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Spherical fuzzy programming approach to optimize the transportation problem

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Abstract

In any real-life problem, decision-making plays a very important role. It is always observable that uncertainty, hesitation, vagueness, etc involves in real-life situations. The existence of such factors results in increases the difficulty for the decision maker(s) to decide the accurate/precise, crisp value of parameters involved in the specific problem. In the present study, the uncertainty included in the transportation problem deals with the proposed method based on the derived accuracy function. The function is derived with the centroid method and is further used to convert the fuzzy number into a crisp value in the proposed approach. The applicability and validity are presented with the numerical illustration. The superiority and comparative study are shown by applying it to the real-life transportation problem as a case study of Dinanagar city, India.

Keywords

Fuzzy sets; Spherical fuzzy sets; transportation problem; spherical fuzzy optimization approach

1. Introduction

The transportation problem is initially introduced by the author of [1]. The goal of [2] study is to handle a symmetric trapezoidal fuzzy number-related fuzzy linear programming issue. It is possible to solve fuzzy linear programming problems without first turning them into crisp linear programming problems according to several significant and intriguing findings.The authors of [3] provide a model of estimating uniformity and demonstrate the system's capabilities for monitoring and rescheduling. The resulting method can handle situations requiring poorly defined expertise, provide plans that are roughly consistent, and modify the execution of strategies to account for unexpected events. In [4] authors formulate the problem and utilize triangular intu-

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itionistic fuzzy numbers to deal with uncertainty and hesitation. In [5] authors define a transportation problem in which costs are represented as triangular intuitionistic fuzzy numbers. The ideal solutions to the stated problem are determined using two methods in article [6], namely intuitionistic fuzzy programming and goal programming, and the optimal solutions are then contrasted. In study [7], a multi objective nonlinear transportation problem formulated in terms of fuzzy parameters.The solution method involves neutrosophic compromise programming approach, based on neutrosophic decision set has been investigated which contains the concept of indeterminacy degree along with truthiness membership and falsity membership degree of different goals. The fuzzy programming approach of Zimmermann and the concept of neutrosophic sets serve as inspirations for the development of a new compromise method for the multi-objective transportation problem (MO-TP) in this [8] paper. In the paper [9], authors used Interval-valued spherical fuzzy AHP method has been applied to public transportation problem. The results are demonstrated and analyzed in detail and the step-by-step description of the procedure might foment other applications of the model. The authors of [10] introduced Fuzzy Analytic Hierarchy Process to the model with two extensions which are Intutionistic Fuzzy Sets and Spherical Fuzzy Sets to evaluate the solution set and also provided with a traditional AHP in order to check the robustness of the former methods. In [11] work, authors investigated the solution of the spherical fuzzy transportation problem (SFTP) and presented three different models of the spherical fuzzy transportation problem. In [12] an efficient algorithm is developed by the authors with the help of combination theory and combined fuzzy TOPSIS method to choose the best suitable alternative out of all possible single and hybrid energy resources in Turkey. In [13] the authors proposed an approach to determine the light business jet aircraft would provide long-range, less travel time, cozy seating arrangements, on-board lavatory facility, other aesthetic ambiance (audio systems, light systems and temperature-noise control) and appliances at reasonable flight cost.

2. Preliminaries

2.1. *Fuzzy set*

Definition 2.1. Ordinary Fuzzy set : [14] Let $\mathbf{U}$ be the universe of discourse then a fuzzy set $\widetilde{X_f}$ in $\mathbf{U}$ is defined as follows:

$$\widetilde{X_f} = \{(x, t_{\widetilde{X_f}}(x)) \,|\, x \in \mathbf{U}\}$$

such that $t_{\widetilde{X_f}}(x) : \mathbf{U} \longrightarrow [0, 1]$ is the membership function and $0 \leq t_{\widetilde{X_f}}(x) \leq 1 \;\forall x \in \mathbf{U}$, represents the membership degree of each $x \in \mathbf{U}$ to $\widetilde{X_f}$.

Definition 2.2. Triangular fuzzy number: The ordered triplets $\widetilde{X_f}(t_1, t_2, t_3)$ denoting lower value, middle value & upper value of a m.f, is said to be triangular fuzzy number if its m.f is defined as:

$$t_{\widetilde{X_f}}(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1} & if\ t_1 \leq x \leq t_2 \\ \dfrac{t_3 - x}{t_3 - t_2} & if\ t_2 \leq x \leq t_3 \\ 0 & otherwise \end{cases} \tag{1}$$

Defuzzification of triangular fuzzy number

In literature, there are various methods available to defuzzify the fuzzy number [15]. Among all, the centroid method is most widely used as it gives a deterministic value on the basis of center of gravity of fuzzy numbers. In this article, the same method is used to obtain the defuzzified version of the triangular fuzzy number which is defined as follows:

$$def(\widetilde{X}) = \frac{\int_x x t_{\widetilde{X}}(x)dx}{\int_x t_{\widetilde{X}}(x)dx} \tag{2}$$

where x is the output variable and $t_{\widetilde{X}}(x)$ is the m.f.

Hence, by calculating the integrals of (2), the defuzzified version of the TFN $\widetilde{X}(t_1, t_2, t_3)$ is:

$$def(\widetilde{X}) = \frac{(\frac{t_3 - t_1}{2})(\frac{3t_1 + t_2 + 3t_3}{3})}{(\frac{t_3 - t_1}{2})} = \frac{3t_1 + t_2 + 3t_3}{3} \tag{3}$$

Definition 2.3. Pythagorean fuzzy set: [16] Let $\mathbf{U}$ be the universe of discourse then a fuzzy set $\widetilde{X_p}$ in $\mathbf{U}$ is defined as follows:

$$\widetilde{X_p} = \{(x; t_{\widetilde{X_p}}(x), f_{\widetilde{X_p}}(x)) \,|\, x \in \mathbf{U}\}$$

such that $t_{\widetilde{X_p}}(x) : \mathbf{U} \longrightarrow [0,1]$ and $f_{\widetilde{X_p}}(x) : \mathbf{U} \longrightarrow [0,1]$ are the truthiness m.f and falsity m.f respectively. Also $0 \leq t^2_{\widetilde{X_p}}(x) + f^2_{\widetilde{X_p}}(x) \leq 1\ \forall x \in \mathbf{U}$, represents the membership degree of each $x \in \mathbf{U}$ to $\widetilde{X_p}$.

Definition 2.4. Spherical fuzzy sets: [17] Let $\mathbf{U}$ be the universal set then a fuzzy set $\widetilde{X}$ in $\mathbf{U}$ is defined as follows:

$$\widetilde{X_s} = \{(x; t_{\widetilde{X_s}}(x), i_{\widetilde{X_s}}(x), f_{\widetilde{X_s}}(x)) \,|\, x \in \mathbf{U}\}$$

such that $t_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0,1]$,$i_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0,1]$ and $f_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0,1]$ are the truthiness m.f , indeterminacy m.f and falsity m.f respectively. Also $0 \leq t^2_{\widetilde{X_s}}(x) + i^2_{\widetilde{X_s}}(x) + f^2_{\widetilde{X_s}}(x) \leq 1\ \forall x \in \mathbf{U}$, represents the membership degree for each element $x \in \mathbf{U}$ to $\widetilde{X_s}$.

Definition 2.5. Spherical triangular fuzzy number(STFN) The spherical triangular fuzzy number $\widetilde{X_s} = (t, i, f) = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ s.t $t, i, f \in [01]$

The m.f for t, i, and f can be defined by using (1):

$$t_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1} & if\ t_1 \le x \le t_2 \\ \dfrac{t_3 - x}{t_3 - t_2} & if\ t_2 \le x \le t_3 \\ 0 & otherwise \end{cases} \tag{4}$$

$$i_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x - i_1}{i_2 - i_1} & if\ i_1 \le x \le i_2 \\ \dfrac{i_3 - x}{i_3 - i_2} & if\ i_2 \le x \le i_3 \\ 0 & otherwise \end{cases} \tag{5}$$

$$f_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x - f_1}{f_2 - f_1} & if\ f_1 \le x \le f_2 \\ \dfrac{f_3 - x}{f_3 - f_2} & if\ f_2 \le x \le f_3 \\ 0 & otherwise \end{cases} \tag{6}$$

3. Ranking of STFN

In the literature, the [18] authors proposed ranking functions for ordering the SFNs but the procedure is not clear, so the existing ranking functions are not universal and cannot be used for ordering or defuzzify the SFNs. To overcome this situation, we develop a new score function using the centroid method (2) and used this to develop an algorithm to optimize the transportation problems.

Definition 3.1. Score function & Accuracy function Let $\widetilde{X_s} = (t, i, f) = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ such that $t, i, f \in [01]$ be a STFN. The score functions for the m.f $t_{\widetilde{X_s}}(x), i_{\widetilde{X_s}}(x), \text{and} f_{\widetilde{X_s}}(x)$ are denoted and defined respectively as follows:

$$Sc(t_{\widetilde{X_s}}) = \frac{3t_1 + t_2 + 3t_3}{3}; Sc(i_{\widetilde{X_s}}) = \frac{3i_1 + i_2 + 3i_3}{3}; Sc(f_{\widetilde{X_s}}) = \frac{3f_1 + f_2 + 3f_3}{3} \tag{7}$$

Now, the accuracy function of $\widetilde{X_s}$ is denoted and defined by:

$$\begin{aligned} Acc(\widetilde{X_s}) =& \frac{Sc(t_{\widetilde{X_s}}) + Sc(i_{\widetilde{X_s}}) + Sc(f_{\widetilde{X_s}})}{3} \\ =& \frac{(3t_1 + t_2 + 3t_3) + (3i_1 + i_2 + 3i_3) + (3f_1 + f_2 + 3f_3)}{9} \end{aligned} \tag{8}$$

Example 3.2. Let $\widetilde{X_s} = (2.5, 3, 4.5; 2.4, 3, 4.8; 2.3, 3, 5)$ and $\widetilde{Y_s} = (4.5, 5, 6.3; 4.3, 5, 6.5; 4, 5, 6.7)$ be the two STFNs, then their respective accuracy functions using proposed method (8) are 2.5000 and 3.9222.

Theorem 3.3. *The score functions for truthiness, indeterminacy, & falsity are linear functions and accuracy function is the average of their score functions. The accuracy function is also a linear function.*

Proof. Let $\tilde{X} = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ & $\tilde{Y} = (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)$ are any two STFNs. Then for any scalar a, we have

$$\begin{aligned}
Acc(a\tilde{X} + \tilde{Y}) =& Acc(a(t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3) + (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)) \\
=& Acc((at_1, at_2, at_3; ai_1, ai_2, ai_3; af_1, af_2, af_3) \\
&+ (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)) \\
=& Acc(at_1 + t'_1, at_2 + t'_2, at_3 + t'_3; ai_1 + i'_1, ai_2 + i'_2, ai_3 + i'_3; \\
& af_1 + f'_1, af_2 + f'_2, af_3 + f'_3) \\
=& \{(3(at_1 + t'_1) + (at_2 + t'_2) + 3(at_3 + t'_3)) + (3(ai_1 + i'_1) + (ai_2 + i'_2) + 3(ai_3 \\
&+ (3(af_1 + f'_1) + (af_2 + f'_2) + 3(af_3 + f'_3))\}/9 \\
=& \frac{(3t_1 + t_2 + 3t_3) + (3i_1 + i_2 + 3i_3) + (3f_1 + f_2 + 3f_3)}{9} \\
&+ \frac{(3t'_1 + t'_2 + 3t'_3) + (3i'_1 + i'_2 + 3i'_3) + (3f'_1 + f'_2 + 3f'_3)}{9} \\
=& aAcc(\tilde{X}) + Acc(\tilde{Y})
\end{aligned}$$

Hence, Acc() is linear function.

□

Definition 3.4. Ordering of STFNs using accuracy function
Let $\tilde{X} = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ & $\tilde{Y} = (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)$ are any two STFNs. Then,

(1) If $Acc(\tilde{X}) \geq Acc(\tilde{Y})$ then $\tilde{X} \geq \tilde{Y}$
(2) If $Acc(\tilde{X}) \leq Acc(\tilde{Y})$ then $\tilde{X} \leq \tilde{Y}$
(3) If $Acc(\tilde{X}) = Acc(\tilde{Y})$ then $\tilde{X} = \tilde{Y}$
(4) If $\tilde{X} \geq \tilde{Y}$ then $\max(\tilde{X}, \tilde{Y}) = \tilde{X}$
(5) If $\tilde{X} \leq \tilde{Y}$ then $\min(\tilde{X}, \tilde{Y}) = \tilde{X}$

Definition 3.5. General form of optimization problem The general form of optimization problem(OP) mainly consist of Objective function(s) which has to be optimize(maximize or minimize), constraints related to the problem, and non-negative restrictions in problem which are called as decision variables. The mathematical formulation is as follows:

$$\begin{aligned}
Optimize\ Z =& (Z_1, Z_2, ..., Z_k) \\
\text{subject to} \quad g_j(x) \leq b_j, &\quad j = 1, 2, 3, ..., m \\
x_i \geq 0, &\quad i = 1, 2, 3, ...n
\end{aligned} \tag{9}$$

where $k, m,$ and, n are the number of objectives, constraints, and variables respectively.

4. Spherical fuzzy optimization problem(SFOP)

The extension of (9) by introducing SF concept named as SFOP and can be expressed as:

$$\begin{aligned} Optimize\ \tilde{Z} =& (\tilde{Z}_1, \tilde{Z}_2, ..., \tilde{Z}_k) \\ \text{subject to} \quad & \tilde{g}_j(x) \leq \tilde{b}_j, \quad j = 1, 2, 3, ..., m \\ & x_i \geq 0, \quad i = 1, 2, 3, ...n \end{aligned} \tag{10}$$

where $k, m, \&,\ n$ represents the number of objectives, number of constraints, and number of variables respectively. **Model-I:Partial fuzzified OP-** When one or more but not all parameters involved in optimization problem can be considered as fuzzy. **Model-II:Full fuzzified OP-** When all parameters involved in optimization problem can be considered as fuzzy.

5. Formulation of proposed SFTP

In general TP are concerned with the transporting of goods from different sources to different destinations to obtain the best(optimal) solution of defined goal(s). Consider, there are m sources having a_i where$i = 1, 2, ..., m$ units of goods availability & to be transported between n destinations having b_j where$j = 1, 2, ..., n$ units of goods demand. The unit transportation cost of transporting goods from source i to destination j is c_{ij}. Let x_{ij} as quantity (known as decision variables & $m \times n$ in number) which are to be transported from all the sources to all the destinations in such a way that the total transportation cost is minimum. The mathematical formulation is given as:

$$\begin{aligned} \mathbf{TP_1}: \qquad & MinZ = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1, 2, ..., m \\ & \sum_{i=1}^{m} x_{ij} \leq b_j, \quad j = 1, 2, ..., n \\ & x_{ij} \geq 0 \quad \forall\, i \,\&\, j \end{aligned}$$

Further, $\sum_{i=1}^{m} a_i$ and $\sum_{j=1}^{n} b_j$ are the total availability and total demand, which may be either equal(called as balanced) or not(called as unbalanced). If balanced then it is standard TP otherwise nonstandard. We develop a method for solving standard TP so in case of unbalanced, must be converted to standard TP by using additional dummy source or destination.

In reality, some or all the parameters involved in TPs may be fuzzy in nature due to various uncontrollable factors such as fuel rates, traffic jams, inexactness of supply-demand, poor decision making, fluctuation in market prices, environmental conditions, consumer's behavior etc. In such situations the fuzzy parameters are more reliable to achieve the prescribed objective(s). The fuzzified version of $\mathbf{TP_1}$: can be as follows:

$$\mathbf{TP_2}: \quad MinZ = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq \tilde{a}_i, \quad i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} x_{ij} \leq \tilde{b}_j, \quad j = 1, 2, ..., n$$

$$x_{ij} \geq 0 \quad \forall\, i\, \&\, j$$

Further, $\sum_{i=1}^{m} \tilde{a}_i$ and $\sum_{j=1}^{n} \tilde{b}_j$ are the total availability and total demand which must be as per the standard TP(as discussed above in (TP_1)) The parameters involved in TP_2 are considered as STFNs and can be defuzzified by using equation 8 and TP_2 represented as below:

$$\mathbf{TP_3}: \quad MinZ = \sum_{i=1}^{m}\sum_{j=1}^{n} def(\tilde{c}_{ij})x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq def(\tilde{a}_i), \quad i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} x_{ij} \leq def(\tilde{b}_j), \quad j = 1, 2, ..., n$$

$$x_{ij} \geq 0 \quad \forall\, i\, \&\, j$$

where $def(\tilde{c}_{ij}), def(\tilde{a}_i)$, and $def(\tilde{b}_j)$ are the defuzzified versions of $\tilde{c}_{ij}, \tilde{a}_i, \&\tilde{b}_j$ respectively. Further, $\sum_{i=1}^{m} def(\tilde{a}_i)$ and $\sum_{j=1}^{n} def(\tilde{b}_j)$ are the total availability and total demand which must be as per the standard TP(as discussed above in (TP_1))

Transportation table

Since the TP is a special case of general OP, the application of any optimization method would give an optimal solution. But whenever it is possible to represent the OP in the form of TP, it is simpler to express in the form of transportation table, which displays all the values c_{ij}, a_i, b_j associated with the problem

6. Algorithm of proposed SFTP

The step-wise procedure from problem formulation to final optimal solution is summarized below:
Step 1: Define the problem with the available data and information collected from the decision maker(s) after a through discussion. Formulate the problem in transportation table form for better visualization of numeric data.

Step 2: Check whether the problem is in standard form of TP or not. Two cases arises:
sub-step 2(a): If TP is balanced go to Step 3.
sub-step 2(b): If not balanced then convert it into balanced TP and go to next step
Step 3: Formulate the TP in mathematical form as in TP_1.
Step 4: Convert the given TP into SFTP by converting the uncertain parameters in terns of STFNs with through discussion.
Step 5: Compute the score function values for all the STFN parameters represent in tabular form.
Step 6: Convert the score function values into defuzzified values using proposed accuracy function & express in tabular form and as TP_2. Again check for standard TP(make it standard, in case not).
Step 7: Formulate the crisp version of SFTP in mathematical form as TP_3
Step 8: Solve both given TP and SFTP using any appropriate optimization technique or mathematical software for comparison of optimal solution.

7. Numerical illustration

The following TP is considered in tabular form (shown in table 1):

Table 1. Considered transportation problem

	M_1	M_2	M_3	M_4	Origin
W_1	2	2	2	1	$a(1) = 3$
W_2	10	8	5	4	$a(2) = 5$
W_3	7	6	6	8	$a(3) = 7$
Destination	$b(1) = 4$	$b(2) = 3$	$b(3) = 4$	$b(4) = 4$	

The STFN for the considered numerical example are presented in the table 2

The step-wise procedure & solution of defined transportation problem is explained as follows:
Step 1: The Mathematical model of given TP is represented (using the data available in table 1) below as TP_1:

$$
\begin{aligned}
\mathbf{TP_1}: \quad MinZ &= 2x_{11} + 2x_{12} + 2x_{13} + x_{14} \\
&\quad + 10x_{21} + 8x_{22} + 5x_{23} + 4x_{24} \\
&\quad + 7x_{31} + 6x_{32} + 6x_{233} + 8x_{34} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq 3 \\
& x_{21} + x_{22} + x_{23} + x_{24} \leq 5 \\
& x_{31} + x_{32} + x_{33} + x_{34} \leq 7 \\
& x_{11} + x_{12} + x_{31} \leq 4 \\
& x_{12} + x_{22} + x_{32} \leq 3 \\
& x_{13} + x_{23} + x_{33} \leq 4 \\
& x_{14} + x_{24} + x_{34} \leq 4 \\
& x_{ij} \geq 0 \quad \forall\, i = 1,2,3 \,\&\, j = 1,2,3,4
\end{aligned}
$$

Table 2. STFN for Numerical example

$\tilde{2}$	$W_1 \to M_1$	$(1, 2, 3; 0.5, 2, 2.5; 0.3, 2, 2.8)$
$\tilde{2}$	$W_1 \to M_2$	$(0.8, 2, 1.5; 0.5, 2, 2.5; 0.3, 2, 3)$
$\tilde{2}$	$W_1 \to M_3$	$(1, 2, 2.5; 0.7, 2, 2.8; 0.5, 2, 3)$
$\tilde{1}$	$W_1 \to M_4$	$(0.6, 1, 1.4; 0.5, 1, 1.5; 0.3, 1, 1.7)$
$\tilde{10}$	$W_2 \to M_1$	$(8, 10, 11; 7.5, 10, 11.5; 7, 10, 12)$
$\tilde{8}$	$W_2 \to M_2$	$(7, 8, 9; 6.5, 8, 8.2; 6, 8, 8.5)$
$\tilde{5}$	$W_2 \to M_3$	$(4, 5, 5.5; 3.8, 5, 5.8; 3.5, 5, 6)$
$\tilde{4}$	$W_2 \to M_4$	$(3, 4, 5; 2.7, 4, 5.3; 2.5, 4, 5.5)$
$\tilde{7}$	$W_3 \to M_1$	$(6.5, 7, 8; 6, 7, 8.2; 5.8, 7, 8.5)$
$\tilde{6}$	$W_3 \to M_2$	$(5.5, 6, 7; 5.1, 6, 7.2; 4.8, 6, 7.6)$
$\tilde{6}$	$W_3 \to M_3$	$(5.6, 6, 6.3; 5.4, 6, 6.6; 5, 6, 7)$
$\tilde{8}$	$W_3 \to M_4$	$(7.5, 8, 9; 7, 8, 9.3; 6.8, 8, 9.5)$
$\tilde{3}$	$a(1)$	$(2.5, 3, 4.5; 2.4, 3, 4.8; 2.3, 3, 5)$
$\tilde{5}$	$a(2)$	$(4.5, 5, 6.3; 4.3, 5, 6.5; 4, 5, 6.7)$
$\tilde{7}$	$a(3)$	$(6.8, 7, 7.2; 6.5, 7, 7.5; 6.6, 7, 8.7)$
$\tilde{4}$	$b(1)$	$(3.8, 4, 6; 4.5, 4, 6.2; 5.5, 4, 6.5)$
$\tilde{3}$	$b(2)$	$(2.8, 3, 3.2; 2.7, 3, 4; 2.5, 3, 4.4)$
$\tilde{4}$	$b(3)$	$(2.7, 4, 5; 2.8, 5, 5.3; 2.5, 4, 5.5)$
$\tilde{4}$	$b(4)$	$(3.2, 4, 4.5; 3, 4, 4.8; 2.7, 4, 5)$

Step 2: The formulation of given TP_1 in terms of STFN using table 2 as follows:

$$
\begin{aligned}
\mathbf{TP_2}: \quad MinZ = {} & (1, 2, 3; 0.5, 2, 2.5; 0.3, 2, 2.8)x_{11} + (0.8, 2, 1.5; 0.5, 2, 2.5; 0.3, 2, 3)x_{12} \\
& + (1, 2, 2.5; 0.7, 2, 2.8; 0.5, 2, 3)x_{13} + (0.6, 1, 1.4; 0.5, 1, 1.5; 0.3, 1, 1.7)x_{14} \\
& + (8, 10, 11; 7.5, 10, 11.5; 7, 10, 12)x_{21} + (7, 8, 9; 6.5, 8, 8.2; 6, 8, 8.5)x_{22} \\
& + (4, 5, 5.5; 3.8, 5, 5.8; 3.5, 5, 6)x_{23} + (3, 4, 5; 2.7, 4, 5.3; 2.5, 4, 5.5)x_{24} \\
& + (6.5, 7, 8; 6, 7, 8.2; 5.8, 7, 8.5)x_{31} + (5.5, 6, 7; 5.1, 6, 7.2; 4.8, 6, 7.6)x_{32} \\
& + (5.6, 6, 6.3; 5.4, 6, 6.6; 5, 6, 7)x_{33} + (7.5, 8, 9; 7, 8, 9.3; 6.8, 8, 9.5)x_{34} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq (2.5, 3, 4.5; 2.4, 3, 4.8; 2.3, 3, 5) \\
& x_{21} + x_{22} + x_{23} + x_{24} \leq (4.5, 5, 6.3; 4.3, 5, 6.5; 4, 5, 6.7) \\
& x_{31} + x_{32} + x_{33} + x_{34} \leq (6.8, 7, 7.2; 6.5, 7, 7.5; 6.6, 7, 8.7) \\
& x_{11} + x_{12} + x_{31} \leq (3.8, 4, 6; 4.5, 4, 6.2; 5.5, 4, 6.5) \\
& x_{12} + x_{22} + x_{32} \leq (2.8, 3, 3.2; 2.7, 3, 4; 2.5, 3, 4.4) \\
& x_{13} + x_{23} + x_{33} \leq (2.7, 4, 5; 2.8, 5, 5.3; 2.5, 4, 5.5) \\
& x_{14} + x_{24} + x_{34} \leq (3.2, 4, 4.5; 3, 4, 4.8; 2.7, 4, 5) \\
& x_{ij} \geq 0 \quad \forall\, i = 1, 2, 3 \,\&\, j = 1, 2, 3, 4
\end{aligned}
$$

Step 3: For the Defuzzification of STFNs, using score function values table 3, the TP

Table 3. Score function values of STFNs

	M_1	M_2	M_3	M_4	$a(i)$
W_1	(4.0000,3.0000,3.1000)	(2.3000,3.0000,3.3000)	(3.5000,3.5000,3.5000)	(1.6667,1.6667,1.6667)	(7.3333,7.5333,7.6333)
W_2	(21.6667,21.6667,21.6667)	(18.0000,16.7000,16.5000)	(10.5000,10.6000,10.5000)	(8.6667,8.6667,8.6667)	(11.8000,11.8000,11.7000)
W_3	(16.1667,15.8667,15.9667)	(13.8333,13.6333,13.7333)	(13.2333,13.3333,13.3333)	(18.5000,18.3000,18.3000)	(15.6667,15.6667,16.9667)
$b(j)$	(10.4667,11.3667,12.6667)	(6.3333,7.0333,7.2333)	(8.3667,8.7667,8.6667)	(8.3667,8.4667,8.3667)	

Table 4. Accuracy function values of score function values: Defuzzification of STFNs

	M_1	M_2	M_3	M_4	$a(i)$
W_1	1.1222	0.9556	1.1667	0.5556	2.5000
W_2	7.2222	5.6889	3.5111	2.8889	3.9222
W_3	5.3333	4.5778	4.4333	6.1222	5.3667
$b(j)$	3.8333	2.2889	2.8667	2.8000	

is as follows:

$$\begin{aligned}
MinZ = {} & (4.0000, 3.0000, 3.1000)x_{11} + (2.3000, 3.0000, 3.3000)x_{12} \\
& + (3.5000, 3.5000, 3.5000)x_{13} + (1.6667, 1.6667, 1.6667)x_{14} \\
& + (21.6667, 21.6667, 21.6667)x_{21} + (18.0000, 16.7000, 16.5000)x_{22} \\
& + (10.5000, 10.6000, 10.5000)x_{23} + (8.6667, 8.6667, 8.6667)x_{24} \\
& + (16.1667, 15.8667, 15.9667)x_{31} + (13.8333, 13.6333, 13.7333)x_{32} \\
& + (13.2333, 13.3333, 13.3333)x_{33} + (18.5000, 18.3000, 18.3000)x_{34} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq (7.3333, 7.5333, 7.6333) \\
& x_{21} + x_{22} + x_{23} + x_{24} \leq (11.8000, 11.8000, 11.7000) \\
& x_{31} + x_{32} + x_{33} + x_{34} \leq (15.6667, 15.6667, 16.9667) \\
& x_{11} + x_{12} + x_{31} \leq (10.4667, 11.3667, 12.6667) \\
& x_{12} + x_{22} + x_{32} \leq (6.3333, 7.0333, 7.2333) \\
& x_{13} + x_{23} + x_{33} \leq (8.3667, 8.7667, 8.6667) \\
& x_{14} + x_{24} + x_{34} \leq (8.3667, 8.4667, 8.3667) \\
& x_{ij} \geq 0 \quad \forall\, i = 1, 2, 3 \,\&\, j = 1, 2, 3, 4
\end{aligned}$$

Step 4:After checking the condition of standard TP in accuracy function values table 4. The balanced sum of last most column and lower most row of this table is 11.7889.
Step 5: the final representation of given TP_2 in crisp form is as below:

$$\begin{aligned}
\mathbf{TP_3}: \quad MinZ = {} & 1.1222x_{11} + 0.9556x_{12} + 1.1667x_{13} + 0.5556x_{14} \\
& + 7.2222x_{21} + 5.6889x_{22} + 3.5111x_{23} + 2.8889x_{24} \\
& + 5.3333x_{31} + 4.5778x_{32} + 4.4333x_{33} + 6.1222x_{34} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq 2.5000 \\
& x_{21} + x_{22} + x_{23} + x_{24} \leq 3.9222 \\
& x_{31} + x_{32} + x_{33} + x_{34} \leq 5.3667 \\
& x_{11} + x_{12} + x_{31} \leq 3.8333 \\
& x_{12} + x_{22} + x_{32} \leq 2.2889 \\
& x_{13} + x_{23} + x_{33} \leq 2.8667 \\
& x_{14} + x_{24} + x_{34} \leq 2.8000 \\
& x_{ij} \geq 0 \quad \forall\, i = 1, 2, 3 \,\&\, j = 1, 2, 3, 4
\end{aligned}$$

Step 6: Solve the original TP using an appropriate method or optimizing s/w packages

for optimal solution which is shown in table 5
Step 7: Similarly, Solve the crisp version of SFTP using same method or optimizing s/w packages for optimal solution which is shown in table 5

Table 5. Optimal solutions obtained by optimizing method with online Matlab

Solu.	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	Min Z
TP	0	0	3	0	4	1	0	0	0	2	1	4	104
SFTP	0	0	2.5	0	3.8333	0.0889	0	0	0	2.2	0.3667	2.8	59.9464

8. Application

The data related to TP is represented in tabular form in table 6 collected from Municipal corporation of Dinanagar city, India which takes the responsibility of waste management in the city. There are three waste generation points denoted as W_1, W_2, W_3 (waste considered to be segregated at source level) from where the recoverable material is transfer to three MRF(Material Recovery Facility) stations denoted as M_1, M_2, M_3. Due to several uncertainties like variations in fuel rates, traffic jams, weather etc. the transporter is not sure about the transportation cost involved here. Also due to some unavoidable factors like uncertainty in amount of waste generation, waste characteristics as per the season etc., transportation cost involved is unpredictable. According to the previous experience of Decision maker(s) the transportation costs are estimated as STFNs shown in table 8 after a thorough discussion. The MC of Dinanagar city requires optimal solution so the TP cost is minimum. All the parameters are spherical triangular fuzzy numbers and converted into crisp version by applying the proposed accuracy function. Formulate the given TP in terms of crisp values of STFNs i.e; $TP_3$and solve with the help of suitable method of optimization. The mathematical formulation of SFTP after Defuzzification is as follows:

Table 6. Application proposed method on transportation problem: Dinanagar city, India

	M_1	M_2	M_3	M_4	Generated waste
W_1	16	20	12	0	$a(1) = 200$
W_2	14	8	18	0	$a(2) = 160$
W_3	26	24	16	0	$a(3) = 90$
MRF capacity	$b(1) = 180$	$b(2) = 120$	$b(3) = 140$	$b(4) = 10$	

Table 7. STFNs for the application of TP

	t	i	f
		M_1	
W_1	(15,16,16.5)	(14.8,16,16.7)	(14.5,16,17)
W_2	(13.5,14,15)	(13.3,14,15.2)	(13,14,15)
W_3	(25.4,26,26.5)	(25.2,26,26.8)	(25.2,26,27)
		M_2	
W_1	(19,20,21)	(18.8,20,21.2)	(18.6,20,21.5)
W_2	(7,8,9)	(6.6,8,8.2)	(6,8,8.5)
W_3	(23.8,24,24.4)	(23.5,24,24.6)	(23.2,24,24.8)
		M_3	
W_1	(11,12,11.5)	(10.7,12,12.3)	(10.5,12,3)
W_2	(17.5,18,18.6)	(17.3,18,18.8)	(17,18,19)
W_3	(15.8,16,16.4)	(15.5,16,16.8)	(15.2,16,17)
		M_4	
W_1	0	0	0
W_2	0	0	0
W_3	0	0	0
		$a(i)$	
W_1	(198,200,205)	(195,200,210)	(193,200,215)
W_2	(155,160,162.75)	(152,160,164.35)	(150,160,165.5)
W_3	(75,90,90.5)	(75.7,90,90.7)	(80,90,100)
		$b(j)$	
M_1	(170,180,190)	(167,180,193)	(165,180,183)
M_2	(110,120,125)	(108,120,126)	(105,120,128)
M_3	(138,140,145)	(135,140,148)	(132,140,150)
M_4	(9.5,10,11)	(9.3,10,11.2)	(9,10,11.5)

$$\begin{aligned}
\mathbf{TP_3}: \quad & MinZ = 12.0556x_{11} + 15.3444x_{12} + 7.7778x_{13} - 0.2222x_{14} \\
& \quad + 10.7778x_{21} + 5.6889x_{22} + 13.8000x_{23} - 0.2222x_{24} \\
& \quad + 20.0111x_{31} + 18.4778x_{32} + 12.3000x_{33} - 0.2222x_{34} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq 157.111 \\
& x_{21} + x_{22} + x_{23} + x_{24} \leq 123.0667 \\
& x_{31} + x_{32} + x_{33} + x_{34} \leq 66.6556 \\
& x_{11} + x_{12} + x_{31} \leq 138.4444 \\
& x_{12} + x_{22} + x_{32} \leq 91.1111 \\
& x_{13} + x_{23} + x_{33} \leq 109.5556 \\
& x_{14} + x_{24} + x_{34} \leq 7.7222 \\
& x_{ij} \geq 0 \quad \forall\, i = 1, 2, 3 \,\&\, j = 1, 2, 3, 4
\end{aligned}$$

The optimal solution with optimal transportation cost are obtained by proposed method with the help of online Matlab. Thus, the optimal solution and spherical fuzzy transportation cost are: $x_{11} = 66.0000, x_{12} = 91.1111, x_{21} = 5.7888, x_{23} = 109.5556, x_{31} = 66.6556$, rest of the decision variables are zero and $MinZ_{sftp} = 5101.8$. Solution of TP without applying the proposed method is $x_{11} = 170, x_{12} = 30, x_{21} = 10, x_{23} = 140, x_{32} = 90$, rest of the decision variables are zero and $MinZ_{tp} = 8140$. Clearly, $MinZ_{sftp} < MinZ_{tp}$. Hence, the resulting solution using proposed approach is better than the existing methods.

9. Comparative study

The transportation problem involves uncertain parameters has been solved by proposed accuracy function derived with centroid method. The applied approach is based on the spherical fuzzy numbers which comprise with membership functions: truthiness m.f , indeterminacy m.f and falsity m.f. These m.f increases the flexibility for decision making by decision makers. In order to prove the efficiency of proposed method one numerical example is explained step-wise and the same is applied to a real life problem related to TP exists in Dinanagar city India. The obtained optimal solutions are presented in 5 and application is discussed in section 8. The values of objective function of given transportation problem and spherical fuzzy transportation problem are 104 and 59.9464 respectively. Similarly, objective function values of real life application are 8140 and 5101.8 corresponding to given TP and SFTP. Hence, it is concluded that by applying the proposed accuracy function and method, the obtained objective values are optimal than the existing methods applied on original data. This confirms the superiority of SFTP.

10. Conclusions

In this research article, SFTP is proposed. Accuracy function is derived from the centroid method and is used to defuzzify the parameters defined in terms of fuzzy number. The method is applicable to solve any type of TP having partial or fully fuzzified parameters In future research, we are working on finding the solution of multi objective problem using SF optimization technique. The compromised solution will be obtained by hybridizing the SFOP with Teaching Learning Based Optimization technique.

11. Compliance with Ethical standards

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