Bayesian Method of an Estimation for Gamma Distribution Generalized Using Extension of Jeffrey's Prior

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Abstract

Several structural characteristics of the Generalized Gamma Distribution (GGD) have been revealed in this work. Under four distinct loss functions, the Bayesian technique of determine has been used to determine the parameters of GGD using the Jeffrey's & extension of Jeffrey's priors. Through the use of simulated studies with various sample sizes and R software, the estimate so derived was compared with the traditional Maximum Likelihood Estimator employing MSE. Jeffrey's prior work has been extended to include the equation for the survival function.

Keywords: -Likelihood, Bayesian, Generalized Gamma Distribution, R Software, Jeffrey's Prior

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Introduction: Modeling duration requires a flexible family, and Stacy's (1962) Generalized Gamma Distribution (GGD) provides one such example. The GGD presents a variety of different form and hazard functions. Seufert (2021) Due to the fact that investigations into subjectivity Quality of Experience (QoE) utilise biassed assumptions regarding ordinal rating scale, Mean Percentage Score (MOS)-based assessments provide results that are problematic and misleading.

Mishra et al. (2019) Epidemiological, statistical approaches for data processing and categorization are accessible. These methodologies can be applied to each and every specific circumstance. We went through parametric and non-parametric methodologies, their prerequisites, and how to choose appropriate statistical measurement and analysis, as well as interpretation of biological data, in this post. The Senators Sarmento and Costa (2019) The application of statistical software in both business and academic settings has become increasingly common during the past few years. Everyone from students and professors to experts and average users has had some experience with statistical software at some time in their lives. In this study, we make an effort to make access to various theoretical concepts easier by providing a statistical review of such concepts.

Peligrad (2018) conducted research on the exact modest and large eccentricity asymptotic in non-logarithmic technique for linear procedures that have self-governing innovations. Since the linear processes that we investigate are universal in nature, we will refer to them as the "long memory case." Savsani and Ghosh (2017) in this article, we have developed a method for finding the posterior distribution of the Moderate Distribution by making use of the likelihood of a single observation and its ordinates. This method was inspired by Savsani and Ghosh's 2017 study.

In the process of duration analysis in economics, distributions including exponential (Kiefer, 1984), gamma (Lancaster, 1979), &Weibull (Favero et al, 1994), which are all subfamilies of GGD, are used. Jasggia has also used the lognormal distribution, which is thought to be a limiting distribution, in economics (1991). In this section of the essay, Jeffreys' prior distribution is extended in order to estimate the posterior distribution of GGD. In order to get a precise assessment of the scale parameter of GGD, we have tested a number of alternative loss functions.

Posterior density by using postponement of Jeffrey's previous

Let (X_1,X_2……X_n) be an n-piece random model that consumes the probability density function as

$$f(x;\lambda,\beta,k)=\frac{\lambda\beta}{\Gamma k}(\lambda x)^{k\beta-1}e^{-(\lambda x)^{\beta}}, \text{ for } x>0 \text{ and } \lambda,\beta,k>0.$$

Given by is the probability function.

$$L(x;\lambda,\beta,k)=\frac{\lambda^{n(k\beta)}\beta^{n}}{\Gamma^{n}(k)}\prod_{i=1}^{n} x_i^{k\beta-1}e^{-\lambda^{\beta}\sum_{i=1}^{n}x_i}.$$

We study the previousdelivery of λbeing

$g(\lambda)\propto[\det|I(\lambda)|]^{c}, c\in R^{+}$

$$g(\lambda)=\rho\frac{1}{\lambda^{2c}} \qquad (1)$$

where ρ is constant. The following distribution of λ is known by

$$\pi_2(\lambda\mid\underline{x})\propto L(x\mid\lambda)g(\lambda) \qquad (2)$$

Using eq. (1) in eq. (2), we get

$$\pi_2(\lambda\mid\underline{x})\propto\frac{\lambda^{nk\beta-2c}\beta^{n}}{\Gamma^{n}(k)}e^{-\lambda^{\beta}\sum_{i=1}^{n}x_i^{\beta}}\prod_{i=1}^{n}(x_i)^{k\beta-1}$$

$$\pi_2(\lambda\mid\underline{x})=\rho_2\lambda^{nk\beta-2c}e^{-\lambda^{\beta}\sum_{i=1}^{n}x_i^{\beta}} \qquad (3)$$

where ρ_2 is independent of λ .

$$\rho_2^{-1} = \int_0^\infty \lambda^{nk\beta - 2c} e^{-\lambda^\rho \sum_{i=1}^n x_i^\beta} d\lambda$$

On resolving the above appearance, we get

$$\rho_2 = \frac{\left(\sum_{i=1}^n x_i^\beta\right)^{nk - \frac{2c}{\beta} + 1}}{\Gamma\left(nk - \frac{2c}{\beta} + 1\right)}$$

calculating ρ2 in eq. (3), we get the posterior distribution given as below

$$\pi_2(\lambda \mid \underline{x}) = \left(\frac{e^{-\lambda^\beta \sum_{i=1}^n x_i} \lambda^{nk\beta - 2c} \left(\sum_{i=1}^n x_i^\beta\right)^{nk - \frac{2c}{\beta} + 1}}{\Gamma\left(nk - \frac{2c}{\beta} + 1\right)}\right) \tag{4}$$

Squared error loss estimate Function (SELF)

Squared error loss$l_{SI}(\hat{\lambda}, \lambda) = a(\hat{\lambda} - \lambda)^2$for some constant a , the risk function is

$$R(\hat{\lambda}) = \int_0^\infty a(\hat{\lambda} - \lambda)^2 \pi_2(\lambda \mid \underline{x}) d\lambda \tag{5}$$

By using eq. (4) in eq. (5), we take

$$R(\hat{\lambda}) = \int_0^\infty a(\hat{\lambda} - \lambda)^2 \left(\frac{e^{-\lambda^\beta \sum_{i=1}^{x_i} x_i} \lambda^{nk\beta - 2c} \left(\sum_{i=1}^n x_i^\beta\right)^{nk - \frac{2c}{\beta} + 1}}{\Gamma\left(nk - \frac{2c}{\beta} + 1\right)} d\lambda\right.$$

$$R(\hat{\lambda}) = \frac{a\left(\sum_{i=1}^n x_i^\beta\right)^{nk - \frac{2c}{\beta} + 1}}{\Gamma\left(nk - \frac{2c}{\beta} + 1\right)} \left[\hat{\lambda}^2 \int_0^\infty e^{-\lambda^\beta \sum_{i=1}^n x_i^\beta} \left(\lambda^\beta\right)^{nk - \frac{2c}{\beta}} d\lambda + \int_0^\infty e^{-\lambda^\beta \sum_{i=1}^n x_i^\beta} \left(\lambda^\beta\right)^{nk + \frac{2(1-c)}{\beta}} d\lambda \right. \\ \left. -2\hat{\lambda} \int_0^\infty e^{-\lambda^\beta \sum_{i=1}^n x_i} \left(\lambda^\beta\right)^{nk + \frac{(1-2c)}{\beta}} d\lambda \right]$$

Resolving the above appearance, we have

$$R(\hat{\lambda}) = a\hat{\lambda}^2 + a \frac{\Gamma\left(nk - \frac{2c}{\beta} + \frac{2}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{2}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)} - \frac{2a\hat{\lambda}\Gamma\left(nk - \frac{2c}{\beta} + \frac{1}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{1}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)}$$

Now in order to gain Bayesian estimator, we take$\frac{\partial R(\hat{\lambda})}{\partial \hat{\lambda}} = 0$

$$\frac{\partial}{\partial \hat{\lambda}} \left[a\hat{\lambda}^2 + \frac{a\Gamma\left(nk - \frac{2c}{\beta} + \frac{2}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{2}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)} - \frac{2a\hat{\lambda}\Gamma\left(nk - \frac{2c}{\beta} + \frac{1}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{1}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)}\right] = 0$$

$$\hat{\lambda} = \frac{1}{\left(\sum_{i=1}^n x_i\right)} \left\{\frac{\Gamma\left(nk - \frac{2c}{\beta} + \frac{1}{\beta} + 1\right)}{\Gamma\left(nk - \frac{2c}{\beta} + 1\right)}\right\} \tag{6}$$

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Remark: Replacing c=1/2 in eq. (6), the same Bayes estimation is got as in eq. (2) equivalent to the Jeffrey's prior

Estimation Al-Bayyati's loss function (ALF):

Al-Bayyati's loss function $l_{Nl}(\hat{\lambda},\lambda)=\lambda^{c_1}(\hat{\lambda}-\lambda)^2$the risk function is given by

$$R(\hat{\lambda})=\int_0^{\infty}\lambda^{c_1}(\hat{\lambda}-\lambda)^2\pi_2(\lambda \mid \underline{x})d\lambda \tag{7}$$

Using eq. (4) in eq. (7), we devise

$$R(\hat{\lambda})$$

$$=\frac{a\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\left[\begin{array}{c}\hat{\lambda}^2\int_0^{\infty}e^{-\lambda^{\beta}\sum_{i=1}^{\infty}x_i'}\left(\lambda^{\beta}\right)^{nk+\frac{(c_1-2c)}{\beta}}d\lambda+\int_0^{\infty}e^{-\lambda^{\beta}\sum_{i=1}^{\infty}x_j'}\left(\lambda^{\beta}\right)^{nk+\frac{(c_1-2c+2)}{\beta}}d\lambda\\ -2\hat{\lambda}\int_0^{\infty}e^{-\lambda^{\beta}\sum_{i=1}^{\infty}x_i}\left(\lambda^{\beta}\right)^{nk+\frac{(c_1-2c+1)}{\beta}}d\lambda\end{array}\right]$$

Solving the above expression, we have

$$R(\hat{\lambda})=\frac{\hat{\lambda}^2\Gamma\left(nk+\frac{c_1}{\beta}-\frac{2c}{\beta}+1\right)}{\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{9}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}+\frac{\Gamma\left(nk-\frac{2c}{\beta}+\frac{c_1}{\beta}+\frac{2}{\beta}+1\right)}{\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{902}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}$$
$$-\frac{2\hat{\lambda}\Gamma\left(nk-\frac{2c}{\beta}+\frac{c_1}{\beta}+\frac{1}{\beta}+1\right)}{\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{9+1}{\beta}}\Gamma\left(nk-\frac{1}{\beta}+1\right)}.$$

Now in order to gain Bayesian estimator, we take$\frac{\partial R(\hat{\lambda})}{\partial\hat{\lambda}}=0$ |

$$\frac{\partial}{\partial\hat{\lambda}}\left[\begin{array}{c}\frac{\hat{\lambda}^2\Gamma\left(nk+\frac{c_1}{\beta}-\frac{2c}{\beta}+1\right)}{\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{c_1}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}+\frac{\Gamma\left(nk-\frac{2c}{\beta}+\frac{c_1}{\beta}+\frac{2}{\beta}+1\right)}{\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{c_1+2}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\\ -\frac{2\hat{\lambda}\Gamma\left(nk-\frac{2c}{\beta}+\frac{c_1}{\beta}+\frac{1}{\beta}+1\right)}{\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{c_1+1}{\beta}}\Gamma\left(nk-\frac{1}{\beta}+1\right)}\end{array}\right]=0$$

$$\hat{\lambda}=\frac{1}{\left(\sum_{i=1}^{n}x_i\right)}\left\{\frac{\Gamma\left(nk-\frac{2c}{\beta}+\frac{c_1}{\beta}+\frac{1}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+\frac{c_1}{\beta}+1\right)}\right\} \tag{8}$$

Remark: Exchanging c=1/2 in eq. (8), the same Bayes estimation is got as in eq. (6) equivalent to the Jeffrey's previous.

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Precautionary Loss Function (PLF)

By using precautionary loss function $l_{pr}(\hat{\lambda},\lambda)=\frac{(\hat{\lambda}-\lambda)^2}{\hat{\lambda}}$ the risk function is given

$$R(\hat{\lambda})=\int_0^\infty \frac{(\hat{\lambda}-\lambda)^2}{\hat{\lambda}}\pi_2(\lambda \mid \underline{x})d\lambda \tag{9}$$

Using eq. (4) in eq. (9), we take

$$R(\hat{\lambda})=\frac{\left(\sum_{i=1}^n x_i^\beta\right)^{nk-\frac{2c}{\beta}+1}}{\hat{\lambda}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\left[\begin{matrix}\int_0^\infty e^{-\lambda^\rho\sum_{i=1}^n x_i}\left(\lambda^\beta\right)^{nk+\frac{2(1-c)}{\beta}}d\lambda+\hat{\lambda}_0^\infty e^{-\lambda^\rho\sum_{i=1}^n x_i^\beta}\left(\lambda^\beta\right)^{nk-\frac{2c}{\beta}}d\lambda\Big] \\ -2\int_0^\infty e^{-\lambda^\beta\sum_{i=1}^\infty x_i}\left(\lambda^\beta\right)^{nk+\frac{(1-2c)}{\beta}}d\lambda\end{matrix}\right]$$

On solving the above expression, we have

$$R(\hat{\lambda})=\frac{\Gamma\left(nk-\frac{2c}{\beta}+\frac{2}{\beta}+1\right)}{\hat{\lambda}\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{2}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}+\hat{\lambda}-\frac{2\Gamma\left(nk-\frac{2c}{\beta}+\frac{1}{\beta}+1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{1}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}.$$

Now in order to determine the Bayesian estimator, we have $\frac{\partial R(\hat{\lambda})}{\partial\hat{\lambda}}=0$

$$\frac{\partial}{\partial\hat{\lambda}}\left[\frac{\Gamma\left(nk-\frac{2c}{\beta}+\frac{2}{\beta}+1\right)}{\hat{\lambda}\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{2}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}+\hat{\lambda}-\frac{2\Gamma\left(nk-\frac{2c}{\beta}+\frac{1}{\beta}+1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{1}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\right]=0 \tag{10}$$

Remark: Replacing c=1/2 in eq. (10), the same Bayes estimator is got as in eq. (2) equivalent to the Jeffrey's prior

Quadratic Loss Function (QLF)

By using quadratic loss function $l_{qd}(\hat{\lambda},\lambda)=\left(\frac{\hat{\lambda}-\lambda}{\lambda}\right)^2$ the risk function is given by

$$R(\hat{\lambda})=\int_0^\infty \left(\frac{\hat{\lambda}-\lambda}{\lambda}\right)^2\pi_2(\lambda \mid \underline{x})d\lambda \tag{11}$$

Using eq. (4) in eq. (11), we have

$$R(\hat{\lambda})=\int_0^\infty\left(\frac{\hat{\lambda}-\lambda}{\lambda}\right)^2\frac{e^{-\hat{\lambda}^\beta\sum_{i=1}^{x_i}}\lambda^{nk\beta-2c}\left(\sum_{i=1}^n x_i^\beta\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}d\lambda$$

Solving the above formula yields.

$$R(\hat{\lambda})=\hat{\lambda}^2\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{2}{\beta}}\frac{\Gamma\left(nk-\frac{2(c+1)}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}+1-2\hat{\lambda}\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{1}{\beta}}\frac{\Gamma\left(nk-\frac{(2c+1)}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}.$$

Now in order to determine the Bayesian estimator, we have $\frac{\partial R(\hat{\lambda})}{\partial\hat{\lambda}}=0$

$$\frac{\partial}{\partial\hat{\lambda}}\left[\frac{\hat{\lambda}^2\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{\frac{2}{\beta}}\Gamma\left(nk-\frac{2(c+1)}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}+1-\frac{2\hat{\lambda}\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{\frac{1}{\beta}}\Gamma\left(nk-\frac{(2c+1)}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\right]=0$$

$$\hat{\lambda}=\frac{1}{\left(\sum_{i=1}^{n} x_i\right)}\left[\frac{\left\{\Gamma\left(nk-\frac{(2c+1)}{\beta}+1\right)\right\}}{\left\{\Gamma\left(nk-\frac{2(c+1)}{\beta}+1\right)\right\}}\right] \qquad (12)$$

Remark: Exchanging c=1/2 in eq. (12), the same Bayes estimator is got as in eq. (2) equivalent to Jeffrey's prior.

Estimation of Survival function (SF)

We may determine the survival function by utilising the posterior probability density function, such that

$$\hat{S}_2(\underline{x})=\int_0^{\infty} e^{-(\lambda x)^{\beta}}\pi_2(\lambda \mid \underline{x})d\lambda \qquad (13)$$

$$\hat{S}_1(\underline{x})=\frac{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\left[\int_0^{\infty} e^{-(\lambda x)^{\beta}} e^{-\lambda^{\beta}\sum_{i=1}^{n} x_i^{\beta}}\lambda^{nk\beta-2c}d\lambda\right]$$

Using eq. (4) in eq. (13), we have

$$\hat{S}_2(\underline{x})=\left(\frac{\sum_{i=1}^{n} x_i^{\beta}}{x_i^{\beta}+\sum_{i=1}^{n} x_i^{\beta}}\right)^{nk-\frac{2c}{\beta}+1}. \qquad (14)$$

SIMULATION STUDY OF GENERALIZED GAMMA DISTRIBUTION (GGD)

A simulation research was carried out with the assistance of the R programming language in order to investigate and evaluate the accuracy of the estimations for three distinct sample sizes (n = 25, 50, and 100), which respectively represented a small, medium, and big data collection. Both traditional and Bayesian approaches to estimation are utilised in the process of determining the value of the scale parameter for the generalised gamma distribution. Within the framework of the Bayesian method of estimation, we make use of Jeffrey's & an extension of Jeffrey's former while considering a variety of loss functions. When determining the value for the scales parameter, we looked at = 1.0, 1.5, & 2.0. Jeffrey's extension had values of c equal to 0.5, 1.0, and 1.5 respectively. The following values have been determined for the c1 loss parameter: 1, -1, 2, and -2. Following the calculation of the scale parameter for each method, this process was repeated two thousand times. The findings are summarised in the tables that are shown below.

Table 1: Mean Squared Error for $\hat{\lambda}$ under Jeffrey's prior

n	λ	β	κ	λ_{ML}	λ_{sl}	λ_{NI}			
						C1=1	C1=-1	C1=2	C1=-2
25	1.0	0.5	0.5	1.0834	1.0244	1.2256	0.9843	1.1135	0.5734
	1.5	1.0	1.0	0.6857	0.6852	0.6632	0.7305	0.6413	0.7305
	2.0	1.5	1.0	0.6823	0.6810	0.6786	0.6863	0.6760	0.6863
50	1.0	0.5	0.5	1.1597	1.2941	1.0642	1.0851	1.1223	0.4792
	1.5	1.0	1.0	0.4263	0.4261	0.4152	0.4485	0.4044	0.4485
	2.0	1.5	1.0	0.7523	0.7523	0.7513	0.7544	0.7502	0.7544
100	1.0	0.5	0.5	1.9086	1.9086	1.9019	1.2507	0.9669	0.1199
	1.5	1.0	1.0	0.8329	0.8329	0.8275	0.8437	0.8222	0.8437
	2.0	1.5	1.0	0.7775	0.7775	0.9631	0.9647	0.7764	0.8081

ML=MaximumProbability,Sl=squared mistakeLF,Pr=defensiveLF,qd=quadraticLF,Nl=Al-Bayyati'sLF

The Bayes estimate using Al-loss Bayyati's function and Jeffrey's prior yields the minimum values in the majority of instances, particularly when the loss parameter C1 is set to -2 in the table 1 that was just presented. Therefore, it is possible for us to draw the conclusion that the Bayes estimator works well when Al-loss is used. Comparing Bayyati's function to other loss functions and the traditional estimator

Table 2: Mean Squared Error for $\hat{\lambda}$ under extension of Jeffrey's prior

n	λ	β	κ	**C**	λ_{ML}	λ_{sl}	λ_{pr}	λ_{qd}	λ_{NI}			
									C1=1	C1=-1	C1=2	C1=-2
25	1.0	0.5	0.5	0.5	0.5838	0.2311	0.2950	0.1948	0.2628	0.1948	0.3089	0.0571
				1.0	0.7738	0.2578	0.1734	0.3786	0.3251	0.1366	0.1491	0.0891
				1.5	0.1787	0.1868	0.5601	0.1028	0.3462	0.1028	0.2022	0.0753
	1.5	1.0	1.0	0.5	0.8879	0.8875	0.8666	0.9304	0.8456	0.9304	0.8047	0.6241
				1.0	0.8155	0.8586	0.8367	0.9037	0.8147	0.9037	0.7717	0.4614
				1.5	0.8769	0.9630	0.9411	1.0082	0.9191	0.9883	0.8761	0.2706
	2.0	1.5	1.0	0.5	1.1271	1.2578	1.3295	1.3889	1.5483	1.4604	1.2710	1.0525
				1.0	1.4098	1.1535	1.4532	1.4886	1.6567	1.2551	1.6148	1.075

				1.5	1.3308	1.4242	1.4725	1.2556	1.4919	1.2400	1.4484	1.0278
50	1.0	0.5	0.5	0.5	0.9244	0.4822	0.1015	0.4309	0.6661	0.4309	1.0517	0.1898
				1.0	0.9288	0.2332	1.0292	0.5212	0.2195	0.5212	0.3160	0.1965
				1.5	0.6407	0.8001	0.1095	0.2509	0.1676	0.7764	0.2780	0.1382
	1.5	1.0	1.0	0.5	0.8442	0.8441	0.8336	0.8657	0.8230	0.8657	0.8020	0.7097
				1.0	1.0558	1.0752	1.0655	1.0949	1.0557	1.0949	1.0361	0.9147
				1.5	0.9252	0.9670	0.9565	0.9883	0.9460	0.9883	0.9250	0.8198
	2.0	1.5	1.0	0.5	1.8551	1.8551	1.8434	1.8783	1.8320	1.8783	1.8088	1.6360
				1.0	1.6263	1.6511	1.6388	1.6761	1.6263	1.6761	1.6017	1.2201
				1.5	1.6536	1.7030	1.6907	1.7281	1.6783	1.7281	1.6535	1.2641
100	1.0	0.5	0.5	0.5	1.4077	0.5084	0.7676	0.3731	0.1842	0.2499	0.5785	0.1400
				1.0	1.1779	0.4541	0.2198	0.1993	0.5784	0.8096	0.1011	0.0982
				1.5	1.7517	0.3001	0.6765	0.7517	0.1431	0.5004	0.3364	0.1322
	1.5	1.0	1.0	0.5	0.8333	0.8332	0.8278	0.4557	0.8225	0.8440	0.8118	0.3914
				1.0	0.9041	0.9146	0.9092	0.9251	0.9041	0.9251	0.8937	0.7525
				1.5	0.8830	0.9043	0.8990	0.9150	0.8935	0.9150	0.8830	0.7436
	2.0	1.5	1.0	0.5	1.6398	1.6397	1.6336	1.6521	1.6275	1.6521	1.6153	1.1885
				1.0	1.6123	1.6248	1.6184	1.6370	1.6123	1.6370	1.5998	1.0497
				1.5	1.5403	1.5654	1.5592	1.5782	1.5527	1.5782	1.5403	1.1252

ML=MaximumLikelihood,Sl=squarederrorLF,Pr=precautionaryLF,qd=quadraticLF,Nl=Al-Bayyati'sLF

Generally, especially when loss parameter C1 is -2, Bayes' estimate using Al-loss Bayyati's function under extension of Jeffrey's previous yields the lowest values, regardless of whether the extension of Jeffrey's prior is 0.5, 1.0, or 1.5, as shown in table 2 above. Particularly when loss parameter C1 is -2, this is true. It follows that Al-loss Bayyati's function provides a Bayes estimate that is superior to other loss functions and the conventional estimator.

Conclusion:

To calculate the generalised gamma distribution's scaling parameter, the primary focus of our research was on traditional methods of estimation as well as Bayesian methods. The Mean Squared Error (MSE) method is utilised to analyse the differences between the estimates, and the outcomes are detailed in the tables that are located above.Based on the findingsWhen compared to other loss functions & the classical estimation, we see that the Bayes estimator under Al-loss Bayyati's function has the lowest MSE values for both priors (Jeffrey's and the extension of Jeffrey's prior). The majority of the time, this is the case. As a consequence, we may infer that when the loss parameter C1 is set to -2, the Bayes estimator using Al-loss Bayyati's function is effective.

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