The Use of Queuing Theory Improved the Service of a Restaurant

Payal Goswami [1], G. V. V. Jagannadha Rao [1] and Aloke Verma [2]

[1] Department of Mathematics, Kalinga University, Naya Raipur (CG) India 492101

[2] Department of Physics, Kalinga University, Naya Raipur (CG) India 492101

Corresponding Author Email ID: goswami.payal123@gmail.com

Article Info
Page Number: 51-59
Publication Issue:
Vol. 72 No. 1 (2023)

Abstract

Queuing theory is the mathematical study of waiting lines, sometimes known as queues. Queue lengths and waiting times can be anticipated using a model developed in queuing theory. A key difficulty that almost every well-known restaurant faces is losing customers due to a long wait in line. This highlights the importance of a numerical model in assisting restaurant management in better understanding the issue. This study attempts to demonstrate that queueing theory meets the model when evaluated in a real-world context. For example, data from the Raipur restaurant "Minerva" is utilised to assess the arrival rate, service rate, utilisation rate, line time, and the chance of potential customers baulking. To analyse the given data, Little's Theorem and the M/M/1 queuing model are employed. During our study period, the arrival rate at "Minerva Restaurant" was 3.244 customers per minute (cpm) during its busiest part of the day, while the service rate was 3.28 cpm. The restaurant has an average of 104 customers and a usage period of 0.989 minutes.

Article History
Article Received: *15 October 2022*
Revised: *24 November 2022*
Accepted: *18 December 2022*

Keywords: Customers per minute (cpm), Queuing Theory, Little's Theorem, Kendall's Notation, Waiting Lines, Poisson Process.

1. Introduction

There are several factors that determine whether a restaurant is good or bad. Taste, cleanliness, and the layout and surroundings of the restaurant are all important factors to consider. They will be able to attract a large number of clients if these characteristics are effectively managed. There is, however, another factor to consider, particularly if the restaurant has already been successful in attracting customers. The length of time customers must wait in line is a component of this component. Once we are serviced, our transactions with the service organisation may be quick, pleasant, and complete, but the bitter taste of how long it takes to get attention taints our overall assessments of service quality. In a waiting line system, managers must decide what level of service to provide. A low level of service may be inexpensive in the short run, but it may result in high costs of customer dissatisfaction, such as lost future business and actual complaint processing costs. The study of queues or waiting lines is known as queueing theory. The expected waiting time in the queue, the average time in the system, the expected queue length, the expected number of customers served at one time, the probability of baulking customers, and the probability of the system being in certain states, such as empty or full, are some of the analyses that can be derived using queuing theory [1]. Waiting lines are common in restaurants, particularly during lunch and dinner. As a result, queuing theory is appropriate for use in a restaurant setting because there is an associated queue or waiting line where customers who cannot be served immediately must queue (wait) for service. Previously, researchers used queuing theory to model restaurant operations [2, 3], reduce cycle time in a busy fast-food restaurant [4, 5], and increase throughput and efficiency [6]. This paper employs queuing theory to investigate the waiting lines at Minerva, a Raipur restaurant with 20 tables, some of which have 2 chairs and some of which have 6. At any given time, there are 12 waiters working. On any given day, it serves over 400 customers during the week and over 700 customers on weekends. The purpose of this paper is to demonstrate the utility of applying queuing theory in a real-world situation.

2. Queuing Theory

Agner K. Erlang was asked by the Copenhagen Telephone Company in 1908 to work on the holding times in a telephone switch. He discovered that the number of phone conversations and the amount of time spent on the phone were both Poisson and exponentially distributed. This was the start of the research into queuing theory. In this section, we will go over two fundamental concepts in queuing theory.

3. Little's Theorem

The relationship between through out rate (i.e., arrival and service rate), cycle time, and work in process (i.e., number of customers/jobs in the system) is described by Little's theorem [6]. This relationship has been demonstrated to be valid for a broad range of queuing models. According to the theorem, the expected number of customers (N) for a system in steady state can be calculated using the following equation:

$$L = \lambda T$$

……….. (1)

Here, λ is the average customer arrival rate, and T is the average customer service time. Consider a restaurant where the customer arrival rate (λ) doubles but the customers spend the same amount of time in the restaurant (T). These facts will increase the number of customers in the restaurant by a factor of two (L).

By the same logic, if the customer arrival rate (λ) remains constant but the customer service time doubles, the total number of customers in the restaurant will also double. This means that managerial decisions for any two of the three variables are all that is required to control the three variables. Little's theorem [5] can be used to derive three fundamental relationships:

L increases if λ or T increases.

λ increases if L increases or T decreases.

T increases if L increases or λ decreases.

According to Rust, the Little's theorem can be useful in quantifying the maximum achievable operational improvements as well as estimating the performance change when the system is modified.

ISSN: 2094-0343
2326-9865

4. Queuing Models and Kendall's Notation

The customer and the server are the primary actors in a queuing situation. When they arrive at a service facility, they can begin service right away or wait in line if the facility is busy. In the context of queue analysis, customer arrival is represented by the inter arrival time between successive customers, and service is described by the service time per customer. Customers' queuing behaviour is taken into account in waiting line analysis.

Human customers may jump from one queue to another in the hope of shortening their wait time. They may also refuse to join a queue entirely due to the anticipated long delay, or they may renege from a queue because they have been waiting for too long. In most cases, queuing models can be distinguished by the following characteristics:

1) Arrival Time Distribution: Inter-arrival times typically follow one of three distribution patterns: a Poisson distribution, a Deterministic distribution, or a general distribution. Inter-arrival times, on the other hand, are frequently assumed to be independent and memoryless, which are characteristics of a Poisson distribution.

2) Service Time Distribution: The time distribution of the service can be constant, exponential, hyper-exponential, hypo-exponential, or general. The service time is not affected by the inter-arrival time.

3) Number of Servers: The queuing calculations differ depending on whether the queue is served by a single server or by multiple servers. A single server queue is served by a single server. This is the typical situation in a bookstore, where there is a line for each cashier. A multiple server queue is analogous to the situation in a bank where a single line waits for one of several tellers to become available.

4) Queue Lengths: A system's queue can be modelled as either infinite or finite in length. This includes customers who are waiting in line.

5) Queuing Discipline: There are several options for the sequence of customers to be served, including FIFO (First In First Out, i.e., in order of arrival), random order, LIFO (Last In First Out, i.e., the last one to arrive will be the first to be served), SIRO (Second In First Out), and SIRO (Second In Last Out) (Service in Random Order).

6) System Capacity: A system's maximum number of customers can range from one to infinity. Kendall proposed a notation system to represent the six discussed above characteristics in 1953. A queue is denoted by the letters A/B/C/D/E/F.

A describes the interarrival time distribution type, B describes the service time distribution type, C describes the number of servers in the system, D describes the maximum length of the queue, E describes the system population size, and F describes the queuing discipline.

5. Minerva Queuing Model

The restaurant's daily visitor count was obtained directly from the restaurant. The data has been recorded by the restaurant as part of its end-of-day routine. In general, we concluded from the data that the queuing model that best illustrates Minerva's operation is M/M/1. This means that the arrival and service times are distributed exponentially (Poisson process). There is only one server in the restaurant system. According to our observations, the restaurant has several waitresses, but there is only one chef to serve all of the customers in the actual waiting queue. The M/M/1 queuing model is depicted in Figure 1.

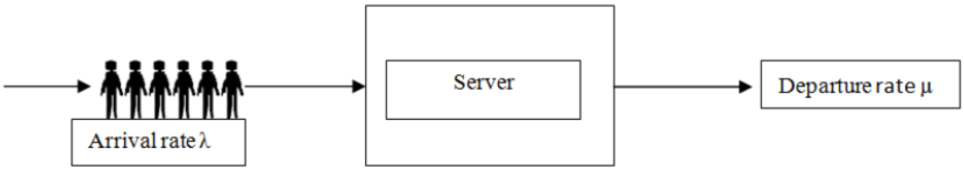

Figure 1. M/M/1 Queuing Model

The following variables will be investigated for the Minerva M/M/1 Queuing Model analysis:

λ: The mean customers arrival rate

μ: The mean service rates

ρ: utilization factor (λ/μ)

Probability of zero customers in the restaurant (P_o) is given by

$$P_o = 1 - \rho$$

………… (2)

P_n: The probability of there being n customers in the restaurant.

$$P_n = P_{o\rho n} = (1 - \rho)\rho_n$$

………… (3)

L: The average number of diners in the restaurant.

$$L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu - \lambda}$$

………….. (4)

L_q: average number of customers in line

$$L_q = \frac{\rho^2}{1-\rho}$$

………….. (5)

W: The average amount of time spent in Minerva, including waiting time.

$$W = \frac{1}{\mu - \lambda}$$

………… (6)

W_q: The average length of time spent in the queue.

$$W_q = \frac{L_q}{\lambda}$$

………… (7)

6. Data Analyzation

The one-week analysis is shown by graph below analyzed by us at dinner slot.

Table 1. One Week Analysis of Number of Customers arriving in restaurant.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1st week	309	395	310	375	319	549	530
2nd week	335	327	321	321	382	545	575
3rd week	349	335	332	323	303	590	582
4th week	380	349	375	360	330	581	603

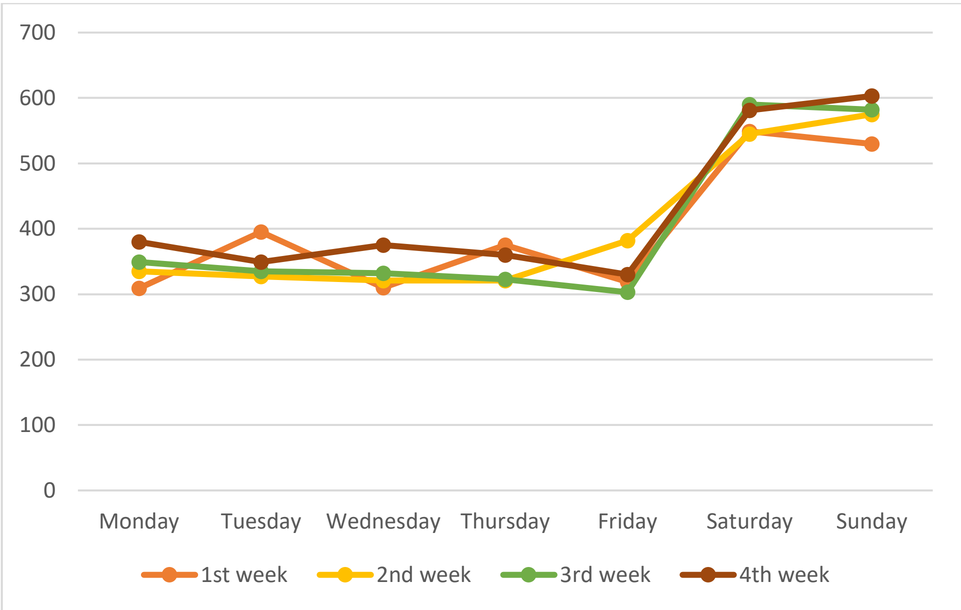

Figure 2. One Week Analysis of Number of Customers arriving in restaurant.

As shown in Figure 2, the number of customers is double on Saturdays and Sundays. The restaurant's busiest time is on weekends during dinner. As a result, we will concentrate our analysis in this time frame. The restaurant was examined by the authors between the hours of 10 and 12.

Figure 3. Minerva Restaurant, Raipur.

7. Calculation

The research was carried out by our teams during dinner time. On average, 292 people visit the restaurant during the one and half hour dinner period. The arrival rate can be calculated as follows:

$$\lambda = \frac{292}{90}$$

$$= 3.244 \text{ customer per minute (cpm)}$$

We also discovered, through observation and discussion with the manager, that each customer spends an average of 32 minutes in the restaurant (W), the queue length is approximately 17 people (L_q.) on average, and the waiting time is approximately 12 minutes. It can be demonstrated using (7) that the observed actual waiting time does not differ significantly from the theoretical waiting time, as shown below.

$$W_q = \frac{L_q}{\lambda}$$

$$= \frac{17 \text{ customers}}{3.244 \text{ cpm}}$$

$$= 5.240 \text{ minutes}$$

Next, using the previously calculated values, compute the average number of people in the restaurant.

$$L = 3.244 \text{ cpm} \times 32 \text{ minutes}$$

$$= 104 \text{ customers approx}$$

After determining the average number of customers in the restaurant Minerva, calculate the service rate as follows:

$$\mu = \frac{\lambda(1 + L)}{L}$$

$$= \frac{3.244(1 + 104)}{104}$$

$$= 3.28 \text{ cpm(approx)}$$

Calculate the Traffic Intensity or Utilization Factor now.

$$\rho = \frac{\lambda}{\mu} = \frac{3.244}{3.28} = 0.989$$

The probability of zero customers in the restaurant or the probability that the system is idle can be calculated using the high utilization rate of 0.989 during dinner time (2)

$$P_0 = 1 - \rho = 0.011$$

The following is a generic formula for calculating the probability of experiencing n customers in the restaurant:

$$P_n = (1 - \rho)\rho^n = (1 - 0.989)0.989^n$$

$$= (0.011)(0.989)^n$$

Assume that when more than 15 people are already queuing for the restaurant, potential customers will begin to baulk, and that the maximum queue length that a potential customer can tolerate is 20 people. Because the restaurant has a capacity of 100 people when fully occupied, we can calculate the probability of 15 people in the queue as the probability when there are 120 people in the system (104 in the restaurant and 15 or more queuing) as follows: Customers leaving = P (more than 15 people in line) = P (more than 120) people in the restaurant.

$$P_{104-120} = \sum_{n=104}^{120} (0.011)(0.989)^n = 5.43\%$$

8. Analysis

The utilization is proportional to the average number of customers. It follows that as utilization rises, so will the average number of customers. The restaurant has a very high utilization rate of 0.989. This, however, is only the utilization rate on Saturdays and Sundays during lunch and dinner. On weekdays, the utilization rate is nearly half of what it is on weekends. This is due to the fact that the number of visitors on weekdays is only half that of visitors on weekends.

Furthermore, the number of waiters or waitresses remains constant regardless of whether it is peak or off-peak hours. If the customer's waiting time is shorter, or if we waited for less than 15 minutes, the number of customers who can be served per minute will increase. When the service rate is higher, the utilization is lower, which reduces the likelihood of customers leaving.

9. Benefits

This research can assist Minerva in improving their QOS (Quality of Service) by anticipating if there are a large number of customers in the queue. This paper's findings may be used as a reference to analyze the current system and improve the next system. Because the restaurant can now forecast how many customers will wait in line and how many will leave each day. By anticipating the large number of customers coming and going throughout the day, the restaurant can set a daily profit target, and the formulas used to complete the research are applicable for future research and could be used to develop more complex theories.

10. Conclusion

The application of queuing theory at Minerva Restaurant was discussed in this study. The authors used two very common decision variables as a medium for introducing and illustrating all of the concepts in this case. According to the findings, the rate at which customers arrive in the queuing system is 3.244 customers per minute, the service rate is 3.28 cpm, and the utilization rate is 0.989. This theory also applies to the restaurant if they want to calculate all of the data on a daily basis. It can be concluded that the arrival rate will be lower and the service rate will be higher on weekdays because the average number of customers is lower than on weekends. The limitations encountered in completing this research were the inaccuracy of the results due to some of the data being based on assumptions or approximation. The authors hope that this research will contribute to the improvement of Minerva restaurant's customer service.

11. Future Outcomes

This research will create a simulation model for the restaurant that will be able to confirm the analytical model's results. Furthermore, a simulation model allows for the addition of more complexity, allowing the model to more closely resemble the actual operation of the restaurant. This study provides a generalized guarantee to stabilize the system from problems such as customers baulking, reneging, jockeying, collusion, or service delays caused by the current way of working in a restaurant. In today's world of accelerating computer technology advancement, it will be beneficial for restaurant managers to install a computer for proper control of service facilities and to keep previous records in order to make forecasting better than good in order to excel in the field.

References

1. Ababneh, K. I., Ponnaiyan, S., ElMelegy, A. R., & Prybutok, V. (2022). Quality Management Journal, 1-21.
2. Ahmed, N., Woishe, M. F., Sultana, N., & Bristy, T. Z. (2022). European Journal of Engineering and Technology Research, 7(2), 55-60.
3. Ahsan, M. M., Islam, M. R., & Alam, M. A. (2014). IOSR Journal of Mechanical and Civil Engineering, 11(6), 31-35.
4. Bhavani, L. K., & Jayalalitha, G. (2021). Annals of the Romanian Society for Cell Biology, 25(6), 4479-4484.

5. Chakraborty, A. K., Chattopadhyay, R., Kaur, I., & Mittra, S. (2022). OPSEARCH, 59(1), 1-19.
6. Chang, Y. Y., & Cheng, C. C. (2022). International Journal of Contemporary Hospitality Management.
7. Chaudhry, M. L., & Gupta, U. C. (2019). In Advances on methodological and applied aspects of probability and statistics (pp. 31-46). CRC Press.
8. Di Martino, B., Colucci Cante, L., & Cerullo, N. (2022). In International Conference on Advanced Information Networking and Applications (pp. 153-162). Springer, Cham.
9. Garimella, B., Shenoy, S., Pai, R. Y., & Shetty, R. (2018). Foundations of Management, 10(1), 75-86.
10. Huang, F., Chuang, Y. C., Zhang, Y. H., Lu, B., & Xu, H. L. (2022). Mathematical Problems in Engineering, 2022.
11. Karniouchina, K., Sarangee, K., Theokary, C., & Kübler, R. (2022). Service Science.
12. Labib, M. Y., & Watfa, M. K. (2022). Journal of Foodservice Business Research, 1-29.
13. Little, J. D. (1961). Operations research, 9(3), 383-387.
14. Molla, Md Al-Amin. (2017). IOSR Journal of Business and Management, 19(2) pp. 93-98.
15. Nair, K. U., Kumar, D. S., & Purani, K. (2022). Marketing Intelligence & Planning.
16. Newell, C. (2013). Springer Science & Business Media.
17. Novitzky, S., Pender, J., Rand, R. H., & Wesson, E. (2019). SIAM Journal on Applied Dynamical Systems, 18(1), 279-311.
18. Goswami, P., & Rao, G. V. V.J. (2022). Int. J of All Research Edu. and Sci. Methods, 10(3), pp. 1182-85.
19. Rochana, W. R. W., Mokan, A., & Mustav, A. W. (2022). In International Conference of Business and Social Sciences (pp. 1318-1330).
20. Ruan, W. Q., & Zhang, S. N. (2022). Service Business, 1-39.
21. Rust, R. T., & Chung, T. S. (2006). Marketing science, 25(6), 560-580.
22. Saaty, T. L. (1961). New York: McGraw-Hill.
23. Suleiman, S., Burodo, M. S., & Ahmed, Z. (2022). Asian Journal of Probability and Statistics, 16(2), 1-9.
24. Sundari, S. M., & Srinivasan, S. (2012). Analysis of M/M/I queueing model for ATM facility. Global Journal of Theoretical and Applied Mathematics Sciences, 2(1), 41-46.
25. Swink, M., Hu, K., & Zhao, X. (2022). Production and Operations Management.
26. Sztrik, J. (2012). University of Debrecen, Faculty of Informatics, 193, 60-67.
27. Ülkü, S., Hydock, C., & Cui, S. (2022). Management Science.
28. Wong, I. A., Huang, J., Lin, Z. C., & Jiao, H. (2022). International Journal of Contemporary Hospitality Management.
29. Zhang, X., & Ahmed, R. R. (2022). Automation in Construction, 137, 104221.