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C_4 Free Detour Distance

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Abstract

For every connected graph G, the square free detour distance SFD (u, v) is the length of a longest u- v square free path in G, where u, v are the vertices of G. A u-v square free path of length SFD(u, v) is called the u-v square free detour. It is found that the square free detour distance differs from the distance , monophonic distance and detour distance. The square free detour distance is found for some standard graphs. Their bounds are determined and their sharpness is checked. Certain general properties satisfied by them are studied.

1. Introduction

For basic graph theoretic terminologies the research refers to [2, 7]. For terminology related to distance and detour distance in graphs Chartrand et.al. [2, 5, 7] are referred to. For any two vertices u and v in a connected graph G, the distance d(u, v) is the length of a shortest u-v path in G. A u-v path of length d(u,v) is called a u-v geodesic in G.

For any two vertices u and v in a connected graph G, a u-v path P is a u-v monophonic path if P contains no chords. The monophonic distance d_m(u, v) from u to v is defined as the length of a longest u-v monophonic path in G. A u-v monophonic path of length d_m(u, v) is called a u-v monophonic.

For any two vertices u and v in a connected graph G, the detour distance D(u, v) is the length of a longest u - v path in G. A u- v path of length D(u, v) is called a u-v detour in G. The following theorem is used in the article

Theorem:1.1[3]An edge e of a graph G is a bridge iff e lies on no cycle of G.

2. C_4 Free Detour Distance

Definition:2.1

Let u and v represent any two of the connected graph G's vertices. A uv path P is referred to be a uv square free path if none of its four vertices cause a cycle C_4 to occur in G. The sfd distance, abbreviated $SDF(u, v)$, is the length of the longest uv sqaure free path in G. The term "$uv\ sfd$" refers to a uv square free path with length $SDF(u, v)$.

Example:2.1

Take a look at the graph in figure:2.1.1. The u_1, u_5 paths $P_1: u_1, u_5$ and $P_2: u_1, u_2, u_5$ are the u_1, u_5 square free paths, however the u_1, u_5 paths $P_3: u_1, u_2, u_3, u_5$ and $P_3: u_1, u_2, u_3, u_{4,} u_5$ are not. Here, $d(u_1, u_5) = 1$, $d_m(u_1, u_5) = 1$, $D_\Delta(u_1, u_5) = 1$, $D(u_1, u_5) = 4$ and $SFD(u_1, u_5) = 2$. As a result, the sfd distance is distinct form distance, monophonic distance, triangle free detour distance and detour distance.

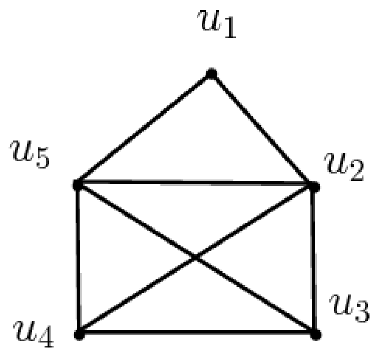

Figure:2.1.1

Result:2.1

Suppose that any two vertices in a connected graph $G$are u and v. Then

(i) $SFD\ (u, v) = 0$ iff $u = v$.

(ii) $SFD\ (u, v) = 1$, then uv is an edge.

Remark:2.1

$SFD\ (u, v) = 1$, doesn't need to be true if uv is an edge. Take a look at Figure: 2.1.2, where uv is an edge but $SFD\ (u, v)$ equals 2.

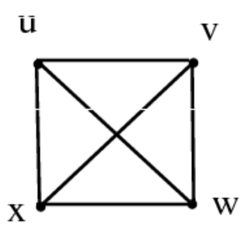

Figure:2.1.2

Note:2.1

On the vertex set of a connected graph, the monophonic distance is not a metric, although the shortest distance d and the detour distance D are. On the vertex set of a connected graph, the triangule free detour distance is not a metric.

Result:2.2

On the vertex set of a connected graph, the sfd distance is not a metric. Take a look at the graph in the figure: 2.1.3, $SFD(u_4, u_3) = 4$, $SFD(u_4, u_9) = 2$, $SFD(u_9, u_3) = 1$. Consequently, $SFD(u_4, u_3) > SFD(u_4, u_9) + SFD(u_9, u_3)$. In light of this, the sfd distance is not a metric.

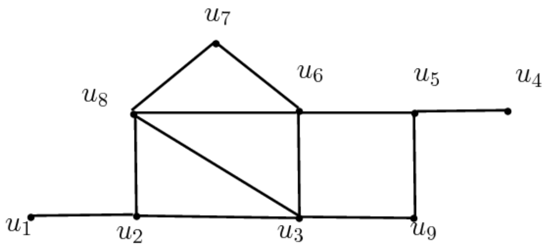

Figure:2.1.3

In a square free connected graph, $D(u, v) = SFD(u, v)$ applies to any two vertices, regardless of which one they are, u and v. Thus, on the vertex set of a square free connected graph, the sfd distance is a metric.

Theorem:2.1

Let G be a graph of order n. If there is any pair of v and $w$vertices in G, then $O \le d(v, w) \le d_m(v, w) \le SFD(v, w) \le D(v, w) \le n - 1$.

Proof:

It is sufficient to prove, $d_m(v, w) \le SFD(v, w) \le D(v, w)$

Assume that P is the longest $v - w$ path in G. Consider the case where P lacks a chord in G, then $d_m(v, w) = SFD(v, w) = D(v, w)$. Consider the case where P has a chord in G,

Subcase (i): If P fails to induce a cycle C_4 in G, then $d_m(v, w) < SFD(v, w) = D(v, w)$

Subcase (ii):If P induce a C_4 in G, then $d_m(v, w) \le SFD(v, w) < D(v, w)$

Remark:2.2

The theorem 2.1's bounds are sharp. Assuming $v = w$, $d(v, w) = d_m(v, w) = SFD(v, w) = D(v, w) = 0$. Further, if G is a path and v and w are its end vertices, $d(v, w) = d_m(v, w) = SFD(v, w) = D(v, w) = n - 1$. For the graph \$G\$ depicted in the figure: 2.1.4, $d(v_1, v_4) = 2, d_m(v_1, v_4) = 3$, $SFD(v_1, v_4) = 4, D(v_1, v_4) = 5$ and $n - 1 = 6$. Thus $d(v_1, v_4) < d_m(v_1, v_4) < SFD(v_1, v_4) < D(v_1, v_4) < n - 1$.

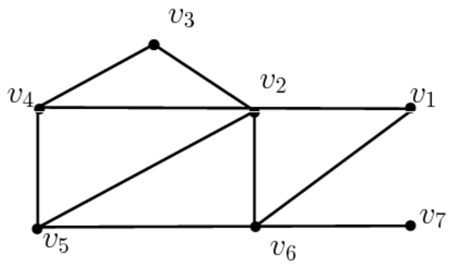

Figure: 2.1.4

Theorem 2.2. Let G be a graph of order n and u and v be any two vertices of G. If uv is a bridge, then $SFD(u,v) = 1$.

Proof. Assume that uv is a bridge. We have to show that $SFD = 1$.

Suppose $SFD(u,v) > 1$. Let $SFD(u,v) = m$ where $n > m > 1$.

Then ∃ a square free path $P = u\, u_1\, u_2 u_3 \ldots .\, u_{m-1} v$. Combining the edge uv along with P, we obtain a cycle with m vertices. Therefore uv lies on a cycle. This contradicts the theorem 1.1 . Thus $SFD(u,v) = 1$.

Remark 2.3:

It's not necessary for the converse of theorem 2.2 to be true. Take a look at the graph in figure 2.1.5, $SFD(u_1, u_2) = 1$, however, $u_1\, u_2$ is not a bridge.

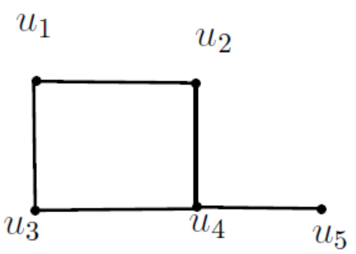

Figure:2.1.5

Remark 2.4:

In a connected graph G, Gary Chartrand et al.[3] demonstrated that |d(u, x) − d(v, x)| ≤ 1 for every vertex x of G holds true for two neighbouring vertices u and v.The sfd distance is an exception to this inequality. Regarding the graph in figure 2.1.6, $SFD(v_2, v_1) = 1, SFD(v_6, v_1) = 3$. But $|SFD(v_2, v_1) - SFD(v_6, v_1)|$ =2 > 1.

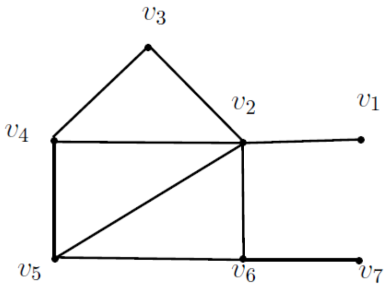

Figure:2.1.6

Remark 2.5

As demonstrated by Gary Chartrand et al. [3], if d(u, x) = k for some u, x ∈ V (G) and if v is a neighbour of u. As a result, d(v, x) is k−1, k or k+1. The sfd distance does not support this. Regarding the graph in figure 2.1.6, SFD(v_2, v_1)) = 1, But SFD(v_6, v_1)= 3.

Theorem 2.3. Let G be a graph of order n. If SFD(u, v) = n−1,

then G contains a Hamiltonian u − v path.

Remark 2.6. The theorem: 2.3's converse doesn't need to be

true. Take a look at the graph in the figure: 2.1.7 shows that G has

a Hamiltonian path$u_1 - u_5$, yet SFD(u_1, u_5) = 2 ≠ n − 1, where

n = 5.

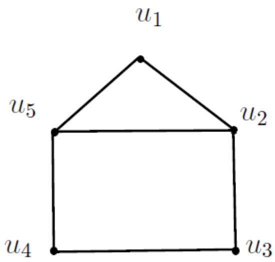

Figure :2.1.7 G

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