Mathematical Modeling and Profit Analysis of a Standby System with Priority for Preventive Maintenance

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Abstract

In this study, RPGT was used to model the system parameters for a profit analysis of a two-unit standby system with priority for preventative maintenance and server failure. It is commonly believed that service facilities never breakdown or degrade in performance while performing their duties, but in reality, service facilities can fail or degrade in performance as a result of an accident caused by improper handling of the system, ignorance of the function of the units or systems, carelessness on the part of the server, electric shock, or something else outside of their control, etc.While doing its intended tasks, the server could experience an error and need to be fixed before continuing. For boosting failure and repair rates, profit analysis tables for system parameters are created. Corresponding graphs are created, and this table and graph are used to write an analysis or trend of the parameters.

Keywords: Availability, MTSF, Busy Period

1. Introduction

In this study, RPGT was used to model the system parameters for a profit analysis of a two-unit standby system with priority for preventative maintenance and server failure. It is commonly believed that service facilities never breakdown or degrade in performance while performing their duties, but in reality, service facilities can fail or degrade in performance as a result of an accident caused by improper handling of the system, ignorance of the function of the units or systems, carelessness on the part of the server, electric shock, or something else outside of their control, etc.In these circumstances, the server needs some form of assistance, such as some instruction for a specific task or specialized repair of the service facility, in order to continue its duties and minimize system downtime and customer wait times. In the dependability modeling of diverse systems, researchers Kumar (2019), Rajbala (2022), Goel (2019), and others have analyzed a number of systems with repairable servers under various scenarios. Once the failed service facility has been recovered or repaired.The system as a whole makes money as the wait time for a failed unit to be repaired and for the system to be in operational condition both decrease. A unit is only repaired when it entirely fails by the server. While doing its intended tasks, the server could experience an error and need to be fixed before continuing. Two units make up the system; one is operational while the other is kept in cold standby mode for preventive maintenance. The only two operating modes for an online unit or a cold standby unit are full capacity operation and total failure.As was said before, a single server is available for unit maintenance and fault-finding. Preventive maintenance is only permitted for online units before failure. Preventive maintenance is given priority over the repair of a unit since it helps to lower the rate of deterioration of online units in various functioning states. A unit is only fixed when it has completely failed. The random variables related to unit failure and repair, servers, preventive maintenance, and server downtime are statistically independent and also have unique probability distributions. In their research on system modeling and analysis, Rajbala et al. [2019] looked at a case study of an EAEP manufacturing facility.The comparative analysis of the subsystem failed simultaneously was discussed by Shakuntla et al. [2011]. In their study, Kumar et al. [2018] investigated a 3:4:: outstanding system plant's sensitivity analysis. PSO was used by Kumari et al. [2021] to research limited situations. Using a heuristic approach, Rajbala et al. [2022] investigated the redundancy allocation problem in the cylinder manufacturing plant.A study of the urea fertilizer industry's behavior was conducted by Kumar et al. [2017]. Mathematical formulation and profit function of a comestible oil refinery facility were investigated by Kumar et al. in [2017]. In a paper mill washing unit, Kumar et al. [2019] investigated mathematical formulation and behavior study. Shakuntla et al [2011] discussed the behavior analysis of polytube using supplementary variable technique the behavior of a bread plant was examined by Kumar et al. in [2018]. In order to do a sensitivity analysis on a cold standby framework made up of two identical units with server failure and prioritized for preventative maintenance, Kumar et al. [2019] used RPGT. The current paper is divided into two half, one of which is being used and the other of which is on cold standby. The main distinctions between online and cold standby equipment are the excellent and fully failed modes. Semi-Markov processes and RPGT with various system dependability parameters are used to model the system.

RPGT is used to construct the phrases for significant reliability features. For boosting failure and repair rates, profit analysis tables for system parameters are created. From these tables and graphs, analysis and trend of parameters are documented, and corresponding graphs are generated.

2. Assumption, Notations and Transition Diagram

- Repairman is always available 24*7
- Repair/Failure rates are Constant.

A/B/S : - Unit A/ cold standby unit B/ server are noble.

B_p : - Cold standby unit undergoing PM.

A_{wp}/B_{wp}: - Unit A/ Unit B are waiting for PM.

a_{ws}/ b_{ws}: - Unit A/ Unit B failed/functioning for repair.

a_{wr}/ b_{wr}: - Unit A/ Unit B unsuccessful

s_e : - Server is unsuccessful

a_{us}/b_{us} : - Unit A/ B failed and under repair.

a_{ur}/b_{ur} : - Unit A/ B failed and under unceasing.

Attractive into attention the upstairs assumptions and notations the Transition Diagram of the organization is assumed in Figure 1.

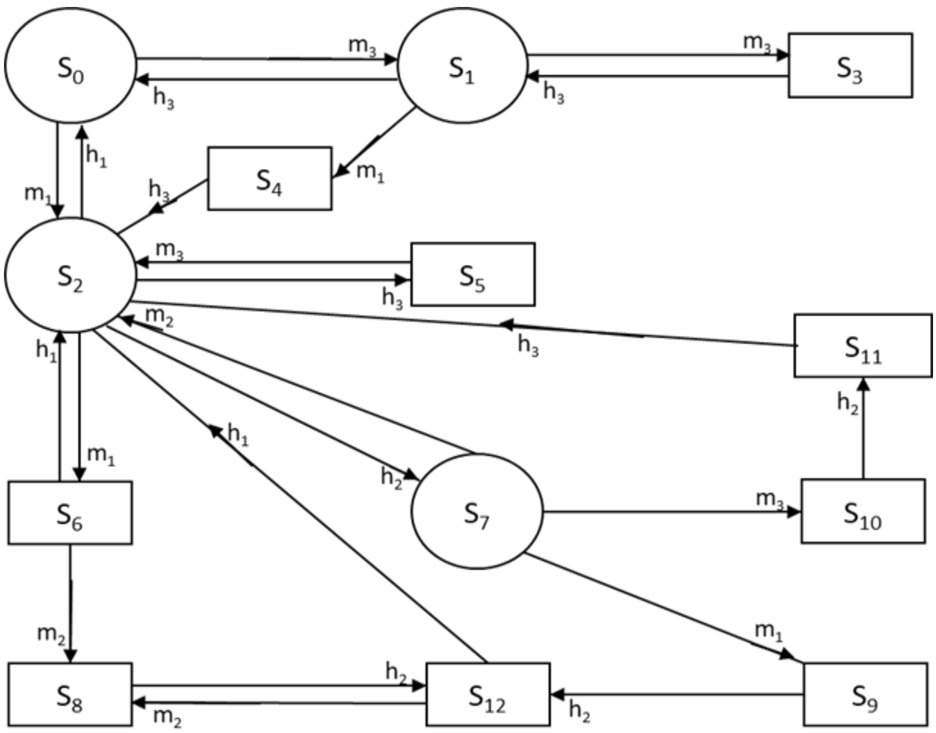

Figure1: Transition Diagram

$S_0 = A(B)S$, $S_1 = AB_pS$, $S_2 = a_{us}BS$, $S_3 = A_{wp}B_pS$,
$S_4 = a_{us}B_pS$ or $a_pb_{ws}S$, $S_5 = a_{ws}B_pS$, $S_6 = a_{ur}B_{ws}S$, $S_7 = a_{ws}Bs_u$,
$S_8 = a_{wr}b_{wr}s_t$ or $a_{wr}b_{wr}S_t$, $S_9 = a_{wr}b_{us}s_t$, $S_{10} = a_{wr}B_{wp}s_t$,
$S_{11} = a_{wr}B_ps$, $S_{12} = a_{wr}b_{us}S$

3. Model Description

There are two somewhat comparable units named "A" and "B." In the initial state S0, unit "A" is online and unit "B" is kept in cold standby mode. Server "S" fixes the broken units and plans preventative maintenance for the working units. The diagram shows that the system is in the initial working state S_0, with unit "A" online and in the working state, unit "B" in the cold by state, and server "S" in the good state. As of this moment, server "S" is in fine condition while cold standby unit "B" is in a cold by state.As of this moment, the cold standby unit "B" is functional.

As a result, the system enters state S1. In this state, the system is still operating at full capacity because unit "A" is operational, unit "B" is undergoing preventative maintenance, and the server is in good condition.In state S_1 if the unit "B" receives preventive maintenance, it is restored at the restoration rate "h_3," so after restoration system rejoins state "S_0," the unit "B" does not receive preventive maintenance and online unit "A" is scheduled for preventive maintenance with rate "m_3," then the system enters state "S_3" as there is no working unit, where the continues policy of PM is followed, as the unit "B" is under PM it will get PM service first and "A". It should be noted that although units "A" and "B" are identical, they are here in the research represented by distinct alphabets just for the sake of system description. If a working unit fails in state S_1 with a continuous failure rate of m_1, the system enters the failed state S_4. The system will enter the state S_2 after PM Service because the analysis of the system gives precedence to PM over the repair of a failing unit.From the starting state S_0, if working unit "A" fails at constant failure rate m_1, the system will also enter state S_2.

If working unit "A" is taken offline for PM at rate m_3, the system then enters either S_5 or S_6, from the state after the unit is repaired at constant repair rate h_1. The system reaches operational state S_2 (full capacity) once more. The system reaches the failure state S_8 when a server mails at a rate of m_2, similar to when a unit is being repaired.The S_{12} capital of a failed state enters the system. After repairing a unit under continuous repair in state S_{12}, the system once more enters functional state S_2

since the suffix "r" indicates that continuous repair from the previous state is being done. The system may enter the working state S_7 if the online working unit fails at failure rate m_1, but if it does so in state S_7 while the server is engaged fixing the failed unit; the system enters the failed state S_9. The server is now being treated in priority, and following that, the system enters state S_{12}.

Additionally, if an online working unit is scheduled for PM at a rate of m_3, while the system is still in the working state S_7, the system enters the failed state S_{10}, where the server is down and must provide PM and repair for the failed unit(s), which is done first before continuing with the repair. The system enters state S_{11}, and from there, upon PM of unit "B," the system enters state S_2, which is the working state.

4. Transition Probability (TP)

$q_{i,j}(t)$ **:**P.d.f. of the first passage periodafter a reformative state 'i' to a reformative state 'j' or to a unsuccessful state 'j' devoid of visiting someadditionalreformative state in (0,t].

$p_{i,j}$**:** Steady state TP from a reformative state 'i' to a reformative state 'j' deprived ofstaying any other reformative state. $p_{i,j} = q_{i,j}^{*}(0)$; anywhere * denotes Laplace transformation.

Transition Probabilities

$q_{i,j}(t)$

$q_{0,1}(t) = m_3 e^{-(m_1+m_2)t}$

$q_{0,1}(t) = m_1 e^{-(m_1+m_2)t}$

$q_{1,0}(t) = h_3 e^{-(m_1+m_3+h_3)t}$

$q_{1,3}(t) = m_3 e^{-(m_1+m_3+h_3)t}$

$q_{1,4}(t) = m_1 e^{-(m_1+m_3+h_3)t}$

$q_{2,0}(t) = h_1 e^{-(m_1+m_2+m_3+h_1)t}$

$q_{2,5}(t) = m_3 e^{-(m_1+m_2+m_3+h_1)t}$

$q_{2,7}(t) = m_2 e^{-(m_1+m_2+m_3+h_1)t}$

$q_{3,1}= h_3 e^{-h_3 t}$

$q_{4,2}= h_3 e^{-h_3 t}$

$q_{5,2}= h_3 e^{-h_3 t}$

$q_{6,2}(t) = h_1 e^{-(h_1+m_2)t}$

$q_{6,8}(t) = m_2 e^{-(h_1+m_2)t}$

$q_{7,2}(t) = h_2 e^{-(m_2+m_3+h_2)t}$

$q_{7,9}(t) = m_2 e^{-(m_2+m_3+h_2)t}$

$q_{7,10}(t) = m_3 e^{-(m_2+m_3+h_2)t}$

$q_{8,12} = h_2 e^{-h_2 t}$

$q_{9,12} = h_2 e^{-h_2 t}$

$q_{10,11} = h_2 e^{-h_2 t}$

$q_{11,2} = h_3 e^{-h_3 t}$

$q_{12,2}(t) = h_1 e^{-(m_2+h_1)t}$

$q_{12,8}(t) = m_2 e^{-(m_2+h_1)t}$

$\mathbf{P_{ij} = q^*_{i,j}(0)}$

$p_{0,1} = m_3/(m_1+h_3)$

$p_{0,2} = m_1/(m_1+h_3)$

$p_{1,0} = h_3/(m_1+ m_3+h_3)$

$p_{1,3} = m_3/(m_1+ m_3+h_3)$

$p_{1,4} = m_1/(m_1+ m_3+h_3)$

$p_{2,0} = h_1/(m_1+ m_2+ m_3+h_1)$

$p_{2,5} = m_3/(m_1+ m_2+ m_3+h_1)$

$p_{2,7} = m_2/(m_1+ m_2+ m_3+h_1)$

$p_{3,1} = 1$

$p_{4,2} = 1$

$p_{5,2} = 1$

$p_{6,2} = h_1/(h_1+ m_2)$

$p_{6,8} = m_2/(h_1+ m_2)$

$p_{7,2} = h_2/(m_2+ m_3+h_2)$

$p_{7,9} = m_2/(m_2+ m_3+h_2)$

$p_{7,10} = m_3/(m_2+ m_3+h_2)$

$p_{8,12} = 1$

$p_{9,12} = 1$

$p_{10,11} = 1$

$p_{11,2} = 1$

$p_{12,2} = h_1/(m_2+h_1)$

$p_{12,8} = m_2/(m_2+h_1)$

4.1 Mean Sojourn Times (MST)

$R_i(t)$: Reliability of the system at time t, given that the system in regenerative state i.

μ_i : MST spent in state i, before visiting any other states;

Mean Sojourn Times

$R_i(t)$

R_0(t)= $e^{-(m_1+m_3)t}$

R_1(t)= $e^{-(m_1+m_3+h_3)t}$

R_2(t)= $e^{-(m_1+m_3+h_1)t}$

$R_3(t)$= $e^{-h_3 t}$

$R_4(t)$= $e^{-h_3 t}$

$R_5(t)$= $e^{-h_3 t}$

R_6(t)= $e^{-(h_1+m_2)t}$

R_7(t)= $e^{-(m_2+m_3+h_2)t}$

$R_8(t)$= $e^{-h_2 t}$

$R_9(t)$= $e^{-h_2 t}$

$R_{10}(t)$= $e^{-h_2 t}$

$R_{11}(t)$= $e^{-h_3 t}$

R_{12}(t)= $e^{-(m_2+h_1)t}$

$\mu_i=R_i^*(0)$

$\mu_0 = 1/(m_1+m_3)$

$\mu_1 = 1/(m_1+m_3+h_3)$

$\mu_2 = 1/(m_1+m_3+h_1)$

$\mu_3 = 1/h_3$

$\mu_4 = 1/h_3$

$\mu_5 = 1/h_3$

$\mu_6 = 1/(h_1+m_2)$

$\mu_7 = 1/(m_2+m_3+h_2)$

$\mu_8 = 1/h_2$

$\mu_9 = 1/h_2$

$\mu_{10} = 1/h_2$

$\mu_{11} = 1/h_3$

$\mu_{12} = 1/(m_2+h_1)$

5. Evaluation of Parameters

Applying RPGT and utilizing "0" as the initial-state of the system, the MTSF and all other significant system parameters are evaluated in steady-state conditions as follows: All states that are attainable from the beginning state ξ' = '0' have the following TP factorsLikelihoods from state '0' to dissimilar vertices are assumed as

$V_{0,0} = 1$

$V_{0,1} = p_{0,1}/(1-p_{1,3}p_{3,1})$

$V_{0,2} = p_{0,2}/(1-p_{2,5}p_{5,2})(1-p_{2,7}p_{7,10}p_{10,11}p_{11,2})+p_{0,1}p_{1,4}p_{4,2}/(1-p_{1,3}p_{3,1})(1-p_{2,5}p_{5,2})$
$(1-p_{2,7}p_{7,10}p_{10,11}p_{11,2})$

$V_{0,3}$ = ….continuous

The TPissues of all the accessible states after the base state 'ξ' = '1' are: Likelihoods from state '1' to dissimilar vertices are assumed as

$V_{1,0} = p_{1,0}/\{(1-p_{0,2}p_{2,0})/(1-p_{2,5}p_{5,2})(1-p_{2,7}p_{7,10}p_{10,11}p_{11,2})\}+p_{1,4}p_{4,2}p_{2,0}/(1-p_{2,5}p_{5,2})$
$(1-p_{2,7}p_{7,10}p_{10,11}p_{11,2})$

$V_{1,1} = 1$

$V_{1,2}$ = ……continuous

6. Mathematical Modeling and Results

MTSF(T_0): The reformative un-failed states to which the organization can transit(original state '0'), before incoming any unsuccessful state are: 'i' = 0,1,2,7attractive 'ξ' = '0'.

$$\text{MTSF } (T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{\text{pr}\left(\xi\xrightarrow{\text{sr (sff)}}i\right)\right\}\mu i}{\Pi_{m_{1\neq\xi}}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[1-\sum_{sr}\left\{\frac{\left\{\text{pr}\left(\xi\xrightarrow{\text{sr (sff)}}\xi\right)\right\}}{\Pi_{m_{2\neq\xi}}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$T_0 = (V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,7}\mu_7)/[1-(0,1,0)-(0,2,0)]$

Availability of the System(A_0): The reformative states at which the organization is accessible are 'j' = 0,1,2,7 and the reformative states are 'i' = 0 to 12 taking 'ξ' = '1'

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\}fj,\mu j}{\Pi_{m_{1\neq\xi}}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{m_{2\neq\xi}}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$A_0 = (V_{1,0}\mu_0+V_{1,1}\mu_1+V_{1,2}\mu_2+V_{1,7}\mu_7)/D_1$

Where $D_1 = (V_{1,i}\mu_i)$

Where i =0 to 12

Busy Period of the Server: The reformative states where attendant j = 1 to 12 and reformative states are 'i' = 0 to 12, attractive ξ = '0', the full fraction of period for which the attendant remains busy is

$$B_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\},nj}{\Pi_{m_{1\neq\xi}}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{m_{2\neq\xi}}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$B_0 = \left[\sum_j V_{\xi,j}, n_j\right] \div \left[\sum_i V_{\xi,i}, \mu_i^1\right]$

$B_0 = (V_{0,i}\mu_i)/D$; i =1 to 12

Where $D = (V_{0,j}\mu_j)$; j =0 to 12

Expected Number of Inspections by the repair man: Reformative states anywhere the overhaul man do this job j = 3 to12 reformative states are i = 0 to 8, Captivating 'ξ' = '0'

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\}}{\Pi_{k_{1\neq\xi}}\{1-V_{\overline{k_1 k_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{k_{2\neq\xi}}\{1-V_{\overline{k_2 k_2}}\}}\right\}\right]$$

$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}, \mu_i^1\right]$

$V_0 = (V_{0,i})/D$; i = 3 to 12

7. PROFIT FUNCTION

Profit Functionis obtained by utilizing the optimization function

$P_0 = C_1A_0 - C_2B_0 - C_3V_0$

C_1= 1000;

C_2= 500;

C_3 = 300

Table 1: Profit Analysis

m \ h	0.80	0.90	1.0
0.10	400.29	412.23	419.79
0.20	350.77	359.91	365.65
0.30	303.36	310.18	325.18

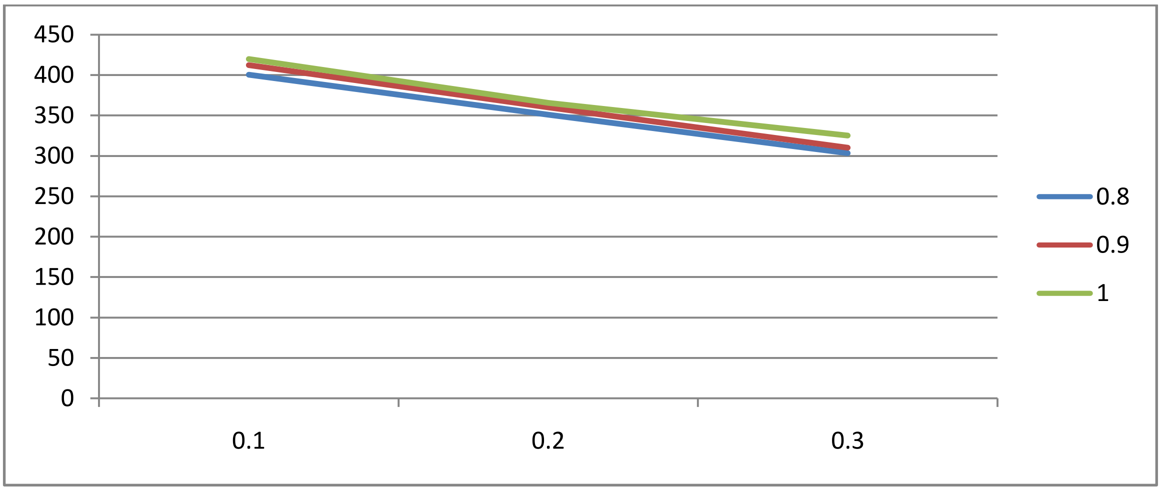

Figure 2: Profit Analysis

8. Conclusion

Obtaining the best values for system parameters requires regulating unit failure and repair rates in relation to financial resources rather than market conditions. We can infer from graph 2 and table 1 that unit failure and repair rates completely determine the profit function of the system. When failure rates are low and maintenance costs are high, profits are at their highest.

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