Fuzzy Filters of a Po Ternary Γ- Semi Groups

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Abstract - We developed the notions of fuzzy filters, PO left (right) filter, fuzzy filters generated by a fuzzy subset and showed that C is a nonempty PO left filter iff h_C, the characteristic function is FLF and the disjoint collection of family of fuzzy filters is also fuzzy filter. We also showed if h is a FLF, then $h' = (1-h)$ is a FPRI, any FF h is a fuzzy n-system and the FF generated by h is the collection of FFs containing h.

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1. INTRODUCTION

The idea of fuzzy subset of a set had established by L. Zadeh [9]. X.Y.Xie was established the fuzzy theory of semi groups [12]. The ideals concept in semi groups had established by Anjaneyulu. A [1].

J. M. Pradeep, A. Gangadhara Rao, P. Ramyalatha, C. Srimannarayana [4] was established the fuzzy filters of a PO ternary semi group. We develop the concept of fuzzy filters of a PO ternary Γ - semi groups. We denote completely prime fuzzy right ideal as CPFRI, fuzzy prime right ideal as FPRI, fuzzy semi prime ideal as FSPI and fuzzy prime ideal as FPI throughout the paper.

2. PREREQUISITES

Definition 2.1: "A semi group T has an ordered relation $\leq$ is known as PO ternary Γ - semi group if T is a POSET such that $q \leq r \Rightarrow q\gamma q_1\delta q_2 \leq r\gamma q_1\delta q_2$, $q_1\gamma q\delta q_2 \leq q_1\gamma r\delta q_2$, $q_1\gamma q_2\delta q \leq q_1\gamma q_2\delta r \ \forall \ q, r, q_1, q_2 \in \mathrm{T}$".

Note 2.2: We consider T as PO Ternary Γ - semi group.

Definition 2.3: "Let $\phi \neq \mathrm{C} \subseteq \mathrm{T}$. the characteristic mapping $h_{\mathrm{C}} : \mathrm{T} \to [0,1]$ is defined as $h_{\mathrm{C}}(t) = \begin{cases} 1 & \text{if } t \in \mathrm{C} \\ 0 & \text{if } t \notin \mathrm{C} \end{cases}$. then h_{C}, is a Fuzzy Subset (FS)".

Definition 2.4: "A mapping $h : \mathrm{T} \to [0,1]$ is said to be a FS of T when T itself is a FS of T $\ni \mathrm{T}(t) = 1\ \forall t \in \mathrm{T}$ and it denotes T or 1".

Definition 2.5: Suppose u, v, w is FS. For each $t \in \mathrm{T}$, the product $u\Gamma v\Gamma w$ as defined by

$$(u\Gamma v\Gamma w)(t) = \begin{cases} \underset{t \le q\gamma r\delta s}{\vee} u(q) \wedge v(r) \wedge w(s) & \text{if } t \le q\gamma r\delta s \text{ exists} \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.6: "Let h be FS of T is known as fuzzy sub semi group if $(i)\, q \le r$ then $h(q) \ge h(r)$ (ii) $h(q\gamma r\delta s) \ge h(q) \wedge h(r) \wedge h(s), \forall q, r, s \in \mathrm{T}, \gamma, \delta \in \Gamma$".

Definition 2.7:"A FS h of T is known as fuzzy PO left ideal if $(i)\, q \le r$ then $h(q) \ge h(r)$ (ii) $h(q\gamma r\delta s) \ge h(s), \forall q, r, s \in \mathrm{T}, \gamma, \delta \in \Gamma$".

Definition 2.8:"A FS h of T is known as fuzzy PO right ideal if $(i)\, q \le r$ then $h(q) \ge h(r)$ (ii) $h(q\gamma r\delta s) \ge h(q), \forall q, r, s \in \mathrm{T}, \gamma, \delta \in \Gamma$".

Definition 2.9: A FS h of T is known as fuzzy ideal (FI) if $(i)\, q \le r$ then $h(q) \ge h(r)$ (ii) $h(q\gamma r\delta s) \ge h(s)$, $h(q\gamma r\delta s) \ge h(q)$, $h(q\gamma r\delta s) \ge h(r)$.

Definition 2.10:"A FI h is known as completely prime fuzzy ideal (CPFI) if for each three OFPs $q_l, r_m, s_n \in T$, $\forall\, l, m, n \in (0,1]$ such that $q_l \Gamma r_m \Gamma s_n \subseteq h$ then $q_l \subseteq h$ *or* $r_m \subseteq h$ *or* $s_n \subseteq h$"

Definition 2.11:"Let h be a FS of T, h is termed as a fuzzy d-system if $z_r \subseteq h \Rightarrow z_r{}^n \subseteq h$ for all $n \in N$, n is odd natural number".

Definition2.12:"A FS h of T is known as fuzzy m-system provided if $h(q) \ge m_1,\ h(r) \ge m_2\ h(s) \ge m_3 \Rightarrow \exists l, m, n \in T \ni h(l) \ge m_1 \vee m_2 \vee m_3$ and $l \le q\gamma m\delta r\mu n\rho s$".

Definition2.13:"Let h be a FS of T, h is termed as fuzzy n-system provided if $h(r) > t \Rightarrow \exists q \in T, s \in T \ni h(q) > t$ *and* $q \le r\,\Gamma\, s\,\Gamma\, r$".

3. FUZZY FILTERS

Definition 3.1: Let H be a sub semi group of T, H is known as PO left filter if

1) $q, r, s \in T, q\,\Gamma\, r\,\Gamma\, s \in H \Rightarrow q \in H$ 2) $q, r \in T, q \le r$ *and* $q \in H \Rightarrow r \in H$.

Definition 3.2: A fuzzy sub semi group h is known as fuzzy left filter (FLF) 1) $q \le r \Rightarrow h(q) \le h(r)$ 2) $h(q\,\Gamma\, r\,\Gamma\, s) \le h(s), \forall\, q, r, s \in T$.

Note 3.3: In a similar way, we can define fuzzy right filter (FRF).

Theorem 3.4: Let $\varnothing \neq C \subseteq T$. Then C is PO left filter $\Leftrightarrow$ h_C, the characteristic function is FLF.

Corollary 3.5: The disjoint collection of any two FLFs in T is a FLF.

Proof: Suppose *h, g* is 2 FLFs. Take $q \le r$

Consider

$$(h \cap g)(q) = h(q) \wedge g(q) \le h(r) \wedge g(r) = (h \cap g)(r) \Rightarrow (h \cap g)(q) \le (h \cap g)(r)$$

Then $(h\cap g)(q\Gamma r\Gamma s)=h(q\Gamma r\Gamma s)\wedge g(q\Gamma r\Gamma s)\leq h(s)\wedge g(s)=(h\cap g)(s)$.

Hence $h\cap g$ is a FLF.

Theorem 3.6: The disjoint collection of family of FLFs is also FLF.

Proof: Consider $\{h_\alpha\}_{\alpha\in\Delta}$ is a collection of a family of FLFs.

Take $H=\bigcap_{\alpha\in\Delta} h_\alpha = h_1\cap h_2\cap---$.

Let $q,r,s\in T$ such that $q\leq r$

Consider $H(q)=\bigcap_{\alpha\in\Delta} h_\alpha(q)=h_1(q)\wedge h_2(q)\wedge h_3(q)---$

$\leq h_1(r)\wedge h_2(r)\wedge h_3(r)---$

$=\bigcap_{\alpha\in\Delta} h_\alpha(r)=H(r)\Rightarrow H(q)\leq H(r)$

Consider $H(q\Gamma r\Gamma s)=\bigcap_{\alpha\in\Delta} h_\alpha(q\Gamma r\Gamma s)=h_1(q\Gamma r\Gamma s)\wedge h_2(q\Gamma r\Gamma s)\wedge h_3(q\Gamma r\Gamma s)\wedge---$

$\leq h_1(s)\wedge h_2(s)\wedge h_3(s)---$

$=\bigcap_{\alpha\in\Delta} h_\alpha(s)=H(s)$

$\Rightarrow H(q\,\Gamma\, r\,\Gamma\, s)\leq H(s).$

Hence H is a FLF.

Theorem 3.7: A fuzzy sub semi group *h* is FLF $\Leftrightarrow h'=(1-h)$ is CPFRI.

Proof: Consider *h* is FLF.

Let $q,r,s\in T \ni q\leq r \;\Rightarrow h(q)\leq h(r)\Rightarrow h'(q)\geq h'(r)$

Consider $h'(q\,\Gamma\, r\,\Gamma\, s)=1-h(q\,\Gamma\, r\,\Gamma\, s)\geq 1-h(q)=h'(q) \Rightarrow h'(q\,\Gamma\, r\,\Gamma\, s)\geq h'(q)$

$\Rightarrow h'$ is a fuzzy right ideal.

Assume $q_{l,}r_{m,}s_n$ are three OFPs such that $l,m,n\in(0,1]$

Consider $q_l\Gamma r_m\Gamma s_n \subseteq h'$. Let $q_l \not\subseteq h', r_m \not\subseteq h'$ *and* $s_n \not\subseteq h'$

$\Rightarrow q_l \supset 1-h\ , r_m \supset 1-h\ \textit{and}\ s_n \supset 1-h$

$\Rightarrow 1-q_l \subseteq h\ , 1-r_m \subseteq h\ \textit{and}\ 1-s_n \subseteq h$

$\Rightarrow (1-q_l)\vee(1-r_m)\vee(1-s_n)\subseteq h \Rightarrow 1-(q_l\wedge r_m\wedge s_n)\subseteq h$

But $(q_l\Gamma r_m\Gamma s_n)\subseteq h'=1-h\Rightarrow 1-(q_l\Gamma r_m\Gamma s_n)\supset h$

$\Rightarrow h\subset 1-(q_l\Gamma r_m\Gamma s_n)\subseteq 1-(q_l\wedge r_m\wedge s_n)$, a contradiction.

$\therefore \textit{either}\ q_l\subseteq h'\ \textit{or}\ r_m\subseteq h'\ \textit{or}\ s_n\subseteq h'$

Therefore h' is a CPFRI.

Conversely, Assume h' is a CPFRI.

Let $q\leq r\ \textit{then}\ h'(q)\geq h'(r)\Rightarrow h(q)\leq h(s)$ Since $h'(q\Gamma r\Gamma s)\geq h'(q)\Rightarrow h(q\Gamma r\Gamma s)\leq h(q)$

Therefore *h* is a FLF.

Corollary 3.8: Let *h* is a FLF. Then $h'=(1-h)$ is a FPRI if $h'\neq\emptyset$.

Proof: From the above, h' is CPFRI.

Every CPFI is FPI.

Hence if *h* is FLF then h' is FPRI.

Definition 3.9: A fuzzy ternary sub semi group h of T is known as a fuzzy filter (FF) if $(i)\, q \le r \Rightarrow h(q) \le h(r),\ (ii)\, h(q\,\Gamma\, r\,\Gamma\, s) \le h(q) \wedge h(r) \wedge h(s), \forall\, q, r, s \in T.$

Lemma 3.10: Let $\varnothing \ne C \subseteq T.$ Then C is PO filter $\Leftrightarrow$ h_C, characteristic function is FF.

Definition 3.11: A FF h is known as proper FF if $h \ne T$.

Theorem 3.12: The disjoint collection of any two FFs is also FF.

Proof: Suppose h, g are any two FFs.

Take $q \le r$

Consider

$(h \cap g)(q) = h(q) \wedge g(q) \le h(r) \wedge g(r) = (h \cap g)(r) \Rightarrow (h \cap g)(q) \le (h \cap g)(r)$

Then $(h \cap g)(q\Gamma r\Gamma s) = h(q\Gamma r\Gamma s) \wedge g(q\Gamma r\Gamma s)$

$\le h(q) \wedge h(r) \wedge h(s) \wedge g(q) \wedge g(r) \wedge g(s)$

$\le h(q) \wedge g(q) \wedge h(r) \wedge g(r) \wedge h(s) \wedge g(s)$

$\le (h \cap g)(q) \wedge (h \cap g)(r) \wedge (h \cap g)(s)$

Hence $h \cap g$ is a FF.

Theorem 3.13: The disjoint collection of a family of FFs is also a FF.

Proof: Suppose $\{h_\alpha\}_{\alpha \in \Delta}$ is a collection of FFs.

Take $H = \bigcap_{\alpha \in \Delta} h_\alpha = h_1 \cap h_2 \cap ---$

Let $q, r, s \in T$ such that $q \le r$

Consider $H(q) = \bigcap_{\alpha \in \Delta} h_\alpha(q) = h_1(q) \wedge h_2(q) \wedge h_3(q) \wedge ---$

$\le h_1(r) \wedge h_2(r) \wedge h_3(r) \wedge ---$

$= \bigcap_{\alpha \in \Delta} h_\alpha(r) = H(r) \Rightarrow H(q) \le H(r)$

Consider $H(q\Gamma r\Gamma s) = \bigcap_{\alpha \in \Delta} h_\alpha(q\Gamma r\Gamma s) = h_1(q\Gamma r\Gamma s) \wedge h_2(q\Gamma r\Gamma s) \wedge h_3(q\Gamma r\Gamma s) \wedge ---$

$\le h_1(q) \wedge h_1(r) \wedge h_2(q) \wedge h_2(r) \wedge h_3(q) \wedge h_3(r) \wedge h_1(s) \wedge h_2(s) \wedge h_3(s) ---$

$\le (h_1(q) \wedge h_2(q) \wedge h_3(q) \wedge ---) \wedge (h_1(r) \wedge h_2(r) \wedge h_3(r) \wedge ---) \wedge (h_1(s) \wedge h_2(s) \wedge h_3(s) \wedge ---)$

$= \bigcap_{\alpha \in \Delta} h_\alpha(q) \wedge \bigcap_{\alpha \in \Delta} h_\alpha(r) \wedge \bigcap_{\alpha \in \Delta} h_\alpha(s) = H(q) \wedge H(r) \wedge H(s)$

$\Rightarrow H(q\,\Gamma\, r\,\Gamma\, s) \le H(q) \wedge H(r) \wedge H(s).$

Hence the disjoint collection of FFs is FF.

Corollary 3.14: A fuzzy sub semi group h is a FF $\Leftrightarrow$ $h' = 1 - h$ is a CPFI.

Corollary 3.15: If h is a FF then $h' = 1 - h$ is a FPI.

Lemma 3.16: Let T be commutative. A FS h is a filter $\Leftrightarrow$ $h' = 1 - h$ is a FPI.

Corollary 3.17: Any FF h is a fuzzy m-system.

Lemma 3.18: Any FF h is a fuzzy d-system.

Corollary 3.19: If h is FF then $h' = 1 - h$ is a FSPI.

Theorem 3.20: Any FF h is a fuzzy n-system.

Proof: Let h be a FF. By above corollary, h' is FSPI.

Hence, $h'' = h$ is fuzzy n-system.

Definition 3.21: Let h be a FS. The smallest FF containing h is called FF generated by h. It is denoted by $\langle h \rangle$.

Theorem 3.22: The FF generated by *h* is the collection of FFs containing *h*.

Proof: Consider Δ is collection of FFs containing *h*.

We have T itself is a FF containing *h*, $T \in \Delta$, $\Delta \neq \varnothing$

Let $H^* = \bigcap_{f \in \Delta} f$, here f is FF containing *h*.

Since $h \subseteq f, \forall f \in \Delta \Rightarrow h \subseteq H^* \Rightarrow H^* \neq \varnothing$

By Theorem 3.13, H^* is the FF.

Take any other FF J containing *h*.

If $h \subseteq J$, J is the FF $\Rightarrow J \in \Delta \Rightarrow H^* \subseteq J$.

Hence H^* is the smallest FF containing *h*.

Therefore H^* is FF generated by *h*.

4. CONCLUSION

We investigate fuzzy left(right) filters, fuzzy filters of a PO ternary Γ- semi groups and develop some theorems. In a similar way, we can develop more results in near future.

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