Thermal Convection in a Porous Layer with Magnetic and Variable Gravity Field Effects

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Abstract

The influence of the magnetic field, heat source, and variable gravity field on the stability of convective phenomena in a porous layer is investigated. Three types of gravity variations, such as linear, parabolic, and cubic functions are considered. For linear theory, the method of normal modes has been used to solve governing dimensionless equations which led to an eigenvalue problem. The effects of the physical parameters on the system are studied. It is found that the onset of convection is delayed by increasing the Hartmann number and gravity variation parameter and enhancement of the Darcy number makes the system stable.

Keywords: linear stability, porous layer, variable gravity.

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1. Introduction

Convective instabilities in a porous layer are analyzed by Horton and Rogers [1], and Lapwood [2]. Weakly nonlinear analysis in a rotating porous slab is studied by Bhadauria et al. [3] and showed that the Nussselt number decreases as the Taylor number increases. Homsy and Sherwood [4] observed the convective instabilities in a porous box with through flow. Convective phenomena in a porous material under different phenomena are studied by many authors like Nield [5], Elder [6], Nield and Kuznetsov [7], Nield [8], Suma et al. [9], Seema et al. [10], Babu et al. [11], Yadav [12, 13], Ravi et al. [14, 15] and Kuznetsov et al. [16, 18].

The gravity field on thermal convection was first studied by Pradhan and Samal [19]. Thermal instability in a reactive porous layer of box with magnetic field and variable gravity is examined by Harfash and Alshara [20]. The influence of linear and also non-linear gravity variations on the onset of instability in a layer consisting porous material with the impact of heat source is studied by Rionero and Straughan [21]. Brain Straughan [22] studied the non-linear theory of thermal convective phenomena with the impact ofgravity field. The impact of gravity variation and heat source in a porous bed has been studied by Harfash [23]. Deepika et al. [24] examined the gravity effect on HadleyPrats flow in a porous slab with vertical throughflow using nonlinear energy theory.

Mahabaleshwar et al. [25] deduced in their article that convective phenomena in a porous packed layer with the heat source and also gravity variation, the size, and shape of convective cell are not changing with the change in variable gravity and heat source, further Chand et al. [26] discussed the steady convection and came to know that decreasing gravity parameter has to stabilize effect. Magneto-rotating convective phenomena inthe nanofluid layer is studied by Mahajan and Arora [27]. Mahajan and Sharma [28] extended the work done by Mahajan and Arora [27] by considering variable gravity. Kaloni and Qiao [29] used the non-linear theory to illustrate the gravity effect on convective phenomena in a porous packed medium. Many authors, such as, Alex and Patil [30], Alex et al. [31], Yadav [32], Alex et al. [33], Chand et al. [34], Yadav [35, 36], and Chand et al. [37, 38] studied the gravity field effect.

The organization of current analysis is as follows: Next section describes the governing equations. The linear instability theory is focused on in Section 3 and Section 4 provides an in-detail explanation of the employed numerical method. Section 5 deals with the results. Conclusions are written in the last section.

2. Basic Equations

In this paper, we choose a newtonian fluid saturated porous layer bounded by the planes $z = 0$ and $z = d$. z -axis is taken vertically upward. The thermal difference is ΔT . The governing equations can be formulated as(Fig. 1)

$$\nabla . u = 0, \tag{2.1}$$

$$\frac{\mu}{K} u = -\nabla p + \mu_c \nabla^2 u + \rho_0 \overline{g}(z) \beta (T - T_0) \hat{e}_z + \sigma_1 (u \times B_0 \hat{e}_z) \times B_0 \hat{e}_z, \tag{2.2}$$

$$\sigma \frac{\partial T}{\partial t} + (u \cdot \nabla) T = \chi \nabla^2 T, \tag{2.3}$$

subject to the boundary conditions

$$z = 0 : u = 0, T = T_0 + \Delta T,$$
$$z = 1 : u = 0, T = T_0. \tag{2.4}$$

Here μ -viscosity, μ_c -effective viscosity, K -permeability, p -pressure, ρ density, β -thermal expansion coefficient, σ_1 -electric conductivity, t - time, σ heat capacity ratio, and χ -thermal diffusivity. And also, $\bar{g}(z) = g_0 \left[1 + \delta G(z)\right]$ - variable gravity, g_0 - reference gravity, and δ - gravity variation parameter. The non-dimensional quantities are

$$(x, y, z) = (x^* d, y^* d, z^* d)$$
$$(u, v, w) = \frac{\chi}{d}(u^*, v^*, w^*)$$
$$t = \frac{\sigma d^2}{\chi} t^*, \quad T = (T_0 + T\Delta T) T^*, \tag{2.5}$$

with

$$Ra = \frac{g \rho_0 \beta \Delta T K d}{\xi \mu} \text{ - Rayleigh number,}$$
$$Da = \frac{\mu_c \kappa}{\mu d^2} \text{ - Darcy number,} \tag{2.6}$$
$$Ha^2 = \frac{\sigma_1 B_0^2 \kappa}{\mu} \text{ - Hartmann number.}$$

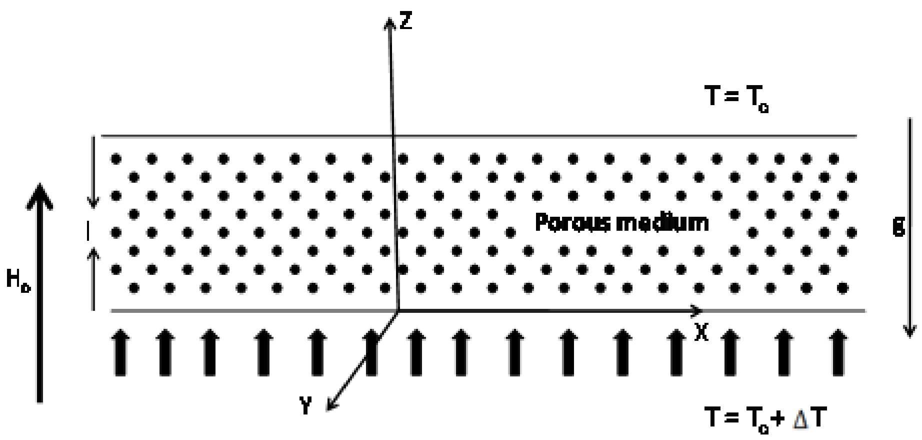

Fig1: Physical Diagram

The non-dimensional governing equations have been formulated as

$$\nabla.u = 0, \tag{2.7}$$

$$u = -\nabla p + Da\nabla^2 u + RaT\left[1+\delta G(z)\right]\hat{e}_z + Ha^2[(u\times\hat{e}_z)\times\hat{e}_z], \tag{2.8}$$

$$\frac{\partial T}{\partial t} + \left(u.\nabla\right)T = \nabla^2 T, \tag{2.9}$$

subject to the boundary conditions

$$z = 0 : u = 0, T = 1$$

$$z = 1 : u = 0, T = 0. \tag{2.10}$$

The basic flow of Eqs. (2.7) - (2.10) is

$$u_b = 0, \tag{2.11}$$

$$T_b = 1 - z. \tag{2.12}$$

3. Linear Instability

The perturbation for the Eqs. (2.11)-(2.13) is given as

$$u = u_b + U,\ p = P_b + P, T = T_b + \Phi. \tag{3.1}$$

On using Eq. (3.1) into Eqs. (2.7)-(2.10), one obtains

$$\nabla.U = 0, \tag{3.2}$$

$$U = -\nabla P + Da\nabla^2 u + Ra\Phi\left[1+\delta G(z)\right]\hat{e}_z + Ha^2[(U\times\hat{e}_z)\times\hat{e}_z], \tag{3.3}$$

$$\frac{\partial \Phi}{\partial t} + U.\nabla T_b + u_b.\nabla\Phi = \nabla^2\Phi, \tag{3.4}$$

$$U = 0, \Phi = 0\ on\ z = 0,1. \tag{3.5}$$

Taking the third component of curl of curl Eq. (3.3),

$$\nabla^2 w - Da\nabla^4 w - Ra\left[1+\delta G(z)\right]\nabla_h^2\Phi + Ha^2 D^2 w = 0, \tag{3.6}$$

$$\frac{\partial \Phi}{\partial t} + U.\nabla T_b + u_b.\nabla\Phi = \nabla^2\Phi, \tag{3.7}$$

$$z = 0,1 :\ w = 0,\ \Phi = 0. \tag{3.8}$$

Now, let introduce the solution in normal modes as

$$(w, \Phi) = \left(W(z), \Phi(z)\right)e^{i(qx\ -\ \omega t)}. \tag{3.9}$$

Where q -wave number and p -growth rate. Eqs. (3.6) - (3.8) become (After using Eq. (3.9))

$$\left((D^2 - q^2) - Da(D^2 - q^2)^2\right)W + Ra\left[1+\delta G(z)\right]q^2\Phi + Ha^2 D^2 W = 0, \tag{3.10}$$

$$\left(D^2-q^2-i\omega\right)\Phi-W\frac{dT_b}{dz}=0, \tag{3.11}$$

$$z=0,1: W=\Phi=0. \tag{3.12}$$

	For Case A			For Case B	
δ	Present R	Rionero and Straughan **DisplayText cannot span m**	δ	Present R	Rionero and Straughan **DisplayText cannot span m**
0	39.478	39.478	0	39.478	39.478
1	77.079	77.020	0.2	41.832	41.832
1.5	132.020	132.020	0.4	44.455	44.455
1.8	189.908	189.908	0.6	47.389	47.389
1.9	212.280	212.280	0.8	50.682	50.682

Table 1: Comparison of Ra_c of the present theory with the results of Rionero and Straughan **Error! Reference source not found.** for $Ha^2=0$, and $Da=0$.

4. Discussion

The influence of variable gravity field and magnetic field on convective instability in a porous layer is studied. The eigenvalue problem for linear theory is examined using the bvp4c routine in MATLAB R2020b[39]-[40]. The effect of Da, Ha and δ on critical Rayleigh number, Ra_c, and critical wave number, q_c are shown in figs. 1-6. The following three cases of gravity field variance are considered.

$$\begin{cases}(A).G(z)=-z,\\(B).G(z)=-z^2,\\(C).G(z)=-z^3.\end{cases}$$

The comparison between present results(for Da=0 andHa=0) with the results of Rionero and Straughan **Error! Reference source not found.** has been provided in Table 1. This Table 1 clearly indicates a good agreement of present results with that of Rionero and Straughan **Error! Reference source not found.**.

Table 2 shows the curve of Ra_c versus Ha2 for distinct values of δ for three cases. From Table 2, it is seen that with an enhance in Ha, the Rac increases. So Q has a stabilizing effect. And it is also observed that with an enhance in the δ, the Ra_c enhances. Hence δ has a stabilizing effect. Finally, we also observe from all these tables that the system become more stable in case A ($G(z)=-z$) and less stable in case C($G(z)=-z^3$).

δ	Ha^2	CaseA	CaseB	CaseC	δ	CaseA	CaseB	CaseC
0.2	0	194.435	185.479	181.307	0.4	218.669	197.251	188.059
0.2	0.1	194.744	185.773	181.595	0.4	219.016	197.564	188.358
0.2	0.2	195.669	186.656	182.457	0.4	220.056	198.502	189.252
0.2	0.3	197.211	188.127	183.895	0.4	221.790	200.067	190.744
0.2	0.4	199.370	190.186	185.909	0.4	224.218	202.257	192.832
0.2	0.5	202.146	192.834	188.497	0.4	227.339	205.072	195.516
0.2	0.6	205.283	195.827	191.423	0.4	230.860	208.250	198.549
0.2	0.7	208.960	199.334	194.852	0.4	234.994	211.979	202.104
0.2	0.8	213.202	203.381	198.807	0.4	239.764	216.282	206.207
0.2	0.9	218.009	207.967	203.290	0.4	245.169	221.158	210.856

Table2:EvaluationofRa$_c$fordistinctvaluesofδandHa^{2}andforDa=0.5.

Table 3 display the impact of Da on Ra_c for $\delta = 0{:}2$ and 0:4 for three cases From Table 3, one canseen that with an enhance in Da the Ra_c increases. So Da has stabilizing effect. And it is also found that with an incriment in δ, the Ra_c increases. Therefore δ, has a stabilizing effect. Furthermore, we also observe from all these tables that the system become more stable in case A ($G(z) = -z$) and less stable in case C($G(z) = -z^3$).

δ	Da	CaseA	CaseB	CaseC	δ	CaseA	CaseB	CaseC
0.2	0	49.189	46.924	45.864	0.4	55.269	49.858	47.559
0.2	0.1	127.715	121.832	119.092	0.4	143.620	129.555	123.522
0.2	0.2	202.146	192.834	188.497	0.4	227.339	205.072	195.516

0.2	0.3	275.436	262.748	256.838	0.4	309.769	279.428	266.405
0.2	0.4	348.726	332.662	325.180	0.4	392.199	353.783	337.294
0.2	0.5	422.016	402.576	393.521	0.4	474.629	428.138	408.182
0.2	0.6	495.306	472.491	461.862	0.4	557.058	502.492	479.070
0.2	0.7	568.597	542.405	530.203	0.4	639.488	576.847	549.959
0.2	0.8	641.887	612.319	598.545	0.4	721.917	651.202	620.847
0.2	0.9	715.177	682.233	666.886	0.4	804.346	725.557	691.735

Table3:EvaluationofRa$_c$fordistinctvaluesofδandDaandforHa=0.5.

5. Conclusions

The onset of magnetoconvection in a porous media with variable gravity have performed numerically using linear theory. We have analysed three cases of gravity field variation namely, and conclusions are listed below:

1. Darcy number, Da, and variable gravity, δ delay the onset of convection.
2. Hartmann number, Ha2 destabilize the system.
3. The flow become more stable for linear variation, and less stable for cubic variation.

6. References

[1] C.W. Horton, and F.T. Rogers, Convection Currents in a Porous Medium, J. Appl. Phys., 16, (1945) 367370.

[2] E.R. Lapwood, Convection of a Fluid in a Porous Medium, Proc. Cambridge Philos. Soc., 44 , (1948) 508521.

[3] B. S. Bhadauria, P. G. Siddheshwar, Jogendra Kumar and Om P. Suthar, Weakly Nonlinear Stability Analysis of Temperature/Gravity-Modulated Stationary RayleighBnard Convection in a Rotating Porous Medium, Transp Porous Med 92, (2012) 633-647.

[4] G. M. Homsy and A. E. Sherwood, Convective Instabilities in Porous Media with Through Flow, AlChE Journal 22(1), (1976) 168-174.

[5] D. A. Nield, and A. Bejan , Convection in porous media, (Springer, New York, 2006) 110-113.

[6] J.W. Elder, , Steady free convection in a porous medium heated from below, Journal of Fluid Mechanics, 27(1), (1967) 29-48.

[7] D.A. Nield and A.V. Kuznetsov, Thermal instability in a porous medium layer saturated by a nanofluid: A revised model, International Journal of Heat and Mass Transfer, 68, (2014) 211-214.

[8] D. A. Nield, Convection in a porous medium with inclined temperature gradient and vertical throughflow, Int. J. Heat Mass Transfer, 41(1), (1998) 241-243.

[9] S. P Suma and Y. H Gangadharaiah , R. Indira , Effect of throughflow and variable gravity field on thermal convection in a porous layer, International Journal of Engineering Science and Technology, 3(10), (2011) 7657-7668.

[10] Seema Kumari and P. V. S. N. Murthy, Convective Stability of Vertical Throughflow of a Non-Newtonian Fluid in a Porous Channel with Soret Effect, Transp Porous Med, 122, (2018) 125-143.

[11] A. Benerji Babu, Ragoju Ravi and S.G. Tagare Nonlinear rotating convection in a sparsely packed porous medium, Commun Nonlinear Sci Numer Simulat 17, (2012) 5042-5063.

[12] Dhananjay Yadav, Numerical investigation of the combined impact of variable gravity field and throughflow on the onset of convective motion in a porousmedium layer, International Communications in Heat and Mass Transfer 108,(2019) 104274.

[13] Dhananjay Yadav, Effects of rotation and varying gravity on the onset of convection in a porous medium layer: a numerical study, World Journal of Engineering,17(6), (2020) 785-793.

[14] Ragoju. Ravi, C. Kanchana and P. G. Ssddheshwar, Effects of second diffusing component and cross diffusion on primary and secondary thermoconvective instabilities in couple stress liquids, Appl. Math. Mech. Engl. Ed., 38(11), (2017) 1579-1600.

[15] Ragoju Ravi, C. Kanchana and G. Janardhana Reddy, and Hussain Basha, Study of Soret and Dufour effects and secondary instabilities on Rayleigh-Benard convection in a couple stress fluid, Eur. Phys. J. Plus 133:513, (2018) 1-14.

[16] A.V.Kuznetsov and D.A. Nield, The onset of double-diffusive nanofluid convection in a layer of saturated porous medium, Transport in Porous Media, 85, (2010) 941-951.

[17] A.V.Kuznetsov and D.A. Nield, Thermal instability in a porous medium layer saturated by a nanofluid: Brinkman Model, Transport in Porous Media, 81, (2010) 409-422.

[18] A.V.Kuznetsov and D.A. Nield, Effect of local thermal non-equilibrium on the onset of convection in a porous medium layer saturated by a nanofluid, Transport in Porous Media, 83, (2010) 425-436.

[19] G. K. Pradhan and P. C. Samal, Thermal Stability of a Fluid Layer under Variable Body Forces, Journal of mathematical analysis and applications, 122, (1987) 487-495.

[20] A. J. Harfash and A. K. Alshara, Chemical reaction effect on double diffusive convection in porous media with magnetic and variable gravity effects, Korean J. Chem. Eng., 32(6), (2015), 1046-1059.

[21] S. Rionero and B . Straughani, Convection in a porous medium with internal heat source and variable gravity effects, int. J. Engng. Sci., 28(6), (1990) 497-503. 13.

[22] Brian Straughan, Convection in a Variable Gravity Field, Journal of mathematical analysis and applications, 140, (1989) 467-475.

[23] A. J. Harfash, Three-Dimensional Simulations for Convection in a Porous Medium with Internal Heat Source and Variable Gravity Effects, Transp Porous Med, 101, (2014) 281-297.

[24] N. Deepika and P. A. L. Narayana, Effects of Vertical Throughflow and Variable Gravity on HadleyPrats Flow in a Porous Medium, Transp Porous Med, 109, (2015) 455-468.

[25] U.S. Mahabaleshwar , D. Basavaraja , S. Wang , G. Lorenzini and E. Lorenzini, Convection in a porous medium with variable internal heat source and variable gravity, International Journal of Heat and Mass Transfer, 111, (2017) 651-656.

[26] Ramesh Chand, G. C. Rana and S. K. Kango, Effect of Variable Gravity on Thermal Instability of Rotating Nanofluid in Porous Medium, FME Transactions 43, (2015) 62-69.

[27] Amit Mahajan and Monika Arora, Convection in rotating magnetic nanofluids, Applied Mathematics and Computation, 219, 62846296, (2013) 6284-6294.

[28] Amit Mahajan and Mahesh Kumar Sharma, The onset of convection in a magnetic nanofluid layer with variable gravity effects, Applied Mathematics and Computation, 339, (2018) 622-635.

[29] P.N. Kaloni and Zongchun Qiao, Non-linear convection in a porous medium withinclined temperature gradient and variable gravity effects, International Journalof Heat and Mass Transfer 44, (2001) 1585-1591.

[30] S. M. Alex and P. R. Patil, Effect of variable gravity field on soret driven thermosolutalconvection in a porous medium, Int. Comm. Hear Mass Transfec, 28(4), (2001) 509-518.

[31] S, M. Alex, P. R.Patil and Vankatakrishan, K.S., Variable gravity effects on thermal instability in a porous medium with internal heat source and inclined temperature gradient, Fluid Dynamics Research, 29, (2001) 1-6.

[32] Dhananjay Yadav, The onset of Darcy-Brinkman convection in a porous medium layer with vertical throughflow and variable gravity field effects, Heat Transfer, 49(5), (2020) 1-10.

[33] S. M. Alex and P.R. Patil, Effect of a variable gravity field on convection in an anisotropic porous medium with an internal heat source and inclined temperature gradient, J. Heat Transf. 124(1), (2002) 144-150.

[34] Ramesh Chand, G.C Rana and S. Kumar, Variable gravity effects on thermal instabilityof nanofluid in anisotropic porous medium, Int. J. of Appl. Mech. And Engg., 18(3), (2013) 631-642.

[35] Dhananjay Yadav, The density-driven nanofluid convection in an anisotropic porous medium layer with rotation and variable gravity field: a numerical investigation, Journal of Applied and Computational Mechanics, 6, (2020) 699-710.

[36] Dhananjay Yadav, Numerical solution of the onset of buoyancy-driven nanofluid convective motion in an anisotropic porous medium layer with variable gravity and internal heating, Heat Transfer, 49(3), (2020) 1170-1191.

[37] Ramesh Chand, Thermal instability of rotating Maxwell visco-elastic fluid with variable gravity in porous medium, Journal of the Indian Math. Soc., 80, (2013) 23-31.

[38] Ramesh Chand, Effect of suspended particles on thermal instability of Maxwell viscoelastic fluid with variable gravity in porous medium, Antarctica J. Math,8(6), (2011) 487-497.

[39] Reddy, G. S. K., and Ragoju, R. Thermal instability of a power-law fluidsaturatedporous layer with an internal heat source and vertical throughflow, 2022;51:2181–2200. Heat Transfer.

[40] Reddy, G. S. K., and Ravi, R. Thermal instability of a Maxwell fluid saturated porous layer with chemical reaction, 13(1):33–47 (2022), Special Topics and Reviews in Porous Media.