Mathematical Modeling and behaviour Analysis of a Milk Food Plant

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Abstract

In the present paper the reliability model for availability analysis of milk food plant is developed in three sub-units. A single repairman who examines and repairs the units as and when need emerge. Availability of milk plant is determined with the assistance of RPGT and accessibility of the arrangement of milk food plant for various values of repair and failure rates of subsystems is additionally determined. Graphs are drawn to depict the behavior of the MTSF, Availability of the system and busy period of the server for a particular case. A milk food plant in Pehowa District Kurkshetra in Haryana has been taken up our study.

Keywords: Availability analysis, MTSF, Availability

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1. Introduction

This paper discusses the behavior analysis to analyze transient behavior of repairable milk food plant using RPGT, based on Markov modeling for modeling system parameters equations. The effect of failure and repair of units is examined to realize optimum level of performance of system parameters. Now a day availability / maintainability analysis of process industries is having increased importance which may benefit the industry with higher productivity and lower maintenance activities with costs. Different performance measures used in process industry are indicates to define the performance of a plant in terms of system parameters. Most of these parameters are linked to operational stage while a few of these are useful to design the units at an early stage. A milk food plant in Pehowa District Kurkshetrain Haryana has been taken up or study. In this chapter a subsystem of the plant (visit is a continuous processing and production system) is analyzed taking pre-emptive resume priority repair policy. Using a heuristic approach, Rajbala et al.

(2022) investigated the redundancy allocation problem in the cylinder manufacturing plant.A case study of an EAEP manufacturing facility was examined by Rajbala et al. (2019) in their work on system modeling and analysis in 2019. A study of the urea fertilizer industry's behavior was conducted by Kumar et al. (2017). The mathematical formulation and profit function of an edible oil refinery facility were investigated by Kumar et al. in 2017. In order to do a sensitivity analysis on a cold standby framework made up of two identical units with server failure and prioritized for preventative maintenance, Kumar et al. (2019) used RPGT. Two halves make up the current paper, one of which is in use and the other of which is in cold standby mode. The good and fully failed modes are the only differences between online and cold standby equipment. In their study, Kumar et al. (2018) investigated a 3:4:: outstanding system plant's sensitivity analysis. PSO was used by Kumari et al. (2021) to research limited situations. In a paper mill washing unit, Kumar et al. (2019) investigated mathematical formulation and behavior study. The behavior of a bread plant was examined by Kumar et al. in (2018).Singla et al. (2022) studied on the mathematical model for finding the availability under the reduced capacity has been proposed using the Chapman Kolmogorov approach with the help of transition diagrams associated with various possible combinations of probabilities. Sinla et al. (2022) studied on the Mathematical analysis of regenerative point graphical technique. Malik et al. (2022) present paper talks over perform ability evaluation for a steam generation system of a Coal Fired Thermal Power Plant (CFTPP) using the concept of the Markov method. Kumari et al. (2021) talked about the benefit examination of an agribusiness harvester plant in consistent state utilizing RPGT.

The failure and repair rates of units are takes as constant, transient probability consideration on under Markov-process are helpful to draw the transient state diagram of the system under steady state. Laplace transformations are used to evaluate mean sojourn times of various stage expressions for system parameters are modeled using RPGT. Keeping failure or repair rates of units fixed while varying other for different units, their effect on system performance parameters is given by drawing tables and graphs, followed by discussions.

2. Assumptions and Notations

1. Facility of single repair is always available.
2. Repairs and failures are statistically independent.
3. Repair is perfect and repaired system is as good as the new one.

$h_{i:}$ Constantrepair rates

g_i: Constantfailure rates

3. System Description

The sub-system and their working is described as given below

1. **Bulk Milk Cooler (A) :**Raw milk is cooled using a bulk milk cooler from 300°C to 40°C, where it can be stored for up to 96 hours.
2. **Pasteurization Unit (B) :** Milk is heated to at least 720C (1600F) for 17 seconds and then quickly cooled to 40C as part of the pasteurisation process. By eliminating all harmful components, this technique renders milk safe for human consumption.
3. **Pouch Making and Bottle Filling Machine (M)** : The milk is delivered to the storage tank after pasteurisation. The pouch-making machine is connected to this tank. These bags of various sizes are created automatically by this machine. The milk is sent to the storage tank after pasteurisation. The bottle filling device is also connected to this storage tank. This device automatically fills milk into bottles of various sizes.

4. Transition Diagrams:

By taking into consideration all the above notations and assumptions, the Transition Diagram of the system is given in Fig. 1.

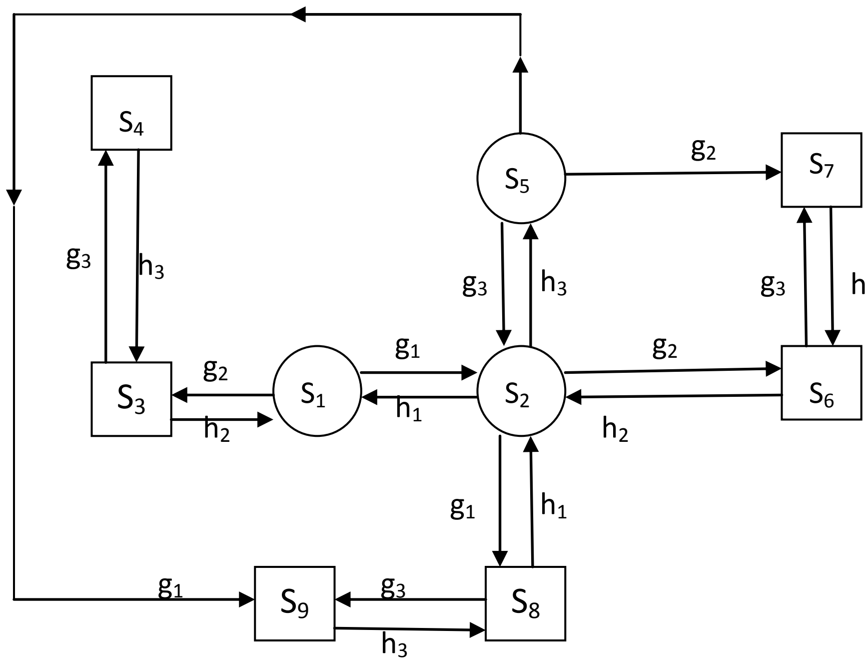

Fig.1: Transition Diagrams

S_1 = A(A)B, S_2 = aAB, S_3 = A(A)b, S_4 = A(A)bM,
S_5 = aABM, S_6 = aAb, S_7 = aAbM, S_8 = aaB,
S_9 = aaBM,

5. Model Description

A milk food plant contains of following sub-units Bulk Milk Cooler (A), Pasteurization Unit (B), Pouch Making and Bottle Filling Machine(M). Significance order to repair the sub-units and server are M > B > A. In the start the sub-unit is in state S_1[A(A)B] where unit 'A', it's cold standby sub-unit, unit 'B' and server are in good operational condition, hence the framework works in full volume. The cold redundant sub-unit when decent is shown in simple bracket which is prepared online directly with the assistance of a perfect switch over framework upon the disappointment of chief sub-unit 'A'. From state S_1 upon the disappointment of online unit 'A', disappointment rate of which is g_1, framework enters the state S_2 [aAB], here framework again works at full capacity as cold standby sub-unit is mode online. From state S_2 upon repair of fizzled sub-unit, repair rate of which is h_1, framework again joins state S_1. In state S_1, if unit 'B' flops of which rate is g_2, upon its repair (repair rates h_2) over the framework re-enters state S_1 while in state S_3 if the attendant (g) fails (failure rate g_3), framework enters the state S_4 [A(A)bM] upon its repair (repair rate h_3) framework re-enters the state S_3. In state S_2 [aAB] if online unit 'A' bombs at rates, the framework enters the state S_8 [aaB], upon repair of unit 'A' at rate h_1. The scheme re-enters state S_2 although in state S_8 if the attendant fails (disappointment rate of which is g_3) structure joins the unsuccessful state S_9 [aaBM] upon its repair or behavior the structure rejoins the state S_8, where its resumes repairing the fizzled sub-unit 'A'. Also in state S_2 if unit 'B' fizzled at rate g_2, the framework takes state S_6 [aAb] upon repair of unit 'B' structure rejoins the state S_2 while in state S_6. If attendant fails at rate g_3, the framework take the state S_7 [aAbM], upon its reparation as it is assumed top priority, the structure rejoins the state S_6. In state S_2 if the server failed the structure joins the state S_5 [aABM], here the structure continues to work of full volume, as the attendant is give top priority in repair, so upon its repair the structure rejoins state S_2,in state S_5if online sub-unit 'A' at rate g_1, the structure joins the state S_9 and if the sub-unit 'B' then the structure joins the state S_7. The transition diagram with possible stable states is given in Fig. 1.

6. Transitional probabilities &Mean Sojourn time.

$q_{i,j}(t)$ **:** p. d. f – "Probability density function of the first passage period from a regenerative state 'i' to a regenerative state 'j' or to a unsuccessful state 'j' deprived of visiting any additional regenerative state in (0,t]".

$p_{i,j}$**:** "Steady state transition probability after a regenerative state 'i' to a regenerative state 'j' deprived of visiting any additional regenerative state. $p_{i,j} = q^*_{i,j}(0)$; where * denotes Laplace transformation.

Table 1: Transition Probabilities

$q_{i,j}(t)$	$p_{i,j} = q^*_{i,j}(t)$
$q_{1,2}(t) = g_1\, e^{-(g_1+g_2)t}$	$p_{1,2} = g_1/(g_1+g_2)$
$q_{1,3}(t) = g_2\, e^{-(g_1+g_2)t}$	$p_{1,3} = g_1/(g_1+g_2)$
$q_{2,1}(t) = h_1\, e^{-(g_1+g_2+g_3+h_1)t}$	$p_{2,1} = g_1/(g_1+g_2+g_3+h_1)$
$q_{2,5}(t) = g_3\, e^{-(g_1+g_2+g_3+h_1)t}$	$p_{2,5} = g_3/(g_1+g_2+g_3+h_1)$
$q_{2,6}(t) = g_2\, e^{-(g_1+g_2+g_3+h_1)t}$	$p_{2,6} = g_2/(g_1+g_2+g_3+h_1)$
$q_{2,8}(t) = g_1\, e^{-(g_1+g_2+g_3+h_1)t}$	$p_{2,8} = g_1/(g_1+g_2+g_3+h_1)$
$q_{3,1}(t) = h_2\, e^{-(h_2+g_3)t}$	$p_{3,1} = h_2/(h_2+g_3)$
$q_{3,4}(t) = g_3\, e^{-(h_2+g_3)t}$	$p_{3,4} = g_3/(h_2+g_3)$
$q_{4,3}(t) = h_3\, e^{-h_3 t}$	$p_{4,3} = 1$
$q_{5,2}(t) = h_3\, e^{-(g_1+g_2+h_3)t}$	$p_{5,2} = h_3/(g_1+g_2+h_3)$
$q_{5,7}(t) = g_2\, e^{-(g_1+g_2+h_3)t}$	$p_{5,7} = g_2/(g_1+g_2+h_3)$
$q_{5,9}(t) = g_1\, e^{-(g_1+g_2+h_3)t}$	$p_{5,9} = g_1/(g_1+g_2+h_3)$
$q_{6,2}(t) = h_2\, e^{-(g_3+h_2)t}$	$p_{6,2} = h_2/(g_3+h_2)$
$q_{6,7}(t) = g_3\, e^{-(g_3+h_2)t}$	$p_{6,7} = g_3/(g_3+h_2)$
$q_{7,6}(t) = h_3\, e^{-h_3 t}$	$p_{7,6} = 1$
$q_{8,2}(t) = h_1\, e^{-(g_3+h_1)t}$	$p_{8,2} = h_1/(h_1+g_3)$
$q_{8,9}(t) = g_3\, e^{-(g_3+h_1)t}$	$p_{8,9} = g_3/(h_1+g_3)$
$q_{9,8}(t) = h_3 e^{-h_3 t}$	$p_{9,8} = 1$
	$p_{1,3} + p_{1,3} = 1$

$p_{2,1} + p_{2,5} + p_{2,6} + p_{,8} = 1$; $p_{3,1} + p_{3,4} = 1$; $p_{5,2} + p_{5,7} + p_{5,9} = 1$; $p_{6,2} + p_{6,7} = 1$

$p_{8,2} + p_{8,9} = 1$

Table 2: Mean Sojourn Times

$R_i(t)$	$\mu_i = R_i^*(0)$
$R_1(t) = e^{-(g_1+g_2)t}$	$\mu_1 = 1/(g_1+g_2)$
$R_2(t) = e^{-(g_1+g_2+g_3+h_1)t}$	$\mu_2 = 1/(g_1+g_2+g_3+h_1)$
$R_3(t) = e^{-(h_2+g_3)t}$	$\mu_3 = 1/(h_2+g_3)$
$R_4(t) = e^{-h_3 t}$	$\mu_4 = 1/h_3$

R_5 (t) = $e^{-(g_1+g_2+h_3)t}$	$\mu_5 = 1/(g_1 + g_2 + h_3)$
R_6 (t) = $e^{-(g_3+h_2)t}$	$\mu_6 = 1/(g_3 + h_2)$
R_7 (t) = $e^{-h_3 t}$	$\mu_7 = 1/h_3$
R_8 (t) = $e^{-(h_1+g_3)t}$	$\mu_8 = 1/(h_1 + g_3)$
R_9 (t) = $e^{-h_3 t}$	$\mu_9 = 1/h_3$

7. Evaluation of Path Probabilities

Applying RPGT and utilizing '1' as initial-state of the framework, we discovery transition probability aspects of all accessible states from first state 'ξ' = '1' . We will discover probabilities after state '1' to various vertices which are defined as follows:

$V_{1,1}$ = 1 (Verified)

$V_{1,2}$ = (1, 2)/ {1 - (2, 5, 2)} [1 - (2, 6, 2)/{1 - (6, 7, 6)}][1 - (2, 8, 2)/{1 - (8, 9, 8)}]

$= p_{1,2}/(1 - p_{2,5}\, p_{5,2})[1 - \{(p_{2,6}\, p_{6,2}/(1 - p_{6,7}\, p_{7,6})\}] [1 - \{(p_{2,8}\, p_{8,2}/(1 - p_{8,9}\, p_{9,8})\}]$

$V_{1,3}$ = ……Continuous

Transition state probabilities from base state '2' are

$V_{2,1}$ = (2, 1)/[{1 - (1, 3, 1)}/{1 - (3, 4, 3)}]

$= p_{2,1}/\{(1 - p_{1,3}\, p_{3,1})/(1 - p_{3,4}\, p_{4,3})\}$

$V_{2,2}$ = 1

$V_{2,3}$ = (2, 1, 3)/[{1 - (1, 3, 1)}/{1 - (3, 4, 3)}]{1 - (3, 4, 3)}

$= p_{2,1}\, p_{1,3}/[\{(1 - p_{1,3}\, p_{3,1})/(1 - p_{3,4}\, p_{4,3})\}(1 - p_{3,4}\, p_{4,3})]$

$V_{2,4}$ =…….Continuous

8. Modeling System Parameters

Mean time to system failure (T_0): Regenerative un-failed states to which the framework can transit (initial state '2'), earlier incoming any fizzled state are: 'i' = 1, 2, 5 attractive 'ξ' = '1'

$T_0 = (V_{1,1}\,\mu_1 + V_{1,2}\,\mu_2 + V_{1,5}\,\mu_5)/ \{1 - (1, 2, 1)\}$

Availability of the system (A_0): Regenerative states at which the framework is accessible are 'i' = 1, 2, 5 attractive 'ξ' = '1' whole fraction of time for which the framework is accessible is assumed by

$A_0 = \left[\sum_j \; V_{\xi,j}\, , f_j\, ,\ \mu_j\right] \div \left[\sum_i V_{\xi,i}\, , f_j\, ,\ \mu_i^1\right]$

$A_0 = (V_{2,1}\,\mu_1 + V_{2,2}\,\mu_2 + V_{2,5}\,\mu_5)/Z_1$

$\therefore Z = V_{1,1}\,\mu_1 + V_{1,2}\,\mu_2 + V_{1,3}\,\mu_3 + V_{1,4}\,\mu_4 + V_{1,5}\,\mu_5 + V_{1,6}\,\mu_6 + V_{1,7}\,\mu_7 + V_{1,8}\,\mu_8 + V_{1,9}\,\mu_9$

$\therefore Z_1 = V_{2,1}\mu_1 + V_{2,2}\mu_2 + V_{2,3}\mu_3 + V_{2,4}\mu_4 + V_{2,5}\mu_5 + V_{2,6}\mu_6 + V_{2,7}\mu_7 + V_{2,8}\mu_8 + V_{2,9}\mu_9$

Server of busy period (B_0): Regenerative states where server is busy are $2 \leq j \leq 9$, attractive ξ = '1', whole fraction of time for which server remains eventful is by equation

$B_0 = \left[\sum_j V_{\xi,j}\, , n_j\right] \div \left[\sum_i V_{\xi,i}\, ,\ \mu_i^1\right]$

$B_0 = (V_{1,2}\mu_2 + V_{1,3}\mu_3 + V_{1,4}\mu_4 + V_{1,5}\mu_5 + V_{1,6}\mu_6 + V_{1,7}\mu_7 + V_{1,8}\mu_8 + V_{1,9}\mu_9)/D$

$= 1 - (\mu_1/ D)$

Expected number of server visit's (V_0): Regenerative states where repair man does this job j = 2, 5 taking 'ξ' = '1', number of visit by repair man is given by

$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}\, ,\ \mu_i^1\right]$

$V_0 = (V_{1,2} + V_{1,5})/ D$

9. Behavior Analysis: Particular Cases: - $h_i = h$; $g_i = g$

MTSF (T_0)

Table 3: MTSF (T_0)

	h = .55	h = .65	h = .75
g = .15	5.32	5.25	5.05
g = .25	4.49	4.42	4.37
g = .35	3.53	3.49	3.42

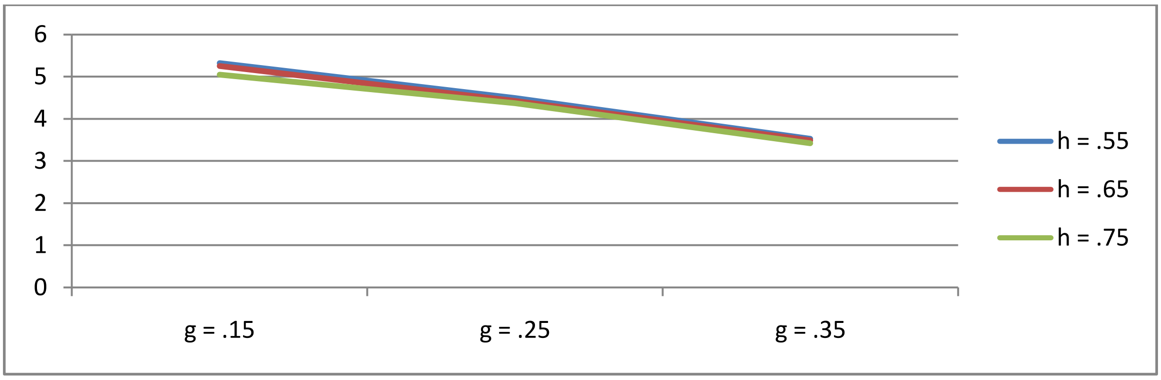

Fig.2: MTSF

From the above chart 2 and table 3demonstrations the performance of MTSF Vs Repair rate of the sub-unit of the framework for various values of the disappointment rate. From the above fig.2 one can determine that MTSF is increasing which must be so once the repair rate amassed and decreases when the disappointment rate rises which should be so in practical situations.

Availability of the system (A_0):

Table 4: Availability of the system

	h = .55	h = .65	h = .75
g = .15	.84	.88	.93
g = .25	.72	.75	.79
g = .35	.62	.67	.72

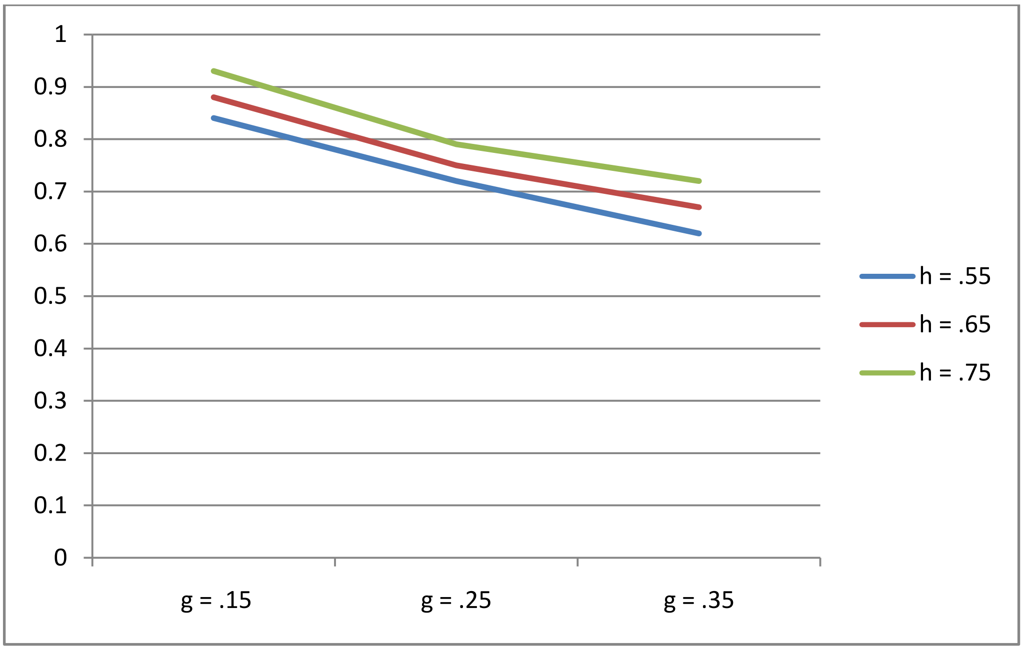

Fig. 3: Availability of the system

The above table 4 shows that the Availability is increasing when the repair rate is increasing and decrease with the rise in disappointment rate, which ought to be actually.

Server of the busy period (B_0):

Table 5: Server of the busy period

	h = .50	h = .60	h = .70
g = .15	.65	.62	.59
g = .25	.69	.65	.62
g = .35	.74	.69	.67

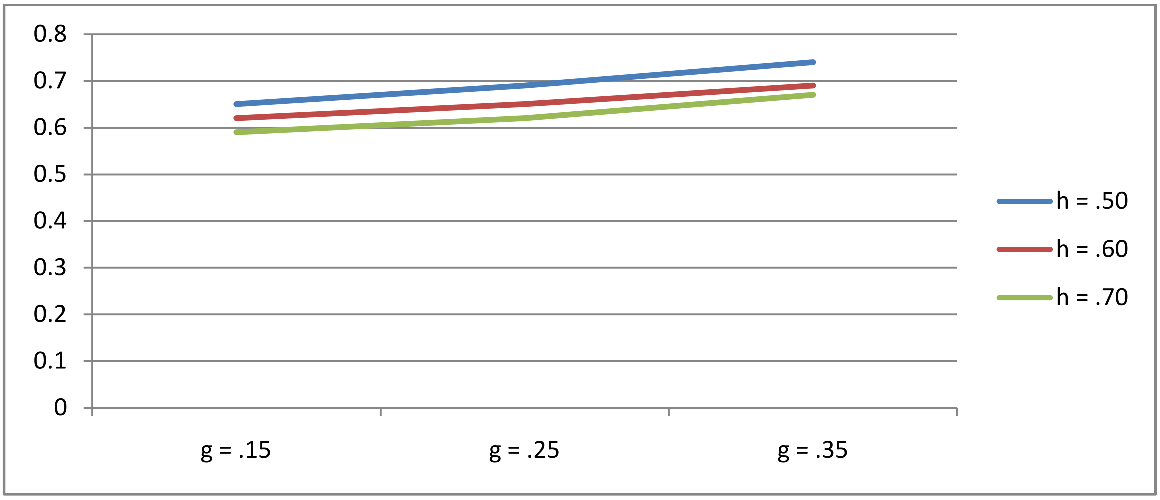

Fig. 4: Server of the busy period

It can be concluded from the above fig. 4 that the values of server of busy period shows the expected trend for various values of disappointment rate, as server of busy period decreases with the rise in the values of repair rate.

Expected number of server visits (V_0): -

Table 6: Expected number of server visits

	h = .55	h = .65	h = .75
g = .15	.34	.38	.42
g = .25	.39	.44	.48
g = .35	.43	.48	.52

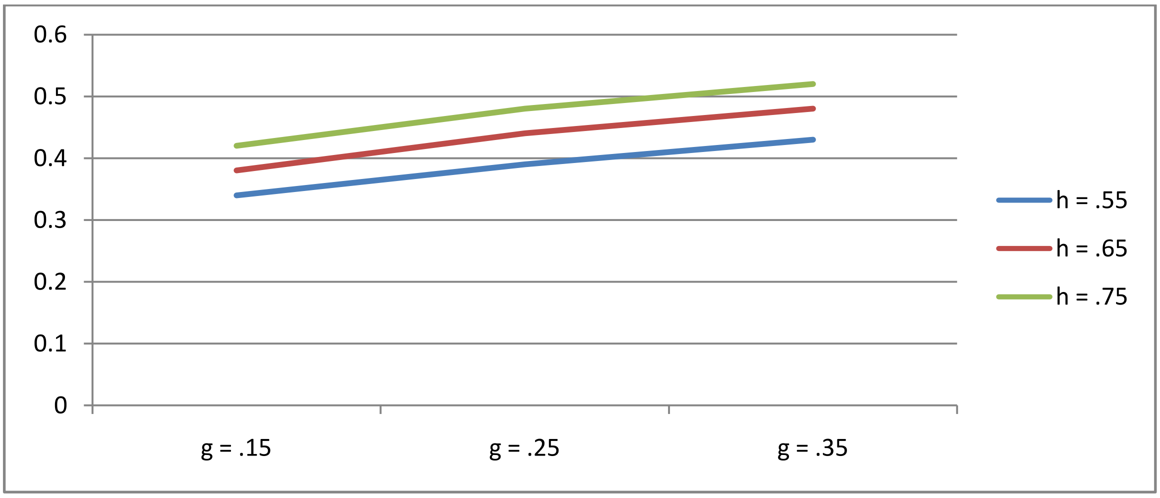

Fig. 5: Expected number of server visits

It can be concluded from the above fig. 5 and table 6that the values of Expected number of server visits demonstrations the expected trend for various values of disappointment rate, as Expected number of server visits increases with the rise in the values of repair rate.

10. Conclusion

To have optimum value of system parameters management may control the failure and repair rates of units depending upon the availability of finances and market circumstances.

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