# Are Background Measurements Able to Remove the Degeneracy between the Dark Energy Models?

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Article Info	Abstract: Many dark energy models aptly describe the present-day
Page Number: 60-73	accelerated expansion. The $\Lambda CDM$ model is the most popular and widely
Publication Issue:	used. This model presents some theoretical challenges and observational
Vol. 72 No. 1 (2023)	inconsistencies. There are many alternatives for this model: the barotropic
	fluid model, the quintessence scalar field models, the tachyon scalar field
	models, etc. These models potentially explain the present-day accelerated
	expansion and satisfy the observational data. In this study, we analyse
	whether the background distance measurement can be used to break this
	degeneracy. We find that if the present value of the dark energy equation of
	state is -1 (a value for cosmological constant), then all the background
	models show the same past evolution. Even if the equation of state deviates
	from $-1$ , the background evolution for the different models differs only at
	higher redshifts.
	Therefore, we need high redshift observations to break the degeneracy
Article History	between models. Studying the effect of the dark energy perturbation on the
Article Received: 15 October 2022	structure formation is also helpful for this purpose.
Revised: 24 November 2022	Keywords: Dark energy, Cosmology, Expansion of universe, ACDM
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#### **1** Introduction

The cosmological observations have established that the Universe, at present, is accelerating. [1-9]. This phenomenon can be explained if the energy budget of the Universe is considered to be dominated by an exotic negative pressure medium. This component of the Universe is called 'the dark energy'. Observations suggest that around two-third of the energy budget of the Universe is due to the dark energy [3, 7-9]. The equation of state (dimension less ratio of pressure to energy density) need to be less than  $-\frac{1}{3}$  [10–13] for accelerated expansion. Now the job of cosmologist is to construct, constrain, and test the models of dark energy. The equation of state parameter is an important quantity for this purpose. Among the many models of dark energy model (*ACDM* model) [14, 15]. Where the constant  $\Lambda$  stands for vacuum energy density. The value of equation of state parameter for this model is -1. Although this model successfully explains many observations, it rises many theoretical challenges. Among them the most discussed ones are 'the fine-tuning problem' and 'the coincident problem'. [15–18]. This model

There are inconsistencies in the estimation of cosmological parameters from independent observations in the light of  $\Lambda CDM$  model [19].

Above facts motivate cosmologist to go on the search for alternatives of *ACDM* model. The simplest alternate is the barotropic fluid model [20–27]. The value of equation of state parameter for this model is not constant in general, but a function of redshift or scale factor. These models lag in the physical motivation. The more physically motivated models are the scalar field models. The quintessence scalar field dark energy models are the well-studied models [28–35]. The present day accelerated expansion is achieved by a slow rolling potential in these models. Another class of scalar field models are the non-conical scalar field models. The example of this class of models includes the tachyon model and k-essence model of dark energy. The tachyon scalar field appears as a decay mode of D-branes in string theory [36–38]. The equation of state for this model becomes dust like in the course of evolution [39–43]. This 'tachyon dust' is considered as a potential candidate for both dark energy and dark matter [37, 38, 44–48].

The aim of this study is to compare the background dark energy models in a way to remove the degeneracy between them. Rajvanshi and Bagla (2019) have shown that it is possible to reconstruct the potential for a scalar field model for a given track of evolution considering parameterization of the equation of state w(z) [49]. Particularly, they have shown the reconstruction of potential for the tachyon, quintessence and interacting dark energy models using the constant w and CPL parameterization, but their method is general. Therefore, in this study we mainly compare the *ACDM* model with parameterized models (fluid models) of dark energy. Although we have compared two classes of models, the conclusions of the study will add to better understanding of other dark energy models.



Figure 1. Evolution of luminosity and angular diameter distance and their dependence on  $\Omega_{m0}$  for flat  $\Lambda CDM$  model. Here,  $D_L$  and  $D_A$  are in the unit of  $cH_0^{-1}$ .

The background cosmological parameter constraints from the data are mainly based on the determination of luminosity distances, angular diameter distances and the rates of expansion measurement. Therefore, models prefer different sets of parameters only if the evolution of these quantities differ. Our aim in this study is to show that background evolution is same in

all the above models if the present value of the equation of state is  $w_0 = -1$ , and they deviate only if  $w_0 \neq -1$ .

In the next section we describe the basic equations of the background cosmology. We introduce models compared in this study in the section 3 and compare them in the section 4. We summarize and conclude our study in the section 5.

#### 2 Background Cosmology

The homogeneous and isotropic universe is represented by FLRW metric given by

$$ds^{2} = -dt^{2} + a^{2}(t)(dx^{2} + dy^{2} + dz^{2})$$

(2.1)

where a(t) is the scale factor of expansion. In this universe the energy momentum tensor  $(T_{\mu\nu})$  is restricted to perfect fluid. In such a universe the dynamics of expansion is governed by the Friedman equations which are given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \qquad \left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}(\rho + 3P)$$

(2.2)

where  $\rho = \rho_m + \rho_r + \rho_{\phi}$ . The equation (2.2) clearly suggests that  $\frac{\ddot{a}}{a} > 0$  (accelerated expansion) if the equation of state  $w = \frac{P}{\rho} < -\frac{1}{3}$ . The energy density of matter (baryonic matter + dark matter)  $\rho_m \propto a^{-3}$ , whereas the energy density of relativistic matter  $\rho_r \propto a^{-4}$ . The  $\rho_{\phi}$  is the energy density of dark energy. We have defined it in the next section for some models.

If we define a dimensionless density parameter  $\Omega_x = \frac{\rho_x}{\rho_{cr}}$ , where the critical density  $\rho_{cr} = \frac{3H^2}{8\pi G}$ , then the first Friedmann equation (2.2) can be written as

$$\frac{\dot{a}}{a} = H_0^2 (\Omega_{r0} a^{-4} + \Omega_{m0} a^{-3} + \Omega_{\phi 0}$$

(2.3)

Here,  $\Omega_{r0}$ ,  $\Omega_{m0}$ , and  $\Omega_{\phi 0}$  are the present values of density parameters of corresponding components. The Hubble parameter  $H = \frac{\dot{a}}{a}$  is the rate of expansion in the Universe. It is an important quantity and its measurement at different redshift z gives us the information of expansion history of the Universe. Using the relation between scale factor and redshift  $1 + z = \frac{a_0}{a}$ , we can find the expression for the Hubble parameter, given by

$$H = H_0 \sqrt{\left[\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\phi 0}\right]}$$

(2.4)

Vol. 72 No. 1 (2023) http://philstat.org.ph The cosmological history of the cosmos is hidden in the relation between distances and redshift. Therefore, the measurement of distances at different redshift are used to constrain the cosmological parameters. There are two types of distances, namely the luminosity distance and the angular diameter distance, which cosmologist used for this purpose. Since the Universe is expanding, the electromagnetic signals get redshifted while reaching to us. The amount of redshift depends on the expansion rate and energy budget of the Universe. These quantities are model dependent; therefore, the luminosity distance and angular diameter distance are also model dependent. For a flat universe, the luminosity distance is given by

$$D_L = \frac{c}{H_0} (1+z) \int_0^z \frac{dz}{E(z)}$$

(2.5)

and the angular diameter distance is given by

$$D_A = \frac{c}{H_0} \frac{1}{1+z} \int_0^z \frac{dz}{E(z)}$$

(2.6)

Where  $E(z) = \frac{H(z)}{H_0}$ . The symbol c represents the speed of light in vacuum and  $H_0$  is the present

value of the Hubble parameter (Hubble constant). Form luminosity distance we can calculate the distance modulus of the object at redshift z. The distance modulus is given by

$$\mu = m - M = 5 \log D_L - 5$$

(2.7)

Here,  $D_L$  is in parsecs. The quantities m and M are the apparent and absolute magnitude of the object.

## 3 The dark energy models

There are many models those can aptly explain the dark energy. The popular ones among them are *ACDM* model, barotropic fluid model, canonical and non-canonical models, etc. In some approach cosmologist also consider modification in the general relativity rather than using these dark energy models. Here, we restrict out study to *ACDM* model and the parameterized models (fluid models).



Figure 2. In the top left panel, we show the phases of evolution of the Universe for the  $\Lambda CDM$  model. The evolution of the density parameters of matter (in red), vacuum (in blue), and radiation (in orange) are shown in top right panel. In these plots, we set the parameter  $\Omega_{m0} = 0.315$  and  $H_0 = 67.4 \ km \ s^{-1}M \ pc^{-1}$  (Planck-2018 best fit values). In the bottom left and right panels, we show comparison of this model with SN-Ia union 2.1 data and direct measurements of Hubble parameter H(z).

3.1 The *ACDM* Model

If we add a constant term  $\frac{\Lambda}{3}$  in the Friedmann equation, we can explain the accelerated expansion of the Universe. The constant  $\Lambda$  is known as 'the Cosmological constant' [50]. Physically  $\Lambda$  stands for the vacuum energy density. In term of  $\Lambda$  the energy density and pressure is given by [10–13]

$$ho_{\Lambda}=rac{\Lambda}{8\pi G}$$
 ,  $P_{\Lambda}=-rac{\Lambda}{8\pi G}$ 

(3.1)

Clearly, the equation of state in the  $\Lambda CDM$  is w = -1, and it is a constant. The Hubble parameter for the  $\Lambda CDM$  is given by

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$$H = H_0 \sqrt{[\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0}]}$$

(3.2)

Where the density parameter for the cosmological constant  $\Omega_{\Lambda} = \rho_{\Lambda}/\rho_{cr}$ .

In the figure 1 the evolution of the luminosity distance and the angular diameter distance with redshift are shown. We find that the luminosity distance is a monotonically increasing function of redshift, while the angular diameter distance first increases, take a maxima, and then decrease. At a certain redshift, both of these distances decrease with increasing value of matter density parameter.

In the top left panel of the figure 2 we show the phases of evolution of the Universe in the *ACDM* model. We clearly see that after decelerating phase of radiation and matter domination, the universe has started accelerating its expansion recently. On the top right panel, we see the evolution of matter density parameters of different component. At very high redshift, dominating component was radiation, then at redshift less than  $10^3$  matter dominates the energy budget almost completely. In near past, the cosmological constant has become the dominating component. It is a dominating component at present z = 0 and go on dominating the energy budget in future z < 0. The agreement of this model with the Supernova-Ia data and H(z) data is shown in the lower panes of the figure 2. This model is successfully explaining the present day accelerated expansion of the Universe, and also shows good agreement with the cosmological data.



Figure 3. Evolution of equation of state with redshift for parameterized models of dark energy. Read, blue and sky-blue colours represent the CPL model, the JBP model and the logarithmic model respectively. For the purpose of this plot, we have set  $w_0 = -1$  and  $w'_0 = 0.05$ .

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Figure 4. A comparison of the evolution of the luminosity distance with redshift for *ACDM* model and the fluid models with different parameterizations. Left panel is for  $\Omega_{m0} = 0.25$  whereas the right panel is for  $\Omega_{m0} = 0.315$ . For the fluid models we set parameters  $w_0 = -1.0$  and  $w'_0 = 0.05$ . The curve for different models are actually overlapped.

#### **3.2 Barotropic Fluid Models**

The simplest alternative to the  $\Lambda CDM$  model are the models with a parametric form of the equation of state w. In these models the equation of state parameter may either be a constant  $w \neq -1$ , or a function of redshift. In the second case there are two important parameters; the present value of the equation of state  $w_0$  and its derivative  $w'_0$ . There are many such models, among them most popular ones are –

The Chevallier-Polarski-Linder (CPL) parameterization [51, 52]. The equation of state parameter in this model is given by

$$w(z) = w_0 + w_0' \frac{z}{z+1}$$

(3.3)

The Jassal-Bagla-Padmanabhan parameterization (JBP) model [53]. The equation of state parameter is given by

$$w(z) = w_0 + w'_0 \frac{z}{(1+z)^2}$$

(3.4)

The logarithmic parameterization [54]. The equation of state parameter is given by

$$w(z) = w_0 + w'_0 \log(1+z)$$

(3.5)

From figure 3 we see that at low redshift all these parameterized models are in agreement with each other, while at higher redshifts they deviate from each other. The energy density of the dark energy in the parameterized models are given by [23]

$$\rho_{de} = \rho_{de0} \exp\left[3 \int_0^z \frac{dz}{1+z} [1+w(z)]\right]$$

(3.6)

and the density parameter of the dark energy is given by

$$\Omega_{de} = \Omega_{de0} \exp\left[3 \int_0^z \frac{dz}{1+z} [1+w(z)]\right]$$

(3.7)

These models satisfy the cosmological data as good as ACDM model [23].

#### 4 Comparison between Background Models

The constraints on the cosmological parameters using the background measurement mainly compare the theoretical evolution of the distances and the rate of expansion of the Universe with the data. In this section we compare the evolution of these quantities for the models mentioned in the section 3.

We start this study by analyzing the degeneracy between models if the present value of the equation of state parameter  $w_0$  is -1 (value for cosmological constant model). In the figure 4 we show the comparison of the evolution of the luminosity distance between the  $\Lambda CDM$  model and fluid models with different parameterizations. We use CPL, JBP and Logarithmic parameterizations for the fluid models. In the left panel we show comparison for  $\Omega_{m0} = 0.25$ , whereas in the right panel we show comparison for  $\Omega_{m0} = 0.315$ . For the fluid models we set  $w_0 = -1.0$  and  $w'_0 = 0.05$ . We clearly see their is absolutely no observable difference between the models.



Figure 5. A comparison of the evolution of the angular diameter distance with redshift for *ACDM* model and the fluid models with different parameterizations. Left panel is for  $\Omega_{m0} = 0.25$  whereas the right panel is for  $\Omega_{m0} = 0.315$ . For the fluid models we set parameters  $w_0 = -1.0$  and  $w'_0 = 0.05$ . The curve for different models are actually overlapped.



Figure 6. A comparison of the evolution of the expansion rate of the Universe with redshift for  $\Lambda CDM$  model and the fluid models with different parameterizations. Left panel is for  $\Omega_{m0} = 0.25$  whereas the right panel is for  $\Omega_{m0} = 0.315$ . For the fluid models we set parameters  $w_0 = -1.0$  and  $w'_0 = 0.05$ . The curve for different models are actually overlapped.

In the figure 5 we show the comparison of the evolution of the angular diameter distance. For this comparison, the values of the parameters are same as mentioned above to generate the figure 4. Clearly, the evolution of the angular diameter distance is also same for the background models. The comparison of the evolution of the expansion rate of the Universe is shown on the figure 6 with the same set of parameters as mentioned above. At fixed redshift the expansion rate varies on varying  $\Omega_{m0}$ . On the other hand, for a given  $\Omega_{m0}$ , if  $w_0 = -1$  then the evolution of the expansion rate does not depend on the specific model. Actually, any effect of dynamical nature of w(z) (with  $w_0 = -1$ ) is effective only in the future (not shown in the figure) when the dark energy dominates.

We also study the case when present value of the equation of state parameter deviates from a cosmological constant like value. In the figure 7 we show the comparison of the theoretical evolution of distances when  $w_0 = -0.95$  and  $w'_0 = 0.05$ . We find that at high redshifts ( $z \ge 2$ ) models deviate form each other. The JBP models deviate the most from  $\Lambda$ CDM model. The CPL and Logarithmic models show same evolution for the given set of parameters. The deviation between models increases if we set  $w_0$  more and more away from -1. We also observe that the models are in agreement with each other at smaller redshift. Therefore, any low redshift observations will not be able to remove degeneracy between dark energy models.

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Figure 7. The comparison of the evolution of the distances when  $w_0 \neq -1$ . We set  $\Omega_{m0} = 0.315$  of all the models. For fluid models we set  $w_0 = -0.95$  and  $w'_0 = 0.05$ . All the models are in agreement at very low redshift while deviate at higher redshift with each other.

#### **5** Summary and Conclusions

In this study we present theoretical comparison on the background dark energy models. The aim of the study is to analyze whether the background measurement are able to remove the degeneracy between models. The constraints on the parameters using background measurement are found by comparing the theoretical value of luminosity distance, angular diameter distance, and the expansion rate of the Universe (Hubble parameter) with data. The luminosity distances are used in terms of distance modulus for the parameter constraining using the supernova-Ia data [1, 23, 39, 55]. The angular diameter distance or the acoustic parameter for constraining using the Baryon acoustic Oscillation (BAO) data [23, 39].

We use  $\Lambda CDM$  model and fluid models with CPL, JBP and logarithmic parameterization. The parameterization of the equation of state w(z) can be used to construct the scalar field potentials [49].

Therefore, conclusions of this study can be extended to have better understanding of other background dark energy models. We find that if the value of the present value of equation of state parameter  $w_0$  is -1 (a cosmological constant like value), then there is no model dependent difference in the background evolution of the past universe. All the models show same track of past background evolution for given sets of the cosmological parameters.

If the  $w_0 \neq -1$  then background models show deviation from each other at higher redshifts. Among the parameterized models, discussed in this report, JBP parameterization shows the largest deviations from the  $\Lambda CDM$  model. At lower redshifts these models are still in agreement with each other. Therefore, it is not possible to remove the degeneracy of the dark energy models with lower redshift measurements. To remove degeneracy, we need data from high redshift measurement and large-scale surveys. The canonical and non-canonical scalar fields are also a potential candidate for dark energy. The scalar field dark energy get clustered as matter does in the Universe if  $w_0 \neq -1$  [40, 56]. The perturbations in dark energy also affect the evolution of the structure in the Universe. The dark energy perturbations also affect the low 1 CMB angular power spectrum through the Integrated Sachs-Wolf Effect (ISW effect). Therefore, the measurement of the ISW effect can be used to break the degeneracy between the dark energy models.

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