Numerical Solution of a Space-Time Fractional by Semi-Linear Diffusion Equation

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Abstract

The objective of the work is to address the underlying limit esteem issue for the semi-direct space-time fragmentary dissemination condition utilizing an implied technique. The fundamental thought behind the methodology is to change over the issue into a logarithmic framework, which makes the calculations simpler. The consistency and security of the methodology are analyzed utilizing the strength framework investigation. Mathematical responses to models of semi straight fragmentary dissemination conditions are given. The acquired information is contrasted with accurate reactions. We inspect the blunder examination of the implied limited contrast plan, assembly, and solidness utilizing reasonable MATLAB models.

Keywords: MATLAB, Stability, Initial Value Problem, Convergence, Diffusion Equation

1. Introduction

In the investigation of numerous biological, chemical, and physical phenomena, fractional differential equations are crucial. Therefore, the theory, methodology, and applications of fractional differential equations are of great interest to many academics. Therefore, it is necessary to research effective and dependable methods for obtaining accurate or approximation fractional differential equation solutions. The scholars have created some numerical methods and achieved approximations for the solutions of fractional differential equations that are both linear and nonlinear.

A number of researchers have analyzed various problems with a number of terms, and conditions having different methods. A few are cited here which are necessary to arrive at the problem in hand.**Kazuaki (2021)** studied onthe functional analytic approach to the problem of Markov process building in probability theory is the focus of this e-book. It is widely known that the difficulty of constructing Markov processes may be simplified to the study of boundary value problems for degenerate elliptic integro-differential operators of second order according to the Hille-Yosida theory of semi groups.**Evgeniya et al. (2021)** studied on theDiffusion processes which are important in a variety of domains, including medicine,metallurgy,physics,chemistry, and others. The problem of charged particle diffusion in a semi-infinite thin tube under the influence of an electromagnetic field is studied and solved in this study.**Wang et al. (2020)** look at a variety of ordinary differential equation models for diffusion of innovation and epidemiological models in this chapter. They review the classic idea of innovation diffusion, focusing on online social networks and examining multiple ordinary differential equation models for diffusion of invention.**Aziz and Khan (2018)** In this study, a HAAR wavelet-based collocation approach for numerical solution of diffusion is devised, one look at one-dimensional & two-dimensional hyperbolic partial method for providing equations. The numerical findings corroborate the correctness, effectiveness, and resilience of proposed technique.The objective of this study, according to **Gunvant (2016),** is to apply a few limited contrast procedures to get a mathematical answer for the underlying limit esteem issue (IBVP) for the semi linear fragmentary dissemination condition of variable request. Utilizing the Fourier methodology, the solidness and union of this procedure are investigated.Finally, the solution to a few numerical examples is investigated and graphically represented using MATLAB.**D'Ambrosio and Paternoster (2014)** the goal of this research is to use progressively adapted numerical solutions computational tools to solve partial differential equations simulating diffusion issue accurately and efficiently. A numerical research show that specific purpose integrates is so much more effective and precise than to use a general-purpose solution across both temporal and spatial.**Bargieł and Tory (2015)** The non - linear diffusing equation may be recast in a way that immediately proceeds towards its stochastic equivalent. The stochastic technique provides a greater understanding of the physical process by simulating the movements of molecules. The parallel version of our approach is incredibly efficient. In order to find linear Convection Diffusion (CD) equation series solutions, Fallahzadeh and Shakibi (2015) employ the hemitrope analysis technique (HAM). The work by **Gurarslan and Sari (2011)** revealedsatisfactory answers for both direct and nonlinear dissemination issues. Certain conditions were tackled utilizing a method with an unmistakable general quadrature approach in space and a versatile soundness protection Runge-Kutta procedure across space. This approach can possibly be applied to extra nonlinear common differential conditions to create undeniably more reasonable models. The one-layered (1D) dispersion condition is a basic illustrative incomplete differential condition (PDE) that concedes voyaging wave arrangements, according to **Griffiths and Schiesser (2012).** This analytical solution is used to assess the numerical solution created using the method of lines (MOL).

This section focuses on determining the numerical solution to the fractional semi linear diffusion problem in space-time. Consider the fractional semi-linear space-time diffusion equation:

$$\frac{\partial^x u}{\partial t^x} = x(a,t)^R D_a^\beta u(a,t) + f(u(a,t),a,t) \qquad (1)$$

$$0 < a < l, x(a,t) > 0, 0 < t \leq T, 0 < a \leq 1, 1 < \beta \leq 2$$

$$withinitialconditionu(a,0) = g(a)(2)$$

$$u(0,t) = 0 = u(l,t) \qquad (3)$$

It is known as the spacetime fractional semi-linear diffusion equation's first initial boundary value problem (IBVP). Keep in mind $\frac{\partial^x u}{\partial t^x}$and ${}^R D_a^\beta u(a,t)$are the fractional derivatives of order Caputo and Riemann−Liouville$\alpha(0 < \alpha \leq 1)$ $and\beta(1 < \beta \leq 2)$ $respectively.$

Implicit Finite Difference Scheme: One discretizes the entire first IBVP (1) in this part (3). For each $\beta(0 \leq n - 1 < n)$according to Riemann-Liouville derivative is real & corresponds to the Grunwald-Letnikow derivative. The mathematical estimate of partial request differential conditions is an extra significant result of the connection between the Riemann-Liouville and Grunwald-Letnikov thoughts. This permits the Riemann-Liouville definition to be utilized during issue plan and the Grunwald-Lettikov definition to be utilized during mathematical arrangement.For${}^R D_a^\beta u(a,t)$,At all-time levels, we use the shifted Grunwald formula to approximate the 2nd order space derivative.

$$RD_a^\beta u(a_i, t_{k+1}) = \frac{1}{h^\beta} \sum_{j=0}^{i+1} g_j u(x_i - (j-1)h, t_{k+1}) + O(h) \qquad (4)$$

where the Grunwald weights are defined as follows –

$$g_0 = 1, g_j = (-1)^j \frac{(\beta)(\beta-1)(\beta-2)\ldots(\beta-j+1)}{j!}, j = 1,2,3, \ldots \qquad (5)$$

Now having equations (3) and (4) in equation (1). we get

$$u(a_i,t_{k+1}) = u(a_i,t_k) + r\sum_{j=0}^{i+1} g_j u\left(a_{i-j+1},t_{k+1}\right) - b_1 u(a_i,t_k) + b_k u(a_i,t_k) +$$

$\sum_{j=1}^{k-1} \left(b_j - b_{j+1}\right)u\left(a_i,t_{k-j}\right) + r_1 f(u(a_i,t_k),a_i,t_k) + R_i^{k+1}$ (6)

Where $r = r(i,k) = \frac{x_i^k \tau^x \Gamma(2-x)}{h^\beta}, r_1 = \tau^x \Gamma(2-x)$

$$\left|R_i^{k+1}\right| \le c_1 \tau^x \left(\tau^{1+x} + h^\beta + \tau\right) \quad (7)$$

Let u_i^kbe the numerical approximation of$u(a_i,t_k)$and let $f_i^k(u_i^k)$be the closest numerical approximation $f\left(a_i,t_k,u(a_i,t_k)\right)$. As a result, the whole discrete form of the initial IBVP (1)-(3) is obtained.

$$(1+\beta r)u_i^1 - r\sum_{j=0,j\neq 1}^{i+1} g_j u_{i+1-j}^1 = u_i^0 - r_1 f_i^0(u_i^0), k = 0$$

$(1+\beta r)u_i^{k+1} - r\sum_{j=0,j\neq 1}^{i+1} g_j u_{i+1-j}^{k+1} = (1-b_1)u_i^k + r_1 f_i^k\left(u_i^k\right) + \sum_{j=1}^{k-1} \left(b_j - b_{j+1}\right)u_i^{k-j} + b_k u_i^0, k > 1$ (8)

$$initialcondition u_i^0 = g_i, i - 0,1,2,\ldots,m-1 \quad (9)$$

$$u_0^k = 0 = u_m^k, k = 0,1,2,\ldots n. \quad (10)$$

Put $k = 0, and i = 1,2,\ldots,m-1$We get a group of (m − 1) calculations from the equation (8), which may be represented in the matrix equation as follows.

$$AU^1 = U^0 + r_2 F^0 \quad (11)$$

$$where U^1 = [u_1^1, u_2^1, \ldots, u_{m-1}^1]^T; U^0 = [u_1^0, u_2^0, \ldots, u_{m-1}^0]^T;$$

$F^0 = [f_1^0(u_1^0), f_2^0(u_2^0), \ldots, f_{m-1}^0(u_{m-1}^1)]^T$

where A is a square matrix of order (m - 1) × (m - 1) like that

$$A = \begin{pmatrix} 1+\beta r & -rg_0 & & & & & \\ -rg_2 & 1+\beta r & -rg_0 & & & & \\ -rg_3 & -rg_2 & 1+\beta r & -rg_0 & & & \\ . & . & . & . & . & . & \\ & & & & & & \\ -rg_{m-1} & -rg_{m-2} & . & & . & -rg_2 & 1+\beta r \end{pmatrix}$$

This can be expressed as

$$A_{i,j} = \begin{cases} 0, & \text{when } j > i+1 \\ 1+\beta r, & \text{when } j = i \\ -rg_{i-j+1}, & \text{otherwise} \end{cases}$$

Also for $k = 1, i = 1,2,\ldots,m-1$, the matrix equation is

$$AU^2 = (1-b_1)U^1 + b_1U^0 + r_1F^1$$

In general, $k \geq 1, i = 1,2,\ldots,m-1$ we can write as

$$AU^{k+1} = (1-b_1)U^k + \sum_{j=1}^{k-1}(b_j - b_{j+1})\,U_i^{k-j} + b_kU^0 + r_1F^k \qquad (12)$$

Where $F^k = [f(u_1^k), f(u_2^k), \ldots, f(u_{m-1}^k)]^T; U^{k+1} = [u_1^{k+1}, u_2^{k+1}, \ldots, u_{m-1}^{k+1}]^T;$

Stability: In this section, we talk about how stable the implicit finite difference scheme is.Let$\bar{u}_i^k$be the approximation of the implicit finite difference scheme (8)–(100), and let $f_i^k(\bar{u}_i^k)$be the approximations of$f(a_i, t_k, u(a_i, t_k)).Setting\epsilon_i^k = u_i^k - \bar{u}_i^k$The round off error equation is obtained.

$$(1+\beta r)\epsilon_i^1 - r\sum_{j=0, j\neq 1}^{i+1} g_j\,\epsilon_{i+1-j}^1 = \epsilon_i^0 - r_1\left(f_i^0(u_i^0) - f_i^0(\bar{u}_i^0)\right), k = 0$$

$$(1+\beta r)\epsilon_i^{k+1} - r\sum_{j=0, j\neq 1}^{i+1} g_j\,\epsilon_{i+1-j}^{k+1} = (1-b_1)\epsilon_i^k + r_1\left(f_i^k\left(u_i^k\right) - f_i^k\left(\bar{u}_i^k\right)\right) +$$

$\sum_{j=1}^{k-1}\left(b_j - b_{j+1}\right)\epsilon_i^{k-j} + b_k\epsilon_i^0, k > 1$ (13)

for $i = 1,2,\ldots,m-1, k = 0,1,2,\ldots,n$. Assuming$\|E^k\|_\infty = \underset{1 \leq i \leq m-1}{max}\left|\epsilon_i^k\right|$

We now examine the stability of the implicit finite difference scheme (8)-(10) using the induction approach. When we enter k = 0 into equation (13), we get ϵ^1

Assume that$|\epsilon_l^1| = max\{|\epsilon_1^1|, |\epsilon_2^1|, \ldots, |\epsilon_{m-1}^1|\}$

$$|\epsilon_l^1| \leq (1+\beta r)|\epsilon_l^1| - r\sum_{j=0, j\neq 1}^{i+1} g_j\,|\epsilon_l^1|$$

$$\leq (1+\beta r)|\epsilon_l^1| - r\sum_{j=0, j\neq 1}^{i+1} g_j\left|\epsilon_{l-j+1}^1\right|$$

$$= \left|\epsilon_{l-1}^0 + r_1\left(f_i^0(u_i^0) - f_i^0(\bar{u}_i^0)\right)\right|$$

$$\leq (1 + r_1L)\left|\epsilon_l^0\right|$$

$$\|E^1\|_\infty \leq C\|\epsilon^0\|_\infty (\because C = 1 + r_1L)$$

Let $\|E^{k+1}\|_\infty = \left|\epsilon_l^{k+1}\right| = max\left\{\left|\epsilon_1^{k+1}\right|, \left|\epsilon_2^{k+1}\right|, \ldots, \left|\epsilon_{m-1}^{k+1}\right|\right\}$ and assume that

$\|E^j\|_\infty \leq C\|\epsilon^0\|_\infty, j = 1,2, \ldots k$we get

$$\left|\epsilon_l^{k+1}\right| \leq (1+\beta r)\left|\epsilon_l^{k+1}\right| - r \sum_{j=0, j\neq 1}^{i+1} g_j \left|\epsilon_l^{k+1}\right|$$
$$\leq \left|(1+\beta r)\epsilon_l^{k+1} - r \sum_{j=0, j\neq 1}^{i+1} g_j \epsilon_l^{k+1}\right|$$
$$\sum_{j=1}^{k-1} (b_j - b_{j+1})\left|\epsilon_i^{k-j}\right| + b_k\left|\epsilon_i^0\right|$$
$$\leq (1-b_1)\left|\epsilon_l^k\right| + r_1 L\left|\epsilon_l^k\right| + (b_1 - b_k)\left|\epsilon_l^k\right| + b_k\left|\epsilon_i^0\right|$$
$$\leq (1+r_1 L)\left|\epsilon_l^0\right|$$
$$\|E^{k+1}\|_\infty \leq C_0\|\epsilon^0\|_\infty, (\because C_0 = C(1+r_1 L))$$

As a result, we can prove the following theorem.

Convergence: The convergence of the implicit finite difference scheme is investigated (8) in this section (10). Let$u(a_i, t_k)$be the exact IBVP (1)–(3) solution at mesh point$(a_i, t_k)$and letu_i^kbe the numerical solution of (8)-(10) calculated with the implicit finite difference technique.Define

$e_i^k = u(a_i, t_k) - u_i^kandE^k = \left(e_1^k, e_2^k, \dots, e_{m-1}^k\right)^T$

$$(1+\beta r)e_i^1 - r \sum_{j=0, j\neq 1}^{i+1} g_j e_{i+1-j}^1 = e_i^0 - r_1\left(f_i^0(u_i^0) - f_i^0(\bar{u}_i^0)\right) + R_i^1, k = 0$$

$$(1+\beta r)e_i^{k+1} - r \sum_{j=0, j\neq 1}^{i+1} g_j e_{i+1-j}^{k+1} = (1-b_1)e_i^k + r_1\left(f_i^k\left(u_i^k\right) - f_i^k\left(\bar{u}_i^k\right)\right) +$$
$$\sum_{j=1}^{k-1} (b_j - b_{j+1})e_i^{k-j} + b_k e_i^0 + R_i^{k+1}, k > 1 \qquad (14)$$

where $i = 1, 2, \dots, m-1, k = 0, 1, 2, \dots, n.$

$$\left|R_l^{k+1}\right| \leq c_1\tau^\alpha\left(\tau^{1+\alpha} + h^\beta + \tau\right) \text{ for } i = 1,2,\dots,m-1, k = 0,1,2,\dots,n$$

Following is a proof that the convergence analysis may be established by the use of mathematical induction. In equation (14), when k is equal to zero, we get e^1

Assuming that $\|e^1\|_\infty = |e_l^1| = \max_{1\leq i\leq m-1}\left|e_i^1\right|$

$$|e_l^1| \leq (1+\beta r)|e_l^1| - r\sum_{j=0, j\neq 1}^{i+1} g_j |e_l^1|$$
$$\leq \left|(1+\beta r)e_l^1 - r\sum_{j=0, j\neq 1}^{i+1} g_j e_{l-j+1}^1\right|$$
$$= \left|e_l^1 + r_1\left(f_i^0(u_i^0) - f_i^0(\bar{u}_i^0)\right) + R_l^1\right| \qquad (15)$$
$$\leq \left|e_l^0\right| + r_1 L\left|e_l^0\right| + |R_l^1|$$
$$\|E^1\|_\infty \leq |R_l^1|$$

$Using e^0 = 0\ and |R_l^1| \leq c_1\tau^x(\tau^{1+x} + h^\beta + \tau)$we have

$$\|e^1\|_\infty \leq b_0^{-1} c_1 \tau^\alpha \left(\tau^{1+\alpha} + h^\beta + \tau\right)$$

Assume that this is the case for $j, \|e^j\|_\infty \leq b_{j-1}^{-1} c_1 \tau^\alpha \left(\tau^{1+\alpha} + h^\beta + \tau\right)$

$j = 1,2, \ldots, k$And $\left|e_l^{k+1}\right| = max\{\left|e_1^{k+1}\right|, \left|e_2^{k+1}\right|, \ldots, \left|e_{m-1}^{k+1}\right|\}$Note that$b_j^{-1} \leq b_k^{-1}$

$$\left|e_l^{k+1}\right| \leq (1+\beta r)\left|e_l^{k+1}\right| - r \sum_{j=0, j\neq 1}^{i+1} g_j \left|e_l^{k+1}\right|$$

$$\leq \left|(1+\beta r)\epsilon_l^{k+1} - r \sum_{j=0, j\neq 1}^{i+1} g_j \epsilon_{l-j+1}^{k+1}\right|$$

$$= | \, (1-b_1)e_i^k + r_1 \left(f_i^k\left(u_i^k\right) - f_i^k\left(\bar{u}_i^k\right)\right) + \sum_{j=1}^{k-1} \left(b_j - b_{j+1}\right)e_i^{k-j} + b_k e_i^0 + R_l^{k+1} \, |$$

$$\leq (1-b_1)\left|e_i^k\right| + r_1 \left|\left(f_i^k\left(u_i^k\right) - f_i^k\left(\bar{u}_i^k\right)\right)\right| + \sum_{j=1}^{k-1} \left(b_j - b_{j+1}\right)\left|e_i^{k-j}\right| + \left|R_l^{k+1}\right|$$

$$\leq (1-b_1)\|e^k\|_\infty + r_1 L \|e^k\|_\infty + (b_1 - b_k)\|e^k\|_\infty + \left|R_l^{k+1}\right|$$

$$\leq b_k^{-1}\{1 + r_1 L\} c_1 \tau^\alpha \left(\tau^{1+\alpha} + h^\beta + \tau\right)$$

$$\|e^{k+1}\|_\infty \leq C_0 k^\alpha \tau^\alpha \left(\tau^{1+\alpha} + h^\beta + \tau\right), (\because (1 + r_1 L)c_1 = C_0)$$

If $k\tau \leq T$ is a finite number, then we can prove the following theorem.

Theorem 1: Letu_i^kbe the close approximation of $u(a_i, t_k)$calculated by making use of an implicit finite difference method, & both the basis term and the situation of Lipschitz are met (3.9). If this is the case, there must be a positive constant C_0such that$\left|u_i^k - u(a_i, t_k)\right| \leq C_0(\tau + h)$

- **Test Problem**

Example 1: Take into consideration the fractional semi-linear space-time diffusion equation.

$$\frac{\partial^{0.9} u}{\partial t^{0.9}} = \frac{\partial^{1.8} u}{\partial a^{1.8}} + u^2 + f\left(a, t, u(a,t)\right), 0 < a < \pi, 0 < t \leq T \qquad (16)$$

$$withinitialconditionu(a, 0) = \ sinx \qquad (17)$$

$$u(0,t) = \ 0 \ = \ u(\pi, t) \qquad (18)$$

Where $f = t^{0.1} sinaE_{1,1.1}(t) - e^t \sin(a + 0.9\pi) - sin^2 a e^{2t}$and exact solution is $u(a,t) = e^t sina$

The following is an example of the discrete form of IBVP (16-18):-

$$(1+\beta r)u_i^1 - r\sum_{j=0,j\neq 1}^{i+1} g_j u_{i+1-j}^1 = u_i^0 - r_1(u_i^0)^2, k=0$$

$$(1+\beta r)u_i^{k+1} - r\sum_{j=0,j\neq 1}^{i+1} g_j u_{i+1-j}^{k+1} = (1-b_1)u_i^k + r_1\left(u_i^k\right)^2 + \sum_{j=1}^{k-1}\left(b_j - b_{j+1}\right)u_i^{k-j} + b_k u_i^0, k>1$$

$$initialconditionu_i^0 = \sin(ih)\,,\ \ i=0,1,2,3,4,5,6.\left(\because\ h=\frac{\pi}{6}\right)$$

$$u_0^k = 0 = u_m^k,\ \ k=0,1,2,\ldots.,n$$

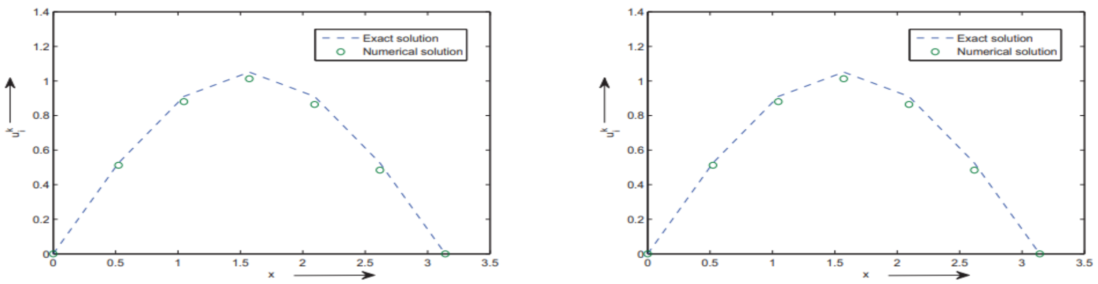

Figure 1: When t equals 0.02 and t equals 0.05, a comparison is made between the exact answer and the numerical solution

Table 1: In the table that follows, a comparison is made between the particularanswer and the numerical answer at the time t = 0.01.

u(a, t)	**I.F.D.M.**	**Particularanswer**	**Absolute Error**	**Relative Error**	**% Error**
$u\left(\frac{\pi}{6},0.01\right)$	0.5064	0.5050	0.0014	0.0028	0.2772
$u\left(\frac{\pi}{3},0.01\right)$	0.8751	0.8747	0.0004	4.573×10^{-4}	0.04578
$u\left(\frac{\pi}{2},0.01\right)$	1.0103	1.0101	0.00024	2.376×10^{-4}	0.0238
$u\left(\frac{2\pi}{3},0.01\right)$	0.8754	0.8747	0.0007	8.0027×10^{-4}	0.08
$u\left(\frac{5\pi}{6},0.01\right)$	0.5064	0.5050	0.0014	0.0028	0.2772

Example 2: Take into consideration the fractional semi-linear space-time diffusion equation.

$$\frac{\partial^a u}{\partial t^a} = \frac{\partial^\beta u}{\partial a^\beta} + sinu, 0 < a < 1, 0 < t \le T \quad (19)$$

$$withinitialconditionu(a,0) = x(1-x) \quad (20)$$

$$u(0,t) = 0 = u(1-t) \quad (21)$$

The discrete IBVP (19-) (21)-

$$(1+\beta r)u_i^1 - r\sum_{j=0,j\neq 1}^{i+1} g_j u_{i+1-j}^1 = u_i^0 - r_1(\sin(u_i^0)), k = 0$$

$$(1+\beta r)u_i^{k+1} - r\sum_{j=0,j\neq 1}^{i+1} g_j u_{i+1-j}^{k+1} = (1-b_1)u_i^k + r_1\big(\sin\big(u_i^k\big)\big) +$$

$$\sum_{j=1}^{k-1}\big(b_j - b_{j+1}\big)u_i^{k-j} + b_k u_i^0, k > 1$$

$$initialconditionu_i^0 = ih(1-ih), \qquad i = 0,1,2,3,4,5. (\because h = 0.2)$$

$$boundaryconditionu_0^k = 0 = (u_m^k), \qquad k = 0,1,2,\ldots,n.$$

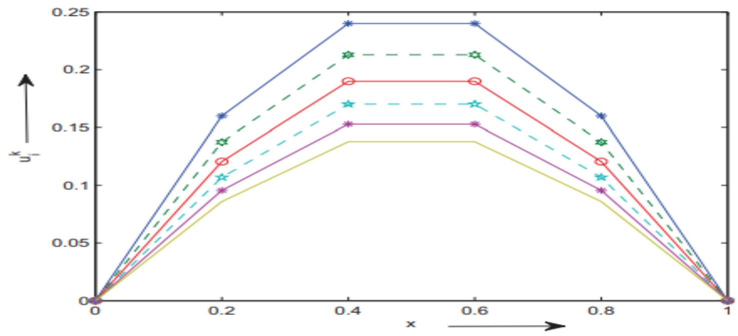

Figure 2: The numerical solution of the function u(a, t) at a variety of time steps for the cases where = 0.9 and = 1.8

2. Conclusions

Finding the mathematical answer for semi direct fragmentary incomplete differential conditions should be possible very successfully utilizing this strategy. The understood limited distinction strategy is made steady and ready to unite with the assistance of the lattice approach. Issues with numbers are utilized to show how the speculations work.

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