Fuzzy Non-Linear Programming Problems with Linear Inequality Constraints and Its Solutions

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Abstract

The problem of fuzzy nonlinear programming is useful for solving problems that are difficult, impossible to solve because of the inaccurate and subjective nature of the problem formulation, or that have a precise solution. We have an objective function here that we have to optimize with certain constraints. This document proposed finding the fuzzy solution to totally fuzzy nonlinear programming problems with inequality constraints using the proposed method, the fuzzy solution to the FFNLP problems with inequality constraints that occur in real-life situations (Loganathan and Lalitha 2017). Fuzzy numbers are known to be the coefficients of nonlinear objective functions and constraints. Numerical examples were illustrated of the method. A new method for finding an optimal solution to fuzzy, nonlinear programming problems is proposed here in this document. Various authors have proposed algorithms which include problems with linear programming with linear constraints. The method proposed is based on previous works available in the literature and is checked by numerical examples.

Keyword: Fuzzy Non-Linear Programming Problems, Linear Inequality Constraints, Solutions

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Introduction:

The nonlinear programming is generally presenting far greater challenges than linear programming. Even in cases where all constraints are linear, and only the objective function is nonlinear, it is often difficult. Nonlinear programming is one of the optimization techniques that is most applicable. The coefficients of the model are accurately known in most real-world situations because the relevant data exist or are scarce, apart from finding; the system is subject to change etc. Thus, Tanaka et al [1984], mathematical programming models for decision support must explicitly take into account the treatment of intrinsic uncertainty associated with the model's coefficients in addition to multiple objective and objective functions. In a confusing environment Bellman and Zadeh[1970] proposed the concept of decision-making. Buckley and Feuring [2000] developed a method for seeking the solution to completely blurred linear programming problems with all parameters and variables as blurred numbers while changing objective function in a multiple objective blurred linear programming problem. Maleki et al [2000] also solved linear programming problems, where all decision parameters are fuzzy numbers when comparing fuzzy numbers. Maleki [2000] introduced a new method to use the classification function to solve linear programming with constraint vagueness. Ganesan and Veeramani [2006] suggested an approach to solving a dubious linear programming problem involving trapezoidal symmetric fuzzy numbers without converting it into crisp linear programming. Kiruthiga and loganathan [2016] solved the multi-objective, nonlinear programming problem of Interval and Fuzzy with necessarily efficient points. Hashemi et al [2009] suggested a two-step approach to finding the optimal class solution to the Fuzzy linear programming problems called Fully Fuzzy linear programming, where the decision parameters and variables are Fuzzy numbers. Jiménez et al [2007] proposed a method for finding a solution to linear programming problems where all the coefficients are Fuzzy Numbers and used a Fuzzy Classification Method to classify the Fuzzy Objective values and manage the Inequality Relationship in the Mahdavi Constraints. -Amiri et al [2009, 2010] implemented the Fuzzy primitive simplex algorithm to solve the Fuzzy linear problem of programming. A new method for finding the fuzzy solution to the FFNLP problems with inequality constraints is proposed in this document. The Fuzzy solution to FFNLP problems with inequality constraints that exist in the real-life solution can be easily obtained using the proposed method. After transforming the Fuzzy nonlinear programming problem (FNLPP) into sharp equality constraints with the aid of the KKT conditions in the nonlinear programming problem with sharp inequality constraints with the aid of the classification function and the trapezoidal fuzzy membership function and its arithmetic operations, we are now using the new form.

Objective:

- To examine the Fuzzy Non-Linear Programming Problems with Linear Inequality Constraints
- To compare the earlier Fuzzy Non-Linear Programming Problems

Method:

Method explored with the support of MATLAB, a new method is proposed to find the optimal fuzzy solution for the following FNLNP algorithm: The steps of the proposed method are as follows:

In problem j, the variables in the objective function are changed from n X to (x) by expressing them as blurry triangular numbers.

1. The Lagrangian function L is acquired by involving the objective function and the blurry limitations.
2. Converting the problem from fuzzy to sharp, using the sort function.
3. Evaluate the fixed point using the appropriate KKT conditions.
4. Check the state of optimisation at the 'xj' fixed points.
5. The optimal value is calculated by entering the goal function xj

Vision:

The vision of this research is to previous survey findings and its comparison. Bellman and. Zadeh, [1970] and the ultimate decision [Zimmermann] used in NLPP to find the best solution introduced the idea of fuzzy decision-making. An optimal solution is defined as a solution that meets both the limitations of the problem and the objective function [Das and Baruah 2004]. We call it a nonlinear programming problem when the objective function or constraints are nonlinear [Swarup et al, 2004]. In addition, this problem has an unstable objective function and fluid variables in the constraints [Wu 2004], [Pandian and Nagarajan 2002] and [Kheirfam and Hasani 2010] where the fluid left and right coefficients in the constraints [Yenilmez et al., 2002] are present. In this article, the Kuhn Tuckers conditions are applied in terms of confusion to solve a nonlinear programming problem in order to find an optimal solution.

Literature Review:

Osman [1993] introduced the notion of solvency set, first and second type stability sets from the previous work, and analyzed those concepts for nonlinear convex parametric programming issues. The qualitative study of the first collection of type stability for parametric fuzzy nonlinear programming problems was introduced by Osman and EleBanna [1993]. In situations with an implicit utility function, Loganathan and Sherali [1987] presented an interactive slice plane algorithm to determine the best compromise solution for a multi-objective optimization problem [Behera and Nayak 2011]. Zimmerman [1978] proposed the first Fuzzy Linear programming formulation. A method for solving linear programming problems was derived by Nasseri [2008] and Amit kumar and Jagdeep Kaur [2010] and Kheirfam [2011] introduced the optimal fuzzy solution to non-linear fuzzy programming problems (FNLP) with inequality constraints. They took all of the coefficients and decision variables as fuzzy numbers in their result, and all equations have to be linear. Here in our article, we assumed as nonlinear the objective function and as linear the constraints of both inequality and the type of equality. Both variables and coefficients are triangular fuzzy numbers [Zadeh 1965]. Rank 2 Fuzzy numbers are a particular type of Fuzzy numbers of type 2. Triangular and trapezoidal shapes can describe those numbers. In this document, perfectly normal trapezoidal type 2 fuzzy numbers were introduced for the first numbers with their left margin and their core, which are normal and convex; therefore, a new type of fuzzy arithmetic operations was proposed for fuzzy trapezoidal type 2 numbers at perfectly normal intervals on the basis of the principle of extension of fuzzy trapezoid type 1 normal numbing The linear programming problems with technological resources and coefficients in this proposal are also perfectly normal fuzzy rank 2 numbers by Srinivasan and G. Geetharamani (1999). A method based on the degree of satisfaction (or degree of possibility) of the constraints was introduced to solve this type of nonlinear Fuzzy programming problem. With the help of the Fuzzy number classification method, compliance with constraints can be measured in this method. When using the Barnes algorithm with the aid of MATLAB the optimal solution is obtained with varying degrees of satisfaction. Finally, an example numerical illustrates the optimal solution procedure. After converting the Fuzzy nonlinear programming problem (FNLPP) into sharp equality constraints using the KKT conditions in the nonlinear programming problem with sharp inequality constraints with the help of the classification function and the trapezoidal fuzzy membership function and its arithmetic operations, we used the new method and provided a confusing approach to the wide range of applications.

Definitions of Fuzzy Nonlinear Programming:

Definition 1 A fuzzy numbers $\bar{a}$ is a fuzzy set on, the real line that satisfies the condition of normality and convexity

Definition 2 The □ □ cut or □ □ level set of a fuzzy set is a crisp setdefined by $A_\alpha = \{x\epsilon\, R \mid \mu_A(x)\} > \alpha\}$

Definition 3 A fuzzy set A is called normal if its core is non-empty. In other words, there is at least one point x∈ R with □ $_A$(x) = 1.

Definition 4 The core of a fuzzy set is the set of all points x in R with □ $_A$(x) = 1.

Definition 5 The support of a fuzzy set A is defined as follow;Sup (A) = {x ∈ R| □ $_A$(x)}.

Definition 6 Let R he the real line, then a fuzzy set A in R is defined to be a set of ordered pairs**.** A = {(x, □ $_A$ (x)) .x ∈ R}, where □ $_A$ (x) is called Fuzzy Set Membership Function. The membership function maps a membership value between 0 and 1 for each element of R.

Definition 7: A fuzzy number Ã = <a, b, c> on R is said to be a triangular fuzzy number if it has the following membership function

$$\bar{a}(x) = \begin{cases} \frac{x-a}{b-a}, x \in [a,b] \\ \frac{x-c}{b-c}, x \in [b,c] \\ 0, oterwise \end{cases}$$

Definition 8 A fuzzy triangular fuzzy number Ã = <a, b, c> is said to be nonnegative triangular fuzzynumber**,** iff a ≥ 0.

Definition 9 A ranking function is a function $\Re : F(R) \to R$ is a set of fuzzy numbers defined in a set of real numbers that maps each fuzzy number into the real line where there is a natural order. Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number then $\Re\,(\tilde{A}) = \frac{a+2b+c}{4}$:

Definition 10 Let $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$be two triangular fuzzy numbers, then

i. $\tilde{A} \leq \tilde{B}\ iff\ a_1 \leq a_2, b_1 - a_1 \leq b_2 - a_2, c_1 - b_1 \leq c_2 - b_2$
ii. $\tilde{A} \geq \tilde{B}\ iff\ a_1 \geq a_2, b_1 - a_1 \geq b_2 - a_2, c_1 - b_1 \geq c_2 - b_2$
iii. $\tilde{A} = \tilde{B}\ iff\ a_1 = a_2, b_1 = b_2, c_1 = c_2$

Definition 11: FNLP Problem's fuzzy optimal solution is a fuzzy number if it meets the following characteristics:

(i) X is a non-negative fuzzy number

(ii) $A \otimes X = b$

(iii) If there exists a non-negative fuzzy number $\tilde{X}'$ such that $\tilde{A} \otimes \tilde{X}'' = \tilde{b}$ then $R(\tilde{C}^T \otimes \tilde{X}^n) > R(\tilde{C}^T \otimes \tilde{X}'')$ for a maximization problem and n T n T / $R(\tilde{C}^T \otimes \tilde{X}^n) < R(\tilde{C}^T \otimes \tilde{X}'')$ for a minimization problem.

Fuzzy Nonlinear Programming with Problem and Its Solution:

Arithmetic operations: Let a (a, b, c) and b (e, f, g) be either two triangular fuzzy numbers identified on the real R set. And there is

(i) $\tilde{a} \oplus \tilde{b} = (a+e, b+f, c+g)$

(ii) $-\tilde{a} = (-c, -b, -a)$

(iii) $\tilde{a} - \tilde{b} = (a-g, b-f, c-e)$

(iv) $$\tilde{a} \otimes \tilde{b} = \begin{Bmatrix} (ae, bf, cg)\ , a \geq 0 \\ (ag, bf, cg), a < 0, c \geq 0 \\ (ag, bf, ce), c < 0 \end{Bmatrix}$$

Theorem 1: Karush, Kuhn-Tucker Conditions (KKT Conditions) [Nasseri 2008]

Let $S = \{x \in R^n \mid \tilde{A} \otimes x \geq_R \tilde{b}, x \geq 0\}$ be nonempty. Then $x^* \in S$ is an optimal solution to the fuzzy linear programming problem; (Minimize: $\tilde{C} \otimes x,$, subject to $\tilde{A} \otimes x \geq_R \tilde{b}$, and $x \geq 0,$) where $\tilde{A} \in F_r(R^{m \times n}),\ \tilde{b} \in F_r(R^m),\ \tilde{c} \in F_r(R^n),\ x \in R^n$ if and only if $(x^*, \omega, r) \in R^n \times R^m \times R^n$ is a solution to the following system:

$$\tilde{A} \otimes x \geq_R \tilde{b}, x \geq 0,$$
$$\omega \otimes \tilde{A} \oplus v \approx_R \tilde{c},\ \omega \geq 0, v \geq 0;$$
$$\omega \otimes (\tilde{A} \otimes x \Theta \tilde{b}) \approx_R \tilde{0},\ vx = 0$$

Nonlinear programming problem

Maximize (or minimize) z= $\sum_{j=1}^{n} c_j x_j{}'$ (n ≥ 2 as the objective function is nonlinear)

Subject to $\sum_{j=1}^{n} a_{ij} x_j{}^n \leq (\geq)\ b_i\ , i = 1, 2, \ldots\ldots, m$ and $x_j \geq 0$ We find programming problem just Nonlinear

The problem of nonlinear programming in Fuzzified form Fuzzy nonlinear programming problem is defined as the fuzzy forms of nonlinear programming problem set out below:

Maximize $[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}] = f\left(\begin{bmatrix} x_1^{(1)}, x_1^{(2)}, x_1^{(3)}, x_1^{(4)} \end{bmatrix}, \begin{bmatrix} x_2^{(1)}, x_2^{(2)}, x_2^{(3)}, x_2^{(4)} \end{bmatrix} \dots \begin{bmatrix} x_n^{(1)}, x_n^{(2)}, x_n^{(3)}, x_n^{(4)} \end{bmatrix}\right)$

Under the constraints

$$g^i\left(\begin{bmatrix} x_1^{(1)}, x_1^{(2)}, x_1^{(3)}, x_1^{(4)} \end{bmatrix}, \begin{bmatrix} x_2^{(1)}, x_2^{(2)}, x_2^{(3)}, x_2^{(4)} \end{bmatrix} \dots \begin{bmatrix} x_n^{(1)}, x_n^{(2)}, x_n^{(3)}, x_n^{(4)} \end{bmatrix}\right) \leq \begin{bmatrix} b_i^{(1)}, b_i^{(2)}, b_i^{(3)}, b_i^{(4)} \end{bmatrix}$$

"where gi 0 s are 'm' real valued functions of 'n' fuzzy variables and bi 0s are 'm' fuzzy constants, and"

$$\begin{bmatrix} x_j^{(1)}, x_j^{(2)}, x_j^{(3)}, x_j^{(4)} \end{bmatrix} \geq 0, \; i = 1, 2, 3, \dots, m; \; j = 1, 2, 3, \dots, n \;\; \& \;\; m < n.$$

Moreover $0 \; (=) \; [-1-\delta, -\delta, \delta, 1+\delta]$, where □ is a small positive number. A feasible solution to the problem of Fuzzy nonlinear programming is a Fuzzy vector that satisfies the conditions of the previous eq.

Numerical Example

FNLP problem are

Maximize $\tilde{x}_1^{\,2} + \tilde{x}_2^{\,2}$

Subject to $\begin{matrix} (0,1,2) \otimes \tilde{x}_1 \oplus (1,2,3)\tilde{x}_2 \leq (1,10,27) \\ (1,2,3) \otimes \tilde{x}_1 \oplus (0,1,2)\tilde{x}_2 \leq (2,11,28) \end{matrix}$ and $\tilde{x}_1, \tilde{x}_2$ are non-negative fuzzy triangular numbers.

Let x1 (x1, y1,z1) and x 2(x2 , y2 ,z2) So we obtained

Maximize $(x_1, y_1, z_1) \otimes (x_1, y_1, z_1) + (x_2, y_2, z_2) \otimes (x_2, y_2, z_2)$ subject to

$(0,1,2) \otimes (x_1, y_1, z_1) \oplus (1,2,3) \otimes (x_2, y_2, z_{12}) \leq (1,10,27)$

$(1,2,3) \otimes (x_1, y_1, z_1) \oplus (0,1,2) \otimes (x_2, y_2, z_{12}) \leq (2,11,28)$ and (x 1, y 1,z 1) and (x2 , y2 ,z2) are non-negative fuzzy triangular numbers.

After the modified program is

Maximize $(x_1^{\,2}, y_1^{\,2}, z_1^{\,2}) \oplus (x_2^{\,2}, y_2^{\,2}, z_2^{\,2})$

Subject to $\begin{matrix} (x_2, y_1 + 2y_2, 2z_1 + 3z_2) \leq (1,10,27) \\ (x_1, 2y_1 + y_2, 3z_1 + 2z_2) \leq (2,11,28) \end{matrix}$

and (x_1, y_1, z_1) and (x_2, y_2, z_2) are non-negative fuzzy triangular numbers.

From The Langrangian function is

$$L(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\lambda}) = (x_1^2, y_1^2, z_1^2) \oplus (x_2^2, y_2^2, z_2^2) \begin{array}{l} -\lambda_1((x_2, y_1+2y_2, 2z_1+3z_2)-(1,10,27)) \\ -\lambda_2((x_1, 2y_1+y_2, 3z_1+2z_2)-(2,11,28)) \end{array}$$

The Lagrangian function [Taha 2005] becomes using ranking function

$$L(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\lambda}) = \frac{x_1^2 + x_2^2 + 2y_1^2 + 2y_2^2 + z_1^2 + z_2^2}{4} - \lambda_1\left(\frac{x_2 - 27 + 2y_1 + 4y_2 - 20 + 2z_1 + 3z_2 - 1}{4}\right)$$
$$-\lambda_2\left(\frac{x_1 - 28 + 4y_1 + 2y_2 - 22 + 3z_1 + 2z_2 - 2}{4}\right)$$

Using the KKT Conditions, we get

X1 =4/3, y1 =4, z1 =6, X2=1, y2 =4, z2 =17/3 , λ1 = 2, λ2 = 8/3

That includes x1= (4/3, 4, 6) and x2= (1, 4,17/3) The optimal value is Z = (25/9, 32, 613/9). If the constraints are of the kind of equality, with the aid of the classification function, you can use the Langrangian method to decide the optimum solution after translating the fuzzy nonlinear programming problem into acute form.

In addition to solve the aforementioned fuzzy membership function for all Kuhn Tucker conditions.

We obtained the optimal fuzzified solution of the above nonlinear programming problem, which are;

$$\begin{aligned} [x_1^{(1)}, x_1^{(2)}, x_1^{(3)}, x_1^{(4)}] &= [41, 42.5, 45.5, 47], \\ [x_2^{(1)}, x_2^{(2)}, x_2^{(3)}, x_2^{(4)}] &= [-196, -97, 101, 200] \\ [\lambda^{(1)}, \lambda^{(2)}, \lambda^{(3)}, \lambda^{(4)}] &= [88, 94, 106, 112] \end{aligned}$$

Since $\lambda \geq 0$ and hence the aim function meaning is $[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}] = [844, 2872, 6928, 8956]$

The optimal solution to both the values of a nonlinear Fuzzy programming problem will be constantly greater than 844 and less than 8956, and the values are most likely between 2872 and 6928. The cost variations with relevance probabilities are shown below.

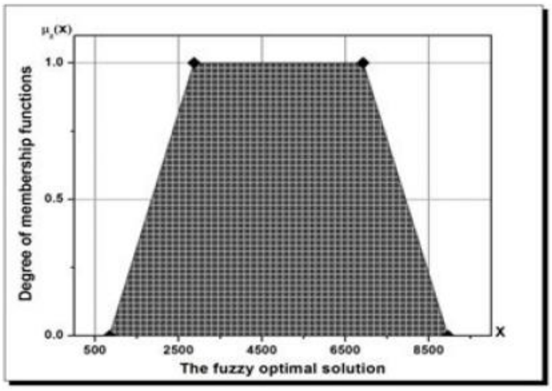

Figure 1: Fuzzy optimum solution μz(X) function of trapezoidal membership

The same results are obtained in the empirical example based on the previous result; it could be suggested that it is optimal to expend the technique proposed from the current methodology. Using the method proposed in this paper, we obtained the optimal fuzzy solution to nonlinear programming problems with linear constraints and tested the correctness of the proposed method through the numerical examples.

Conclusion:

The analysis applies to nonlinear Fuzzy programming problems (FNLP) with numerical coefficients that are fuzzy. Linear programming (LP) has been one of the operational research techniques for a considerable amount of time, which was widely used and achieved many results in both applications and theories. But the strict requirement of LP is that the data must be well-defined and accurate, which is often impossible in problems of real decision. The traditional way of assessing any inaccuracies in an LP model's parameters is through post-optimization analysis, using sensitivity analysis and parametric programming. Neither of these methods, however, is suitable for a general analysis of the effects of imprecise parameters. Another way to handle imprecision is by modeling it according to the probability theory in stochastic programming problems. A third way to address inaccuracy is to use fuzzy set theory which provides a conceptual and theoretical approach to addressing complexity, imprecision, and vagueness. Different TSP and KP models were resolved in multiple applications, in a fuzzy environment. A triangular fuzzy number or a trapezoidal fuzzy number are considered travel cost, time, and other parameters. Various defuzzification methods, such as the need method, the credibility measurement method, the average integration method for classification, and the expectations method, were used to solve the problems. I have been researching some new and updated versions of the solution strategies to address the problems at hand. In all cases, we present a comparison of the results for the different models, taking into account some instances of standard benchmarks to show the algorithm's effectiveness.

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