E-commerce with a target of estimating the store income the Customer Behaviour Hidden Markov Model has been proposed [8]. Eight discrete state Markov chain process is applied to model the Istanbul Stock Exchange 100 (ISE100) index and provided signals to the investors regarding the short-run selling and buying investment strategies [9]. Multivariate models for the analysis of stock market returns were introduced by outlining the EM algorithm for MLE that exploit recursions within Hidden and semi-Markov literature [10]. HMM is used for predicting the daily share price of three equities namely Apple, Google, and Facebook [11]. A multi-step process is designed to choose equities from the global stock market using HMM [12]. Markov chains model has been utilized for predicting the financial time series [13]. The stock index trend of the Prague stock exchange (PX) for the daily closing price was predicted through Markov chain analysis [14]. The variance of excess return depends on the state of the variable generated by the first-order Markov process. Further, mean excess returns move in the inverse direction of the risk level. It is observed there is a negative correlation between moments volatility and excess returns [15]. The Extended coupled Hidden Markov model has introduced the incorporation of new events with the historical trading data to study the methodology of exploring the multi-data sources to improve the performance of the stock prediction [16]. The HMM used for proper understanding of the financial variables in share market and these findings more useful for portfolio managers to make optimal decision [17]. The application of Markov model used for analyzing the moment of share market and forecasting the value of share [18]. Development and analysis of one of the stochastic model i.e., Markov model used for forecasting the share price in stock market [19]

It is observed that most of the studies have considered either classical works of developing new models or applying the available Markov models to speculate the market behaviour. The estimates and predictions are primarily on obtaining the Markov processes for transient cases. Studies on CAPM estimates and HMM are also meant for addressing the concerns of forecasting market demands. Times series modelling with Markov processes is also reported in the literature on how to obtain the expected returns. To a greater extent, the decision-making of selling or purchasing shares have been decided through the stationarity of Markov processes and Kolmogorov’s higher-order transition probabilities.

However, there is little evidence of a study of the transit state’s behaviour when they happen in a sequence or successive happenings of the possible states of variation. In order to assess the market behaviour given to short-term investors, the sequence of different lengths for two, three, four and five days business sessions will be a requirement. There is a need of computing the probabilities of diversified combinations and sequence lengths among *Rise* & *Fall* states on the observed end. It is required for finding the probability distributions of length one, two, three and so on 'n' days sequence of each state. Finding the probability of occurring the state *Rise*/*Fall* for ‘r’ times out of a sequence length ‘n’ ($0 \leq r \leq n$) is essential for the optimal decision-making on the buy/ sale of the shares. This said research gap has motivated us for taking up this study.

This study is on the formulation of a hidden Markov model of two hidden states (*Gain* & *Loss*) among the NSE closing prices that influence two observed states (*Rise* & *Fall*) of closing prices of ICICI bank’s shares. Usual hidden Markov modelling needs three parameters namely, (i) Initial Probability Vector (IPV) speaks about the chances of defined hidden states in the process; (ii) Transition Probability Matrix (TPM) deals with hidden states where the transitions will be within and between states; (iii) Observed Probability Matrix (OPM) explores the transition probabilities from hidden states to observed states. The authors are making their contributions to this study with the objectives of (i) to formulate the probability distribution models for *Rise* and *Fall* states separately; (ii) to derive the explicit mathematical relations of different statistical measures and Pearson's coefficients; (iii) to apply the real-time data of closing prices of NSE and ICICI banks and to perform the sensitivity analysis by finding the HMM parameters; and (iv) to obtain the probability distributions and statistical measures for proper understanding of the market behaviour. The purpose of this study is also to examine how the stock price of ICICI has changed over time through the developed mathematical models to understand the behaviour of stock market transactions.

This study has two major parts. The first part is on the formulation of HMM, derivation of PMF of the two defined states namely *Rise* and *Fall;* and finding the explicit mathematical relations of different statistical properties from the derived PMF. The second part is on focused the sensitivity analysis of the real-time data and understanding the model behaviour of the observed states.

2. Stochastic Model

As the proposed processes that will change the closing prices of NSE and ICICI banks are purely random and non-deterministic, understanding the market behaviour is probabilistic. Many unexplained reasons that influence the patterns prompted the study through stochastic modeling rather than mathematical modeling.

2.1. Markov Model

A random process called a Markov chain is employed in decision-making situations where the likelihood of transitioning to any future state depends only on the present state and not on how that particular state was reached. This can be modelled mathematically as

$$P\{X_{n+1}=j/(X_n=i, X_{n-1}=i-1, X_{n-2}=i-2,..,X_2=2,X_1=1, X_0=0)\}= P\{X_{n+1}=j/X_n=i\}=P_{ij}=a_{ij} \qquad (1)$$

Markov analysis includes examining a system's current behaviour in order to forecast its future behaviour. Russian mathematician Andrey A. Markov introduces this. Markov processes have a general theory that was established by A N Kolmogrov, W Feller, and others. Markov processes are a certain special class of mathematical models that are widely used in decision problems associated with dynamic systems.

2.1.1. Transition Probability Matrix (TPM)

The change of system from one state to another state is called transition and the probability associated with this state transition is called transition probability. The transition of the state 'i' to state 'j' in one step i.e., $(n-1)^{th}$ step to n^{th} step is called one step or unit step or identity step transition probability. Which can be represented as follows $a_{ij}^{(1)} = P\left\{X_{n+1} = j / X_n = i\right\}$. Similarly, the transition of the state 'i' to state 'j' in 'm' steps i.e., n^{th} step to $(m+n)^{th}$ step which is called m^{th} step transition probability is denoted as $a_{ij}^{(m)} = P\left\{X_{m+n} = j / X_n = i\right\}$. In general, every TPM must satisfy some important properties like the matrix must be a square (n×n) matrix, the values in the matrix should be probabilities ($0 \le a_{ij} \le 1$) and each row sum are equal to 1, $\sum_j a_{ij} = 1$.

2.2. Hidden Markov Model (HMM)

The hidden Markov model (HMM) is another type of stochastic model in which the process being described is regarded as a Markov process with hidden (unobserved) states. The HMM is composed of two different types of stochastic processes: an unobserved process of hidden states and an observed process of visible states. The hidden states from a Markov chain and the probability distribution of the visible states depend on the underlying state. Due to this cause, HMM is also known as a double binding stochastic process and it is denoted by λ=(A,B,$\hat{\pi}_i$) , Where 'A' is the TPM among hidden states, 'B' is OPM from hidden states to visible states, '$\hat{\pi}_i$' denotes the initial probability vector for hidden states.

2.2.1. Transition Probability Matrix (TPM) among hidden states

The following TPM shows the transition from one hidden state to the next hidden state, such as from H_1 to H_1, H_1 to H_2, H_2 to H_1, and H_2 to H_2. $A=\left[a_{ij}\right],\ \sum_{j=1}^{2} a_{ij}=1$; for i, j=1,2.

2.2.2. Observed Probability Matrix (OPM) between hidden to observed states

OPM denotes the transition from the hidden state to the visible state, such as from H_1 to V_1, H_1 to V_2, H_2 to V_1, and H_2 to V_2. This indicates the impact of the concealed state on the visible state. $B=\left[b_{ij}\right]$ for i, j=1,2.

2.2.3. Initial Probability Vector for hidden states

The initial probability vector holds the initial probability values of hidden states $\hat{\pi}_i=\left(\hat{\pi}_1,\hat{\pi}_2\right)$ for i=1,2 ;

$$\sum_{i=1}^{2}\hat{\pi}_i=1.$$

2.2.4. Notation and Terminology

The notations and terminology used in the current work are listed below.

$\hat{\pi}_i$	:	Initial Probability for i^{th} hidden state
X_n	:	The resulting value of hidden state at n^{th} trail; n=1,2, 3,…
Y_m	:	The resulting value of visible state at m^{th} trail; m=1,2, 3,…
I	:	Origin's (From/Previous) state; i=1,2
J	:	Destination's (To/Current) state; j=1,2
a_{ij}	:	Transition probability from i^{th} hidden state to j^{th} hidden state; for i=1,2 and j=1,2; $P\{X_{n+1}=j/X_n=i\}\geq 0$
b_{ij}	:	Observed probability from i^{th} hidden state to j^{th} visible state; for i=1,2 and j=1,2; $P\{Y_{m+1}=j/X_n=i\}\geq 0$
H_1	:	Gaining state of NSE's closing price
H_2	:	Losing state of NSE's closing price
V_1	:	Raising a state of ICICI bank's closing price
V_2	:	Falling state of ICICI bank's closing price
$X(\omega_{n1})$	:	A study of a number of times the *Rise* (V_1) state is obtained in the sequence of n days ahead; ω_{n1}=0,1, 2,…,n

$X(\omega_{n2})$: A study of a number of times the *Fall* (V_2) state is obtained in the sequence of n days ahead; ω_{n1} =0,1, 2,…,n

2.3. Schematic diagrams for HMM of *Rise* and *Fall* state occurs in n days ahead sequence

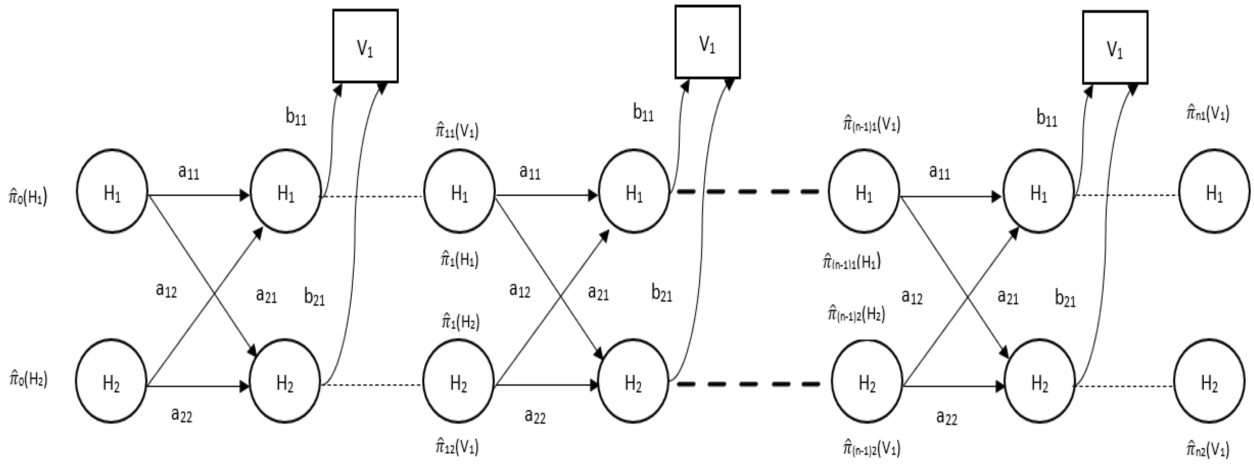

Figure 1: Schematic diagram of HMM of n days ahead for all n days in Rise (V1) State

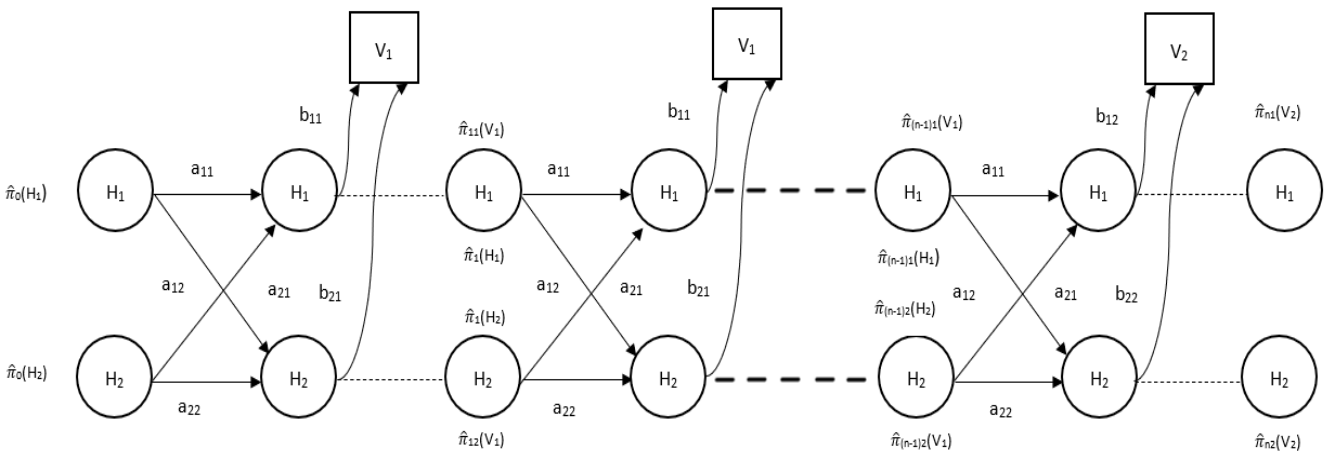

Figure 2: Schematic diagram of HMM of n days ahead for (n-1) days are Rise (V1) and last day Fall (V2) state

And so on,

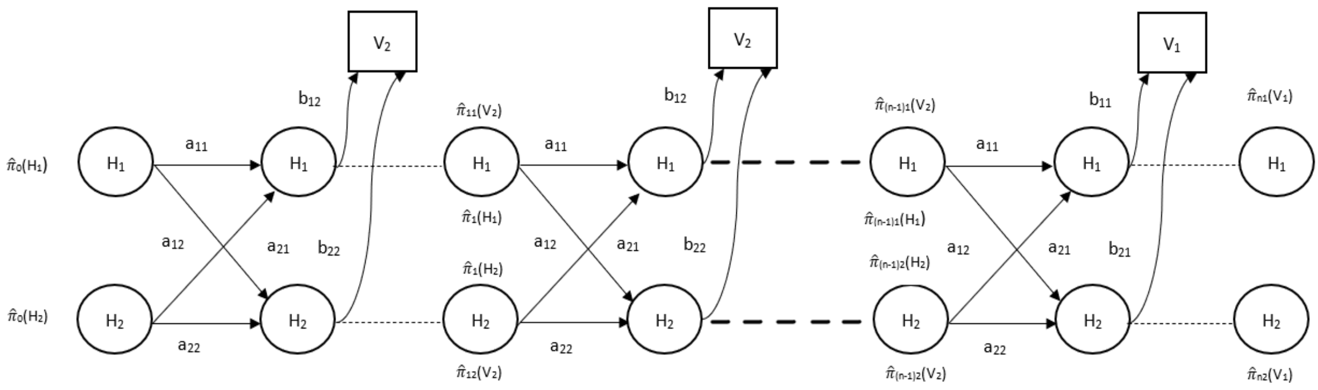

Figure 3: Schematic diagram of HMM of n days ahead for (n-1) days are Fall (V2) and last day Rise (V1) state

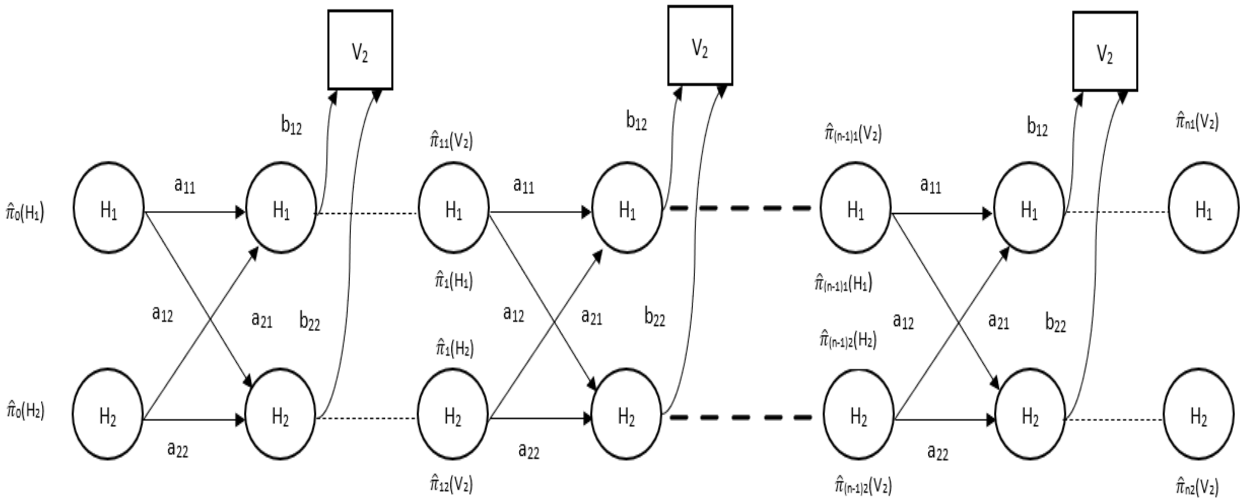

Figure 4: Schematic diagram of HMM of n days ahead for all days are Fall (V2) state

3. PMF for *Rise* state occurs in n days ahead sequence

Assume that $X(\omega_{n1})$ is a random variable that reflects the number of times a *Rise* state of the stock price has occurred. It takes the value '0' if there is no occurrence of *the Rise* state, it takes '1' if one time *Rise* state occurs and so on, it takes value n if all 'n' days occurring *Rise* state in n days ahead sequence study of *Rise* state. The probability mass function of *Rise* the state of 'n' days ahead is

$$P\left[X\left(\omega_{n1}\right)=x\right]=\begin{cases}\prod\limits_{k=1}^{n}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right] & ;\text{ for } x=0\\ \left[\prod\limits_{k=1}^{n-1}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ni}\left(V_1\right)\right]\right]+ & \\ \left[\prod\limits_{k=1,k\neq(n-1)}^{n}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum\limits_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_1\right)\right]\right]+ & \\ \ldots+\left[\prod\limits_{k=2}^{n}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum\limits_{i=1}^{2}\hat{\pi}_{1i}\left(V_1\right)\right]\right] & ;\text{ for } x=1\\ \left[\prod\limits_{k=1}^{n-2}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod\limits_{k=n-1}^{n}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+ & \\ \left[\prod\limits_{k=1,k\neq(n-2)}^{n-1}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod\limits_{k=n-2,k\neq(n-1)}^{n}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+ & \\ \ldots+\left[\prod\limits_{k=3}^{n}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod\limits_{k=1}^{2}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right] & ;\text{ for } x=2\\ \vdots & \\ \prod\limits_{k=1}^{n}\left[\sum\limits_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right] & ;\text{for } x=n\\ 0 & ;\text{otherwise}\end{cases} \tag{2}$$

3.1. Statistical measures for *Rise* state in a sequence of n days ahead

The average number of times a *Rise* state observed

$$\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_1\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_1\right)\right]\right]+$$

$$...+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_1\right)\right]\right]+2\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\right.$$

$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\right]\right\}+ \qquad (3)$$

$$....+n\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]$$

The variance of the *Rise* state is observed

$$\mu^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+(1-\mu)^2\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_1\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_1\right)\right]\right]+\right.$$

$$\left....+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_1\right)\right]\right]\right\}+(2-\mu)^2\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\right.$$

$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\right]\right\}+ \qquad (4)$$

$$....+(n-\mu)^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]$$

The third central moment for *Rise* state is observed

$$-\mu^3\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+(1-\mu)^3\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_1\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_1\right)\right]\right]+\right.$$

$$\left....+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_1\right)\right]\right]\right\}+(2-\mu)^3\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\right.$$

$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\right]\right\}+ \qquad (5)$$

$$....+(n-\mu)^3\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]$$

The mode for *Rise* state is observed

$$
\begin{aligned}
&\mu-\Bigg\{\Bigg\{\mu^{3}\Bigg[\prod_{k=1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Bigg]+(1-\mu)^{3}\Bigg\{\Bigg[\prod_{k=1}^{n-1}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{ni}(V_1)\Big]\Bigg]+\Bigg[\prod_{k=1,k\neq(n-1)}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}(V_1)\Big]\Bigg]+\\
&\ldots+\Bigg[\prod_{k=2}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{1i}(V_1)\Big]\Bigg]\Bigg\}+(2-\mu)^{3}\Bigg\{\Bigg[\prod_{k=1}^{n-2}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=n-1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]+\\
&\Bigg[\Bigg[\prod_{k=1,k\neq(n-2)}^{n-1}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=n-2,k\neq(n-1)}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]+\ldots+\Bigg[\prod_{k=3}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=1}^{2}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]\Bigg\}+\\
&\ldots.+(n-\mu)^{3}\Bigg[\prod_{k=1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]\Bigg\}\Bigg\{\mu^{2}\Bigg[\prod_{k=1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Bigg]+(1-\mu)^{2}\Bigg\{\Bigg[\prod_{k=1}^{n-1}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{ni}(V_1)\Big]\Bigg]+\\
&\Bigg[\prod_{k=1,k\neq(n-1)}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}(V_1)\Big]\Bigg]+\ldots+\Bigg[\prod_{k=2}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{1i}(V_1)\Big]\Bigg]\Bigg\}+(2-\mu)^{2}\\
&\Bigg\{\Bigg[\prod_{k=1}^{n-2}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=n-1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]+\Bigg[\prod_{k=1,k\neq(n-2)}^{n-1}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=n-2,k\neq(n-1)}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]+\\
&\ldots+\Bigg[\prod_{k=3}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=1}^{2}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]\Bigg\}+\ldots.+(n-\mu)^{2}\Bigg[\prod_{k=1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]\Bigg\}^{-1}\Bigg\}
\end{aligned}
\tag{6}
$$

Pearson’s coefficient of skewness for *Rise* state

$$
\begin{aligned}
&\Bigg\{-\mu^{3}\Bigg[\prod_{k=1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Bigg]+(1-\mu)^{3}\Bigg\{\Bigg[\prod_{k=1}^{n-1}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{ni}(V_1)\Big]\Bigg]+\Bigg[\prod_{k=1,k\neq(n-1)}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}(V_1)\Big]\Bigg]+\\
&\ldots+\Bigg[\prod_{k=2}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{1i}(V_1)\Big]\Bigg]\Bigg\}+(2-\mu)^{3}\Bigg\{\Bigg[\prod_{k=1}^{n-2}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=n-1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]+\\
&\Bigg[\Bigg[\prod_{k=1,k\neq(n-2)}^{n-1}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=n-2,k\neq(n-1)}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]+\ldots+\Bigg[\prod_{k=3}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=1}^{2}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]\Bigg\}+\\
&\ldots.+(n-\mu)^{3}\Bigg[\prod_{k=1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]\Bigg\}^{2}\Bigg\{\mu^{2}\Bigg[\prod_{k=1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Bigg]+(1-\mu)^{2}\Bigg\{\Bigg[\prod_{k=1}^{n-1}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{ni}(V_1)\Big]\Bigg]+\\
&\Bigg[\prod_{k=1,k\neq(n-1)}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}(V_1)\Big]\Bigg]+\ldots+\Bigg[\prod_{k=2}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\Big[\sum_{i=1}^{2}\hat{\pi}_{1i}(V_1)\Big]\Bigg]\Bigg\}+(2-\mu)^{2}\\
&\Bigg\{\Bigg[\prod_{k=1}^{n-2}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=n-1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]+\Bigg[\prod_{k=1,k\neq(n-2)}^{n-1}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=n-2,k\neq(n-1)}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]+\\
&\ldots+\Bigg[\prod_{k=3}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\Big]\prod_{k=1}^{2}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]\Bigg\}+\ldots.+(n-\mu)^{2}\Bigg[\prod_{k=1}^{n}\Big[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\Big]\Bigg]\Bigg\}^{-3}
\end{aligned}
\tag{7}
$$

Pearson's coefficient of Kurtosis for *Rise* state

$$\Bigg\{\mu^4\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+(1-\mu)^4\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_1\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_1\right)\right]\right]+\right.$$
$$\left.\ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_1\right)\right]\right]\right\}+(2-\mu)^4\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\right.$$
$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\right]\right\}+$$
$$\ldots.+(n-\mu)^4\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\Bigg\}\Bigg\{\mu^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+(1-\mu)^2\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_1\right)\right]\right]+\right.$$
$$\left.\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_1\right)\right]\right]+\ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_1\right)\right]\right]\right\}+(2-\mu)^2 \tag{8}$$
$$\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\right.$$
$$\left.\ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\right\}+\ldots.+(n-\mu)^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\Bigg\}^{-2}$$

3.2 Moment Generating Function of number of *Rise* states

As the random variable $X(\omega_{n1})$ represents the number of *Rise* states of transactions, the moment generating function of

$$M_{X_{n1}}(t)=\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+e^{t}\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_1\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_1\right)\right]\right]+\right.$$
$$\left.\ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_1\right)\right]\right]\right\}+e^{2t}\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\right.$$
$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\right]\right\}+ \tag{9}$$
$$\ldots.+e^{nt}\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]$$

3.3 Characteristic Function of number of *Rise* states

As the random variable $X(\omega_{n1})$ represents the number of *Rise* states of transactions, the characteristic function of

$$\phi_{X_{n1}}(t)=\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+e^{it}\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_1\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_1\right)\right]\right]+\right.$$
$$\left....+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_1\right)\right]\right]\right\}+e^{2it}\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\right.$$
$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\right]\right\}+ \tag{10}$$
$$....+e^{nit}\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]$$

3.4 Probability Generating Function of number of *Rise* states

As the random variable $X(\omega_{n1})$ represents the number of *Rise* states of transactions, the probability generating function of

$$P_S(t)=\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+S\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_1\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_1\right)\right]\right]+\right.$$
$$\left....+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_1\right)\right]\right]\right\}+S^{2}\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+\right.$$
$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]\right]\right\}+ \tag{11}$$
$$....+S^{n}\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]$$

4. PMF for *Fall* state occurs in n days ahead sequence

Assume that $X(\omega_{n2})$ is a random variable that reflects the number of times a *Fall* state of the stock price has occurred. It takes the value ‘0’ if there is no occurrence of *the Fall* state, it takes ‘1’ if one time *Fall* state occurs and so on, it takes value n if all ’n’ days occurring *Fall* state in n days ahead sequence study of *Fall* state. The probability mass function of *Fall* the state of ‘n’ days ahead is

$$P\left[X(\omega_{n2})=x\right]=\begin{cases}\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right] & ;\ \text{for } x=0\\ \left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}(V_2)\right]\right]+ & \\ \left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}(V_2)\right]\right]+ & \\ \ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}(V_2)\right]\right] & ;\ \text{for } x=1\\ \left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\right]\right]+ & \\ \left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\right]\right]+ & \\ \ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\right]\right] & ;\ \text{for } x=2\\ \vdots & \\ \prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\right] & ;\ \text{for } x=n\\ 0 & ;\ \text{otherwise}\end{cases} \tag{12}$$

4.1. Statistical measures for *Fall* state in a sequence of n days ahead

The average number of times a *Fall* state observed

$$\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}(V_2)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}(V_2)\right]\right]+$$
$$\ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}(V_2)\right]\right]+2\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\right]\right]+\right.$$
$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\right]\right]+\ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_1)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\right]\right]\right]\right\}+ \tag{13}$$
$$\ldots.+n\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}(V_2)\right]\right]$$

The variance of the *Fall* state is observed

$$\mu^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+(1-\mu)^2\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\right.$$
$$...+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\Bigg\}+(2-\mu)^2\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$
$$\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}+ \tag{14}$$
$$....+(n-\mu)^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]$$

The third central moment for *Fall* state is observed

$$-\mu^3\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+(1-\mu)^3\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\right.$$
$$...+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\Bigg\}+(2-\mu)^3\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$
$$\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}+ \tag{15}$$
$$....+(n-\mu)^3\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]$$

The mode for *Fall* state is observed

$$\mu-\left\{\left\{\mu^3\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+(1-\mu)^3\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\right.\right.\right.$$
$$...+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\Bigg\}+(2-\mu)^3\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$
$$\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}+$$
$$....+(n-\mu)^3\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\Bigg\}\left\{\mu^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+(1-\mu)^2\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\right.\right.$$
$$\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+...+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\Bigg\}+(2-\mu)^2 \tag{16}$$
$$\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$
$$\left....+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}+....+(n-\mu)^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\Bigg\}^{-1}\Bigg\}$$

ISSN: 2094-0343
2326-9865

Pearson's coefficient of skewness for *Fall* state

$$\left\{-\mu^3\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+(1-\mu)^3\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\right.\right.$$

$$\left.\ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\right\}+(2-\mu)^3\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$

$$\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}+$$

$$\left.\left.\ldots.+(n-\mu)^3\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right]\right\}^2\left\{\mu^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+(1-\mu)^2\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\right.\right.$$

$$\left.\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\right\}+(2-\mu)^2 \tag{17}$$

$$\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$

$$\left.\left.\ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}+\ldots.+(n-\mu)^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}^{-3}$$

Pearson's coefficient of Kurtosis for *Fall* state

Vol. 71 No. 4 (2022)
http://philstat.org.ph

$$\left\{\mu^4\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+(1-\mu)^4\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\right.\right.$$
$$\left.\ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\right\}+(2-\mu)^4\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$
$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right]\right\}+$$
$$\left.\ldots+(n-\mu)^4\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}\left\{\mu^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+(1-\mu)^2\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\right.\right.$$
$$\left.\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\right\}+(2-\mu)^2 \tag{18}$$
$$\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$
$$\left.\left.\ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}+\ldots+(n-\mu)^2\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right\}^{-2}$$

4.2. Moment Generating Function of number of *Fall* states

As the random variable $X(\omega_{n2})$ represents the number of *Fall* states of transactions, the moment generating function of

$$M_{X_{n2}}(t)=\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right]+e^{t}\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\right.$$
$$\left.\ldots+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\right\}+e^{2t}\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$
$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\ldots+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right]\right\}+ \tag{19}$$
$$\ldots+e^{nt}\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]$$

4.3 Characteristic Function of number of *Fall* states

As the random variable $X(\omega_{n2})$ represents the number of *Fall* states of transactions, the characteristic function of

$$\phi_{X_{n2}}(t) = \left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right] + e^{it}\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\right.$$

$$\left. ...+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\right\}+e^{2it}\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$

$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right]\right\}+ \tag{20}$$

$$....+e^{nit}\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]$$

4.4. Probability Generating Function of number of *Fall* states

As the random variable $X(\omega_{n2})$ represents the number of *Fall* states of transactions, the probability generating function of

$$P_S(t) = \left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\right] + S\left\{\left[\prod_{k=1}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{ni}\left(V_2\right)\right]\right]+\left[\prod_{k=1,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{(n-1)i}\left(V_2\right)\right]\right]+\right.$$

$$\left. ...+\left[\prod_{k=2}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\left[\sum_{i=1}^{2}\hat{\pi}_{1i}\left(V_2\right)\right]\right]\right\}+S^2\left\{\left[\prod_{k=1}^{n-2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+\right.$$

$$\left.\left[\left[\prod_{k=1,k\neq(n-2)}^{n-1}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=n-2,k\neq(n-1)}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]+...+\left[\prod_{k=3}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_1\right)\right]\prod_{k=1}^{2}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]\right]\right\}+ \tag{21}$$

$$....+S^n\left[\prod_{k=1}^{n}\left[\sum_{i=1}^{2}\hat{\pi}_{ki}\left(V_2\right)\right]\right]$$

5. Data Analysis & Discussion

In order to make use of the developed HMM a real-time data on closing prices of NSE and ICICI banks have been considered. Data on closing prices of the stock market about 518 observations are collected for the period from 1st January 2020 to 28th January 2022 from the websites http://in.finance.yahoo.com and http://www.indianfoline.com. The first-order finite differences were calculated using $\Delta_t = U_{t+1} - U_t$. The transit states *Gain* and *Loss* are identified from NSE finite differences. State -H_1(*Gain*) is considered if Δ_t is non-negative ($\Delta_t \geq 0$), whereas, State-H_2 (*Loss*) is considered if Δ_t value is negative ($\Delta_t < 0$). On similar lines, the changes in the closing prices of ICICI bank are considered to identify the visible/observed states as State-V_1 (*Rise*) and State-V_2 (*Fall*).

Sample data format

The collected data from the internet sources was placed in an excel data matrix as below

Table 1: Sample data format for NSE and ICICI bank's closing price

S. No.	Date	NSE closing price X_t	ICICI closing price Y_t	$\Delta X_t = X_{t+1} - X_t$	$\Delta Y_t = Y_{t+1} - Y_t$	NSE stated of X_t (*Gain /Loss*)	ICICI stated of Y_t *(Rise /Fall)*
1	1.1.2020	12182.5	537	-	-	-	-
2	2.1.2020	12282.2002	541	99.70019	4	G	R
3	3.1.2020	12226.65039	539	-55.5498	-2	L	F
4	6.1.2020	11993.04981	526	-233.601	-13	L	F
5	7.1.2020	12052.9502	523	59.90039	-3	G	F
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
514	21.1.2022	17617.15039	805	-139.85	-5	L	F
515	24.1.2022	17149.09961	798	-468.051	-7	L	F
516	25.1.2022	17277.94922	802	128.8496	4	G	R
517	27.1.2022	17110.15039	795	-167.799	-7	L	F
518	28.1.2022	17101.95	781	-8.20039	-14	L	F

The ΔX_t and ΔY_t which are fluctuations of NSE and ICICI are exhibited in the following graphs.

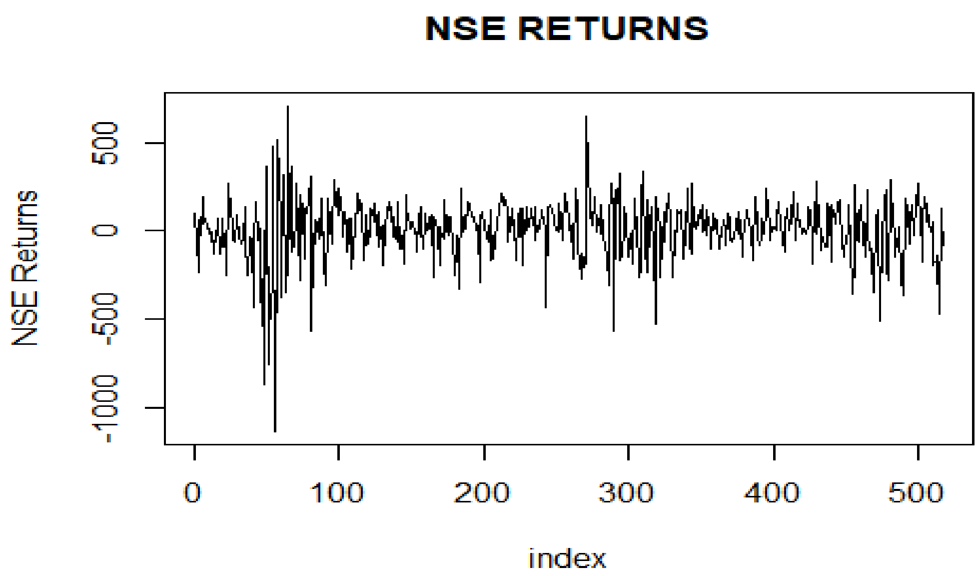

Figure 5: Market trend of NSE

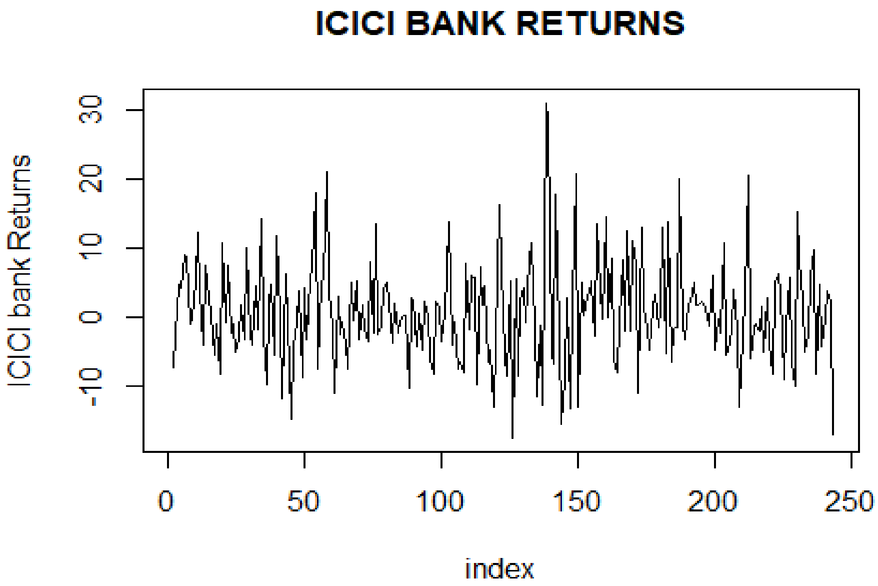

Figure 6: Stock Market Trend of ICICI

Figures 5 and 6 speak about the fluctuations of stock prices in the returns of NSE and ICICI bank respectively.

The transition frequency tables as well as the transition probability matrix, observed probability matrix, and initial probability vector are obtained through MS Excel and R software. We have obtained separate probability distributions for *Rise* and *Fall* after explored the estimates of the HMM parameters $\lambda=(A,B,\hat{\pi}_i)$. The statistical measures such as Mean, Variance, Third and Fourth central moments, measures of skewness, kurtosis, etc. are obtained through the derived mathematical expressions of the said measures using R.

5.1. Transition Probability Matrix (TPM)

The explored TPM from the data set is

$$A = \begin{array}{c} \\ Gain \\ Loss \end{array} \begin{array}{c} \begin{array}{cc} Gain & \quad Loss \end{array} \\ \begin{bmatrix} 0.6122449 & 0.3877551 \\ 0.509009 & 0.490991 \end{bmatrix} \end{array}$$

It is identified that the highest likelihood 61.22% of transition is with NSE’s *Gain* state to NSE’s *Gain* state; followed by the next highest likelihood 50.9% observed with NSE’s *Loss* state to NSE’s *Gain* state. The third likelihood 49.09% is at NSE’s *Loss* state to NSE’s *Loss* state and the least likelihood 38.78% is observed at NSE’s *Gain* state to NSE’s *Loss* state.

5.2. Observed Probability Matrix (OPM)

The explored OPM for the data set from hidden states to visible/observed states

$$B = \begin{array}{c} \\ Gain \\ Loss \end{array} \begin{array}{c} \begin{array}{cc} Rise & Fall \end{array} \\ \begin{bmatrix} 0.7789116 & 0.2210884 \\ 0.2331839 & 0.7668161 \end{bmatrix} \end{array}$$

It is identified that the highest likelihood 77.89% of transition is with NSE's *Gain* state to ICICI's *Rise* state; followed by the next highest likelihood 76.68% observed with NSE's *Loss* state to ICICI's *Fall* state. The third likelihood 23.32% is at NSE's *Loss* state to ICICI's *Rise* state and the least likelihood 22.11% is observed at NSE's *Gain* state to ICICI's *Fall* state.

5.3. Initial Probability Vector

The initial probability vector is

$$\pi_i = \begin{array}{c} \begin{array}{cc} Gain & Loss \end{array} \\ \begin{pmatrix} 0.5686654 & 0.4313346 \end{pmatrix} \end{array}$$

It shows that the NSE of the *Gain* state has the highest likelihood with 56.87%, while the NSE of the *Loss* state has the least likelihood with 43.13 %. On average it reveals that the state of *Gain* is having more advantages than the state of *Loss*.

5.4. Stationarity for Invisible state (NSE)

The Kolmogrov's stationary TPM at 9th order is

$$A^{(9)} = \begin{array}{c} \\ Gain \\ Loss \end{array} \begin{array}{c} \begin{array}{cc} Gain & Loss \end{array} \\ \begin{bmatrix} 0.5676064 & 0.4323936 \\ 0.5676064 & 0.4323936 \end{bmatrix} \end{array}$$

The stationarity was recorded on the 9th consecutive day. In this NSE has the highest likelihood with a 56.76 % possibility of being in a *Gain* state and the least likelihood with a 43.24% possibility of being in a *Loss* state. Hence, the trade of selling the NSE stock on the 9th day is suggestable.

5.5. Stationarity for Invisible-Visible State (NSE to ICICI Bank)

Kolmogrov's stationary OPM

$$B^{(29)} = \begin{array}{c} \\ Gain \\ Loss \end{array} \begin{array}{c} \begin{array}{cc} Rise & Fall \end{array} \\ \begin{bmatrix} 0.513313 & 0.486687 \\ 0.513313 & 0.486687 \end{bmatrix} \end{array}$$

The stationarity was observed on the 29th consecutive day. In this observation that the ICICI bank has the highest likelihood with a 51.13% possibility of being in a *Rise* state and the least likelihood with a 48.66% possibility of being in a *Fall* state; irrespective of *Loss* and *Gain* states at NSE. Hence, it is suggested to sell the ICICI bank's shares on the 29th day make good profits

5.6. Probability distribution for *Rise* state

The following table represents the probability distribution of *the Rise* state of n=1,2,3 days ahead sequence, observed with ICICI stock prices.

Table 2: Probability distribution for *Rise* state

P (Rise state)	0	1	2	3
P [Rise state] n (n=1) day ahead	0.456998	0.543002	-	-
P [Rise state] n (n=2) days ahead	0.2088716	0.4963063	0.2948222	-
P [Rise state] n (n=3) days ahead	0.0954662	0.3402456	0.4042165	0.1600716

From Table 2 it is observed that NSEs of *Rise* state one day ahead is having more chance. The chance of happening of one *Rise* state in a row of two days ahead is more. In the case of three days sequence, happening of *Rise* state twice has more chance. The graphical presentation of the distributions is in Figure 7.

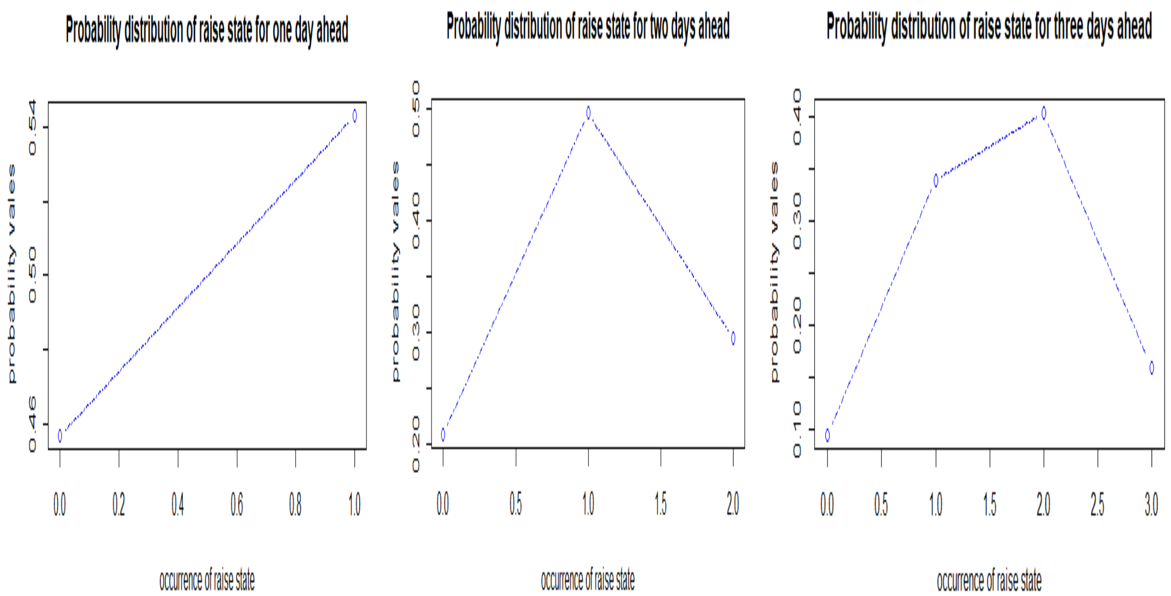

Figure 7: Probability of *Rise* state for one, two and three days ahead

5.7. Probability distribution for *Fall* state

The following table represents the probability distribution of the *Fall* state of n=1,2,3 days ahead sequence, observed with ICICI stock closing prices.

Table 3: Probability distribution for *Fall* state

P (Fall state)	0	1	2	3
P [Fall state] n (n=1) day ahead	0.543002	0.456998	-	-
P [Fall state] n (n=2) days ahead	0.2948222	0.4963063	0.2088716	-
P [Fall state] n (n=3) days ahead	0.1600716	0.4042165	0.3402456	0.0954662

From the above Table 3, it is observed that the occurrence of a *Fall* state one day ahead is having less chance. The chance of happening of one *Fall* state in a row of two days ahead is more. In the case of three days sequence happening of one *Fall* state is having more chance. The graphical presentation of the distributions is in Figure 8.

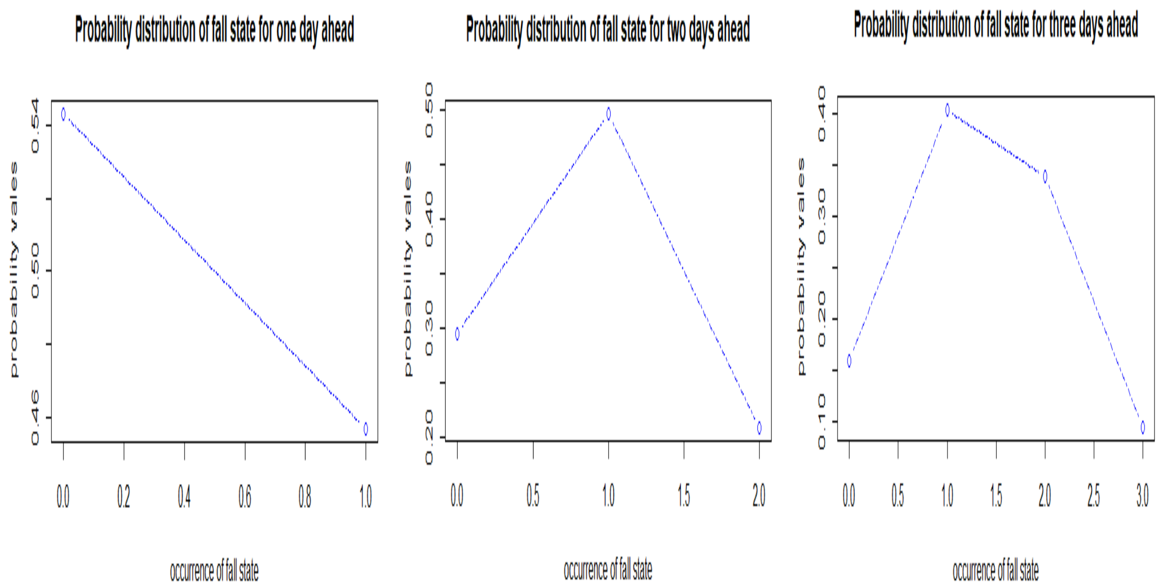

Figure 8: Probability of *Fall* state for one, two and three days ahead

5.8. Statistical Measures of *Rise* state for n (n=1,2,3) days sequence

The computed statistical measures for HMM are presented in Table 4; 1 day a run, 2 days a run and 3 days a run for *Rise* state.

Table 4: Statistical measures for *Rise* state

Statistical Measure	**1 day run**	**2 days run**	**3 days run**
Mean	0.543002	1.0859506	1.6288936
Variance	0.2481508	0.4963062	0.7444621
Mode	0.456998	1	1.5429645
Skewness	0.1726478	0.1220039	0.0995907
Kurtosis	1.0298073	2.014885	2.3432516

5.9. Statistical Measures of *Fall* state for n (n=1,2,3) days ahead sequence

The computed statistical measures of HMM for1 day a run, 2 days a run and 3 days a run for *Fall* state is placed the below Table 5.

Table 5: Statistical measures for *Fall* state

Statistical Measure	**1 day run**	**2 days run**	**3 days run**
Mean	0.456998	0.9140494	1.3711064
Variance	0.2481508	0.4963062	0.7444621
Mode	0.3709939	0.8280989	1.2851774
Skewness	0.1726478	0.1220039	0.0995907
Kurtosis	1.0298073	2.014885	2.3432516

5.10. Findings and Recommendations

Based on the above results obtained in Table 4 and 5; the average happenings in one-day business, the *Rise* state (0.543002$\simeq$ 1) is having more chance than the *Fall* state (0.456998). Hence, it is observed that *Rise* state of ICICI banks share is more likely in one-day business tenure, it may give an indication to the traders that they may prefer to sell the stock to get good returns.

In two days trade tenure, the average happenings of *Rise* state (1.0859506) is more than that of *Fall* state (0.9140494). Hence, it is observed that one day in two days business cycle suggests *Rise* state than the *Fall* state. It gives an indication to the traders that either to buy or to sell ICICI banks stock. However, the sale of shares has a little edge over the purchase of shares, if it is two days business cycle.

Similarly, for three days trade tenure, the average happenings of *Rise* state (1.6288936$\simeq$ 2) more than that of *Fall* state (1.3711064$\simeq$ 1). Hence, it is observed that two days in three days business cycle suggests *Rise* state than the *Fall* state. It gives an indication to the traders that they may prefer to sell the ICICI bank's stock for two days in a length of three days business cycles to get good returns.

The volatility of *Rise* and *Fall* state is (0.2481508) same in one-day business cycle, (0.4963062) same in two days business cycle and (0.7444621) same in three days business tenure. Hence, it may give an indication to the traders that there is more volatility in three days business cycle. It reveals that ICICI bank share prices are more dynamic on *Rise* or *Fall* and vice versa and there is less volatility in one-day business tenure, it reveals that ICICI bank share prices are closely knit across a narrow range. Therefore, trading has more volatility in three days business cycle when compared to other business cycles. It is helpful to traders to portfolio their stocks.

Regarding the mode value in one-day business the *Rise* state (0.6290061) is more when compared to the mean (0.543002) and the third central moment is (-0.021342) negative. It reveals that the *Rise* state of the one-day business cycle is having negatively skewed distribution. The mode of *Fall* state (0.3709939) is less than the mean (0.456998) and the third central moment (0.021342) is non-negative. It reals that the *Fall* state of the one-day business cycle is having positively skewed distribution.

The mode value in two days business, the *Rise* state (1.1719011) is more than the mean (1.0859506) and the third central moment of *Rise* state (-0.042658) is negative. It reveals that the *Rise* state is having negatively skewed distribution. The mode of *Fall* state (0.8280989) is less than the mean (0.9140494) and the third central moment (0.042658) is non-negative. It reals that *Fall* state of the two days business cycle is having positively skewed distribution.

The mode value in three days business the *Rise* state (1.7148226) is more than the mean (1.6288936) and the third central moment (-0.0639709) is negative. It reveals that the *Rise* state distribution of three days business cycle is a negatively skewed distribution. The mode of *Fall* state (1.2851774) is less than the mean (1.3711064) and the third central moment of *Fall* states (0.0639709) is non-negative. It reals that *Fall* state distribution of the three days business cycle is a positively skewed distribution.

The mean value is less than the mode in all business tenure of *Rise* state which means negatively skewed and the mean is more than the mode in all business tenure of *Fall* state which means positively skewed. Hence, it reveals that an investor may expect frequent small gains and a few large losses.

The measures of kurtosis in *Rise* and *Fall* state is (1.0298073<3) same in a one-day business cycle, (2.014885<3) same in two days business cycle and (2.3432516<3) same in three days business tenure. Hence it may give an indication to the traders that there is a platy kurtic distribution of the investment returns indicating the lesser consistency of the outcome. So, that is a small probability that the investment would experience better returns.

6. Summary and Conclusions

The present study is on developing HMM for a daily closing price of the NSE and ICICI bank. The changing states of NSE are considered as hidden and the changing states of NSE to ICICI bank are considered as visible. The parameters are obtained for HMM namely the initial probability vector, transition probability matrix and observed probability matrix for real-time data sets by assuming the discrete Markov chains among hidden states (*Gain* & *Loss* in NSE) and between hidden and observed states (*Rise* & *Fall* in ICICI bank) respectively. The stationarity arrived for both transition and observed probability matrices by computing Kolmogorov's probability matrices, which represent the market stability and it helps the portfolio managers to take the optimal decision when buying/selling the shares. The behaviours of *Rise* and *Fall* states in the closing prices of ICICI bank are explored through explicit mathematical relations of different statistical measures like average, variability, mode and Pearson's coefficients. Numerical illustrations are given for a proper understanding of the developed models. The average occurrence of *Rise* has 2 days out of 3 trading days, which reveals that there is a chance of occurrence of *Rise* of ICICI share price is more in three days sequences and the average occurrence of *Fall* has 1 day out of 3 days, which reveals that there is the chance of occurrence of *Fall* of ICICI share price is less in three days sequence. The variability of one day sequence is less in both the *Rise* and *Fall* states compared to the remaining two. The mean is less compared to the mode in *Rise* state of all sequences which indicates that negatively skewed in all days sequences and the mean is more compared to the mode in *Fall* state of all sequences which indicates that positively skewed in all days sequences. These results will be helpful for short-term business investors for understanding the indicators of when to sell and when to purchase by observing the chances of emission states.

Acknowledgement

We are thankful to the DST INSPIRE. Sarode Rekha (*Corresponding author) is supported by INSPIRE fellowship (IF190741) of Department of Science and Technology (DST), Government of India.

References

1. Adesokan, P. Ngare, and A. Kilishi, "Analyzing expected returns of a stock using the Markov chain model and the capital asset pricing model", 2017.
2. I. Akgül, M. Bildirici, and S. Ozdemir, "Evaluating the nonlinear linkage between gold prices and stock market index using Markov-Switching Bayesian VAR models", Procedia-0Social and Behavioral Sciences, vol. 210, pp. 408-415, 2015.
3. B. Aparna, and S. C. H. Kakaty, "Analysis of stock market price behaviour: a Markov Chain approach", International Journal of Research Scientific Research, vol. 6, no. 10, pp. 7061-7066, 2015.
4. M. K. Bhusal, "Application of Markov chain model in the stock market trend analysis of Nepal", International Journal of Scientific & Engineering Research, vol. 8, no.10, pp. 1733-1745, 2017.
5. M. A. Fridman, two state capital asset pricing model, 1994.
6. C. Gudala, "Some stochastic models for optimal investment policies in portfolio management" (Doctoral dissertation, Department of Statistics, Pondicherry University), 2019.
7. J. C. Huang, W. T. Huang, P. T. Chu, W. Y. Lee, H. P. Pai, C. C. Chuang, and Y. W. Wu, "Applying a Markov chain for the stock pricing of a novel forecasting model", Communications in Statistics-theory and Methods, vol. 46, no. 9, pp. 4388-4402, 2017.
8. Jandera, and T. Skovranek, "Customer Behaviour Hidden Markov Model", Mathematics, vol. 10, no. 8, pp. 1230, 2022.
9. S. B. Kilic, "Estimating Probability of Session Returns for Istanbul Stock Exchange 100 Index as Markov Chain Process", Cukurova Universitesi Sosyal Bilimler Enstitüsü Dergisi, vol. 22, no. 1, pp. 12-20, 2013.
10. Maruotti, A.Punzo, and L. Bagnato, "Hidden Markov and semi-Markov models with multivariate leptokurtic-normal components for robust modeling of daily returns series", Journal of Financial Econometrics, vol. 17, no. 1, pp. 91-117, 2019.
11. N. Nguyen, "Stock Price Prediction using Hidden Markov Model", Editorial Express, March, 21, 2016.
12. N. Nguyen, and D. Nguyen, "Hidden markov model for stock selection", Risks, vol. 3, no. 4, pp. 455-473, 2015.
13. V. Soloviev, V. Saptsin, and D. Chabanenko, "Markov chains application to the financial-economic time series prediction", arXiv preprint arXiv:1111.5254, 2011.
14. M. Svoboda, and L. Lukas, "Application of Markov chain analysis to trend prediction of stock indices", In Proceedings of 30th international conference mathematical methods in economics. Karviná: Silesian University, School of Business Administration, vol. 848, pp. 853, 2012.
15. C. M. Turner, R. Startz, and C. R. Nelson, "A Markov model of heteroskedasticity, risk, and learning in the stock market", Journal of Financial Economics, vol. 25, no. 1, pp. 3-22, 1989.

16. X. Zhang, Y. Li, S. Wang, B. Fang, and P. S. Yu, "Enhancing stock market prediction with extended coupled hidden Markov model over multi-sourced data", Knowledge and Information Systems, vol. 61, no. 2, pp. 1071-1090, 2019.
17. G. F. Dar, T. R. Padi, S. Rekha, and Q. F. Dar, "Stochastic Modeling for the Analysis and Forecasting of Stock Market Trend using Hidden Markov Model", Asian Journal of Probability and Statistics, vol. 18, no. 1, pp. 43-56, 2022.
18. G. F. Dar, T. R. Padi, and S. Rekha, "Stock price prediction using a Markov chain model: a study for TCS share values", Advances and Applications in Statistics, vol. 80, pp. 83-101, 2022.
19. T. R. Padi, G. F. Dar, and S. Rekha, "Stock Market Trend Analysis and Prediction using Markov Chain Approach in the Context of Indian Stock Market", IOSR Journal of Mathematics, vol. 8, no. 4, pp. 40-48, 2022.