$\widehat{D}_S$-Closed Sets in Topological Spaces

S.Sumithra Devi[1] and L.Meenakshi Sundaram[2]

[1]Research Scholar(Reg.No:20212232092008),PG and Research DepartmentofMathematics,V.O Chidambaram College,Tuticorin-628 008. Affliated by ManonmaniamSundaranar University, Tamil Nadu, India.

Mail Id: ssumithradevi27@gmail.com

[2]Assistant Professor,PG and Research Department of Mathematics,V.O Chidambaram College,Tuticorin-628 008. Affliated by ManonmaniamSundaranar University, Tamil Nadu, India.

Mail Id: lmsundar79@gmail.com

Article Info
Page Number: 10442-10449
Publication Issue:
Vol. 71 No. 4 (2022)

Abstract:
In this paper, we introduce a new classes of sets called $\widehat{D}_S$-Closed Sets in Topological spaces and study some basic properties of $\widehat{D}_S$-Closed Sets.

Article History
Article Received: 12 October 2022
Revised: 24 November 2022
Accepted: 18 December 2022

1.Introduction:

A topological space's basic elements are closed sets. The axioms for closed sets or the Kuratowski closure axioms, for instance, can be used to determine the topology on a set. 1970 saw the start of N. Levine's [10] investigation on so-called generalised closed sets. When cl(A)⊂ U whenever A⊂ U and U is open, a subset A of a topological space X is said to be generalised closed. Since generalised closed sets are not just straightforward extensions of closed sets, several topologists have focused a lot of their recent research on this idea. Furthermore, they provide a number of novel topological space features.In this study, we provide new classes of sets for topological spaces known as $\widehat{D}_S$ -closed sets.

2. Preliminaries:

Definition 2.1. Let(X,)beatopologicalspace.AsubsetAofthespaceXissaidtobe

1. Preopen [11] if A ⊆ int(cl(A)) and preclosed if cl(int(A))⊆A.
2. Semi-open [9] if A ⊆ cl(int(A)) and semi-closed if int(cl(A)) ⊆A.
3. α-open [13] if A⊆int(cl(int(A))) and α-closed if cl(int(cl(A)))⊆A.
4. Semipreopen [1] if A ⊆ cl(int(cl(A))) and semi preclosed if int(cl(int(A)))⊆A.

5. Regular-open [16] if A=int(cl(A)) and regular closed if A=cl(int(A)).

Lemma 2.2([1])**.**For any subset A of X, the following relations hold.

1. Scl(A)=A∪int(cl(A)).
2. αcl(A)=A∪cl(int(cl(A))).
3. Pcl(A)=A∪cl(int(A)).
4. Spcl(A)=A∪int(cl(int(A))).

Definition 2.3.A subset A of a space (X,τ) is called a

1. Generalized closed (g-closed) [10]if cl(A)⊆U whenever A⊆U and U is open in (X,τ).
2. generalizedpre-closed(gp-closed)[14]ifpcl(A)⊆UwheneverA⊆UandU isopenin(X,τ).
3. generalizedsemipre-closed(gsp-closed)[6]ifspcl(A)⊆Uwhen-everA⊆UandUisopenin(X,τ)
4. generalized preregular closed (gprclosed) [7] if pcl(A)⊆U whenever A⊆U and U is regular open in (X,τ).
5. pre-generalizedclosed(pg-closed)[12]ifpcl(A)⊆UwheneverA⊆UandUispre-openin(X,τ).
6. g^*-preclosed(g^*p-closed)[19]ifpcl(A)⊆UwheneverA⊆UandUisg-openin(X,τ).
7. μ-preclosed(μp-closed)[20]ifpcl(A)⊆UwheneverA⊆UandUisgα*-openin(X,τ).
8. *g-closed[18]ifcl(A)⊆UwheneverA⊆UandUisω-openin(X,τ).
9. $\tilde{g}$-closed[8]ifcl(A)⊆UwheneverA⊆UandUis#gs-openin (X,τ).
10. ρ-closed[5]ifpcl(A)⊆int(U)wheneverA⊆UandUis$\tilde{g}$-open in(X,τ).
11. ω-closed[17]ifcl(A)⊆UwheneverA⊆UandUissemi-openin(X,τ).
12. πgp-closed[15]ifpcl(A)⊆UwheneverA⊆UandUisπ-openin(X,τ).
13. $\hat{\eta}^*$-closed[2]ifspcl(A)⊆UwheneverA⊆UandUisanω-open in(X,τ)
14. D-closed[3]ifpcl(A)⊆int(U)wheneverA⊆UandUisω -open in(X,τ).
15. $\widehat{D}$-closed[4]ifs pcl(A)⊆int(U)wheneverA⊆UandUisD -open in(X,τ)

The complement of g-closed (resp.gp-closed,gsp-closed,gpr-closed,pg-closed,g^*p-closed,gα*-closed,μp-closed,presemi-closed,*g-closed,#gs-closed,$\tilde{g}$-closed,ρ-closed,ω-closed,$\hat{g}$-closed,πgp-closed,$\hat{\eta}^*$-closed) setissaidtobeag-open(resp.gp-open,gsp-open,gpr-open,pg-open g^*p-open,gα*-open,μp-open,presemi-open,*g-open,#gs-open,open,ρ-open,ω-open,$\hat{g}$-open,πgp-open,$\hat{\eta}^*$-open)set.

3.Basic properties of $\widehat{D}_S$

Definition 3.1.

A subset A of (X; τ) is called an $\widehat{D}_S$ -closed setif scl(A) ⊂ U whenever A ⊂ U and U is $\widehat{D}$- open in (X; τ).The class of all $\widehat{D}_S$-closed sets in (X; τ) is denoted by $\widehat{D}_Sc(\tau)$.

That is $\widehat{D}_Sc(\tau) = \{A \subset X : A$ is $\widehat{D}_S$-closed in $(X; \tau)\}$.

Theorem 3.2.

Every closed(resp. α-closed, semi-closed) set is $\widehat{D}_S$-closed.

Proof.

Let A be any closed set .Let A ⊂ U and U is $\widehat{D}$-open set in X. Then cl(A) ⊂ U. But scl(A) ⊂cl(A ⊂U. Thus A is $\widehat{D}_S$ -closed. The proof follows from the facts that scl(A) ⊂αcl(A)⊂ cl(A).

Remark 3.3.

The converse of the above theorem need not be true as seen from the following example.

Example 3.4.

Let X = {a;b;c;d} and τ = { φ;{a};{a;d};{b;c};{a;b;c};X }. Then the set A = {b;d} is $\widehat{D}_S$ -closed but not closed(resp. α-closed, semi-closed) .

Theorem 3.5.

Every $\widehat{D}_S$ -closed set is gsp-closed but not conversely.

Proof:

Let A be $\widehat{D}_S$ -closed set. Let A ⊂ U and U be any open set . Since every open set is $\widehat{D}$- open and A is $\widehat{D}_S$ –closed , scl(A) ⊂U . Hence spcl(A) ⊂ U, which implies A is gsp -closed.

Theorem 3.6.

Every $\widehat{D}_S$ -closed set is $\hat{\eta}^*$-closed but not conversely.

Proof:

Let A be $\widehat{D}_S$ -closed set. Let A ⊂ U and U be any ω-open set . Since every ω-open set is $\widehat{D}$-open and A is $\widehat{D}_S$ –closed , scl(A) ⊂U . Hence spcl(A) ⊂ U, which implies A is $\hat{\eta}^*$ -closed.

Example 3.7.

Let X = {a;b;c;d} and τ = { φ;{c};{a;b};{a;b;c};X }.Then the set A = {a} is both gsp-closed and $\hat{\eta}^*$-closed but not $\widehat{D}_S$ -closed .

Remark 3.8 . $\widehat{D}_S$ -closedness and pre-closedness are independent concepts as we illustrate by means of the following examples.

Example 3.9.

Let X = {a;b;c;d} and τ = { φ;{c};{a;b};{a;b;c};X }. Then the set A = {a;b} is $\widehat{D}_S$ -closed but not preclosed . A= {a} is preclosed but not $\widehat{D}_S$ -closed in (X, τ).

Remark 3.10. $\widehat{D}_S$ -closedness and g,gp,gpr ,pg,*g,g̃,ω, g*p, μp , ρ, D πgp,gα*-closedness are independent concepts as we illustrate by means of the following examples.

Example 3.11.

Let X = {a;b;c;d} and τ = { φ;{c};{a;b};{a;b;c};X }. Then the set A = {a;b} is $\widehat{D}_S$ -closed but not g,gp,gpr ,pg,*g,g̃,ω, g*p, μp , ρ, D, πgp,gα*-closed .

A= {a;d} is g,gp,gpr ,pg,*g,g̃,ω, g*p, μp , ρ, D, πgp, gα*-closed but not $\widehat{D}_S$ -closed in (X, τ).

Remark 3.12. From the above discussions and known results should be accompanied by a reference we have the following implications A → B (A ↮ B) represents A implies B but not conversely (A and B are independent of each other). See Figure 1.

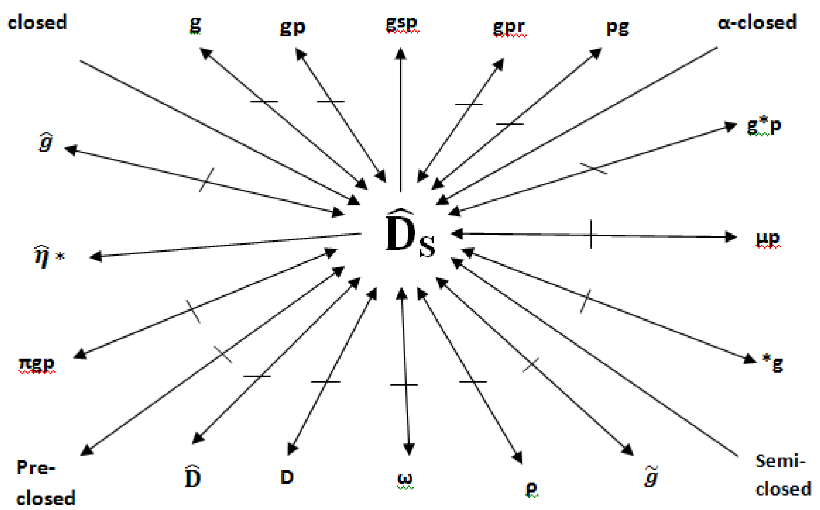

Figure 1: Implications.

Remark 3.13. The union of two $\widehat{D}_S$ -closed sets need not be $\widehat{D}_S$ –closed and the intersection of two $\widehat{D}_S$ -closed sets need not be $\widehat{D}_S$ -closed.

Example 3.14.

Let X = {a, b, c, d} and τ = {φ, {c}, {a, b}, {a, b, c}, X}. Then the set A = {c} and B = {a,b}.

Here A and B are $\widehat{D}_S$ -closed sets. But A∪B = {a, b, c} is not $\widehat{D}_S$ -closed.

Example 3.15.

Let X = {a, b, c, d} and τ = {φ, {a}, {a,d}, {b, c},{a,b,c}, X}. Let A = {b, c} and B = {b, d}. Here A and B are $\widehat{D}_S$ -closed sets. But A ∩ B = {b} is not $\widehat{D}_S$ -closed.

Definition 3.16. The intersection of all $\widehat{D}$-open subsets of (X,τ) containing A is called $\widehat{D}$-kernel of A and denoted by $\widehat{D}$−ker(A).

Theorem 3.17.

If a subset A of (X, τ) is $\widehat{D}_S$-closed then scl(A)⊆$\widehat{D}$-ker(A).

Proof.

Suppose that A is $\widehat{D}_S$ -closed. Then scl(A) ⊆U whenever A ⊆U and U is $\widehat{D}$-open. Let x ∈scl(A). Suppose x ∉ $\widehat{D}$− ker(A).Then there is an $\widehat{D}$-open set U containing A such that x ∉U. Since U is an $\widehat{D}$-open set containing A, x ∉scl(A), which is a contradiction. Hence scl(A) ⊆$\widehat{D}$-ker(A). Conversely , suppose scl(A) ⊆$\widehat{D}$-ker(A). Then scl(A) ⊆∩U_i where U_i is a $\widehat{D}$- open set containing A . Therefore scl(A) ⊆U_i whenever A⊆U and U is $\widehat{D}$- open. Hence A is $\widehat{D}_S$ –closed .

Theorem 3.18.

A set A is $\widehat{D}_S$ -closed then scl(A) − A contains nonon-empty closed set.

Proof.

Let F ⊆scl(A) − A be a non-empty closed set. Then F ⊆ scl(A) and A ⊆X. Since X − F is $\widehat{D}$-open, we get scl(A) ⊆ X − F. Hence F ⊆X − scl(A). Therefore F ⊆scl(A) ∩ (X − scl(A)) = φ, which is a contradiction.

Remark 3.19. The converse of the above theorem need not be true as seen from the following example:

Example 3.20.

Let X = {a, b, c, d} and τ = { φ, {c}, {a, b}, {a, b, c},X}. Let A = {a}. Then scl(A) − A = {b}

contains no non-empty closed set. But A is not $\widehat{D}_S$ -closed.

Theorem 3.21.

A set A is $\widehat{D}_S$ -closed then scl(A) − A contains no non-empty $\widehat{D}$-closed set.

Proof.

It follows from theorem 3.18.

Theorem 3.22.

Let A and B be any two subsets of (X, τ). If A is $\widehat{D}_S$ -closed such that A⊆B⊆ scl(A) then B is $\widehat{D}_S$ -closed.

Proof.

Let U be an $\widehat{D}$-open set of X and B ⊆U. Then A ⊆U. Since A is $\widehat{D}_S$ -closed, scl(A) ⊆U. Now scl(B) ⊆scl(scl(A)) = scl(A) ⊆U. Hence B is $\widehat{D}_S$ –closed.

Example 3.23.

Let X = {a, b, c, d} and τ = { φ, {c}, {a, b}, {a, b, c},X}. Let A = {d} and B={c,d}. Then

A and B are $\widehat{D}_S$ –closed sets but B is not a subset of scl(A).

Theorem 3.24.

If a subset A of (X, τ) is $\widehat{D}$-open and $\widehat{D}_S$ -closed then A is semi-closed in (X, τ).

Proof.

Since a subset A of (X, τ) is $\widehat{D}$-open and $\widehat{D}_S$ -closed, we get, scl(A) ⊆A. But A ⊆scl(A). Hence A is semi-closed in (X, τ).

Theorem 3.25.

A open set A of (X, τ) is g-closed then A is $\widehat{D}_S$ -closed in (X, τ).

Proof.

Let A ⊆ U and U be $\widehat{D}$-open in (X, τ). Since A is open and g-closed we get A is closed. (i.e) cl(A) ⊆ U. We know that scl(A) ⊆ cl(A)⊆ U. Hence A is $\widehat{D}_S$ -closed.

Remark 3.26.

The converse of the above theorem need not be true as seen from the following example:

Example 3.27.

Let X = {a, b, c, d} and τ = { φ, {c}, {a, b}, {a, b, c},X}. Let A = {c}. Then A is open and $\widehat{D}_S$ –closed but not g- closed.

Theorem 3.28.

Let A be $\widehat{D}_S$ -closed in (X, τ). Then A is semi-closed if and only if scl(A) − A is $\widehat{D}$-closed.

Proof.

Let A be semi-closed. Then scl(A) = A. Hence scl(A)−A = φ, which is $\widehat{D}$-closed. Conversely, suppose scl(A)−A is $\widehat{D}$-closed. Since A is $\widehat{D}_S$ -closed and by theorem 3.21, scl(A)−A = φ. Then scl(A) = A. Hence A is semiclosed.

Theorem 3.29.

In a topological space (X, τ), for each x ∈X, {x} is $\widehat{D}$-closed or its complement X − {x} is $\widehat{D}_S$ -closed in (X, τ).

Proof.

Suppose that {x} is not $\widehat{D}$-closed in (X, τ). Then X − {x} is not $\widehat{D}$-open. Hence the only $\widehat{D}$-open set containing X − {x} is X. Thus scl(X − {x}) ⊆X. Hence X−{x} is $\widehat{D}_S$ -closed in (X, τ).

References

1. D.Andrijevic. Semi- preopen sets, Mat. Vesnik, 38 (1), 24-32. 1986.
2. Antony Rex Rodrigo.J, Some Characterizations of ˆη∗-closed setsand ˆη∗-continuous maps in topological and bitopologicalspaces,Ph. D., Thesis, Alagappa University, Karaikudi (2007).
3. J.Antony Rex Rodrigo and K.Dass, A New type of generalized closed sets, IJMA, 3(4)(2012), 1517-1523.
4. K. Dass and G. Suresh,$\widehat{D}$-Closed sets in Topological Spaces, Malaya Journal of Matematik, Vol. S, No. 1, 265-269, 2021
5. Devamanoharan.C, Pious Missier.S and Jafari.S, ρ-closed sets intopological spaces accepted for publication in the European Journal of Pure and Applied Mathematics, ISSN 1307-5543.
6. J. Dontchev. On generalizing semi-preopen sets, Mem.Fac.sci. Kochi Univ.Ser.A.Maths 16, 35-48. 1995.
7. Y. Gnanambal. Generalized Pre-regular closed sets in topological spaces, Indian J. PureAppl. Maths., 28 (3), 351-360. 1997.
8. S. Jafari, T. Noiri, N. Rajesh and M.L. Thivagar. Another generalization of closed sets, KochiJ.Math, 3, 25-38. 2008.
9. N. Levine. Semi-open sets, semi-continuity in topological spaces, Amer Math, Monthly, 70, 36-41. 1963.
10. N. Levine. Generalized closed sets in topology, Rend circ. Math Palermo, 19 (2). 89-96. 1970.
11. A.S. Mashour, M.E. Abd El- Monsef and S.N. El-Deep. On Precontinuous and weak precontinuous mappings, Proc, Math, Phys. Soc. Egypt., 53, 47-53. 1982.
12. H. Maki, J. Umehara and T. Noiri. Every topological space in Pre- T1/2 ,Mem.Fac.Sci, KochiUnivSer.A.Maths. (17). 33-42. 1996.
13. O. Njastad. On some classes of nearly open sets, Pacific J. Math. 15, 961-970. 1965.
14. T.Noiri, H.Maki and J.Umehara. Generalized preclosed functions, Mem.Fac.Sci. Kochi.Univ.SerA.Maths., 19.13-20. 1998.
15. J.H. Park. On πgp-closed sets in topological spaces, Indian J.Pure Appl. Math (to ap-pear).
16. M.H.Stone. Application of the Theory Boolean rings to general topology,Trans.Amer.Math.Soc.,41, 375-381. 1937.
17. Sundaram.P and Sheik John.M, On ω-closed sets in topology,ActaCienc. Indica Math. 4 (2000), 389-392.
18. Veerakumar.M.K.R.S, Beetween g*-closed and g-closed sets, Antarctica J. Maths 3(1)(2006) 43–65.
19. M.K.R.S Veerakumar. g*-preclosedsets,ActaCienciaIndica(mathematics) Meerut,

XXVIII(M) (1). 51-60. 2002.

20. Veerakumar.M.K.R.S, µp-closed sets in topological spaces, Antarctica.J.Mat 2(1),(2005) 31-52.
21. Ravi, R Senthil Kumar, A Hamari Choudhi, Weakly ⊐ g-closed sets, BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE, 4, Vol. 4(2014), 1-9
22. Ravi, R Senthil Kumar, Mildly Ig-closed sets, Journal of New Results in Science, Vol3,Issue 5 (2014) page 37-47
23. Ravi, A senthil kumar R & Hamari CHOUDHİ, Decompositions of Ï g-Continuity via Idealization, Journal of New Results in Science, Vol 7, Issue 3 (2014), Page 72-80.
24. Ravi, A Pandi, R Senthil Kumar, A Muthulakshmi, Some decompositions of πg-continuity, International Journal of Mathematics and its Application, Vol 3 Issue 1 (2015) Page 149-154.
25. S. Tharmar and R. Senthil Kumar, Soft Locally Closed Sets in Soft Ideal Topological Spaces, Vol 10, issue XXIV(2016) Page No (1593-1600).
26. S. Velammal B.K.K. Priyatharsini, R.SENTHIL KUMAR, New footprints of bondage number of connected unicyclic and line graphs, Asia Liofe SciencesVol 26 issue 2 (2017) Page 321-326
27. K. Prabhavathi, R. Senthilkumar, P.Arul pandy, m-$I_{\pi g}$-Closed Sets and m-$I_{\pi g}$-Continuity, Journal of Advanced Research in Dynamical and Control Systems Vol 10 issue 4 (2018) Page no 112-118
28. K. Prabhavathi, R. Senthilkumar, I. Athal, M. Karthivel, A Note on Iβ * g Closed Sets, Journal of Advanced Research in Dynamical and Control Systems11(4 Special Issue), pp. 2495-2502.
29. K PRABHAVATHI, K NIRMALA, R SENTHIL KUMAR, WEAKLY (1, 2)-CG-CLOSED SETS IN BIOTOPOLOGICAL SPACES, Advances in Mathematics: Scientific Journal vol 9 Issue 11(2020) Page 9341-9344