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Radio Geometric Mean Graceful Labeling on Degree Splitting of Cycle Related Graphs

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Abstract

A Radio geometric mean graceful labeling of a connected graph G is a bijection μ from the vertex set V(G) to {1,2,3,…,|V(G)|} such that for any two distinct vertices u and v of G, $d(u,v) + \lceil \sqrt{\mu(u)\mu(v)} \rceil \geq 1 + diam(G)$. A graph which admits radio geometric mean graceful labeling is called radio geometric mean graceful graph. In this paper, we introduced the radio geometric mean graceful labeling on degree splitting of graphs.

Keywords: labeling, radio geometric mean graceful labeling, degree splitting graph.

I. Introduction

Graphs described here is simple, undirected and connected graphs. Let V(G) and E(G) denotes the vertex set and edge set of a graph G respectively. A graph labeling is an assignment of integers to the vertices or edges both based on certain conditions. The concept of radio labeling was introduced by Chartrand et al[1] in 2001. S. Somasundaram and R.Ponraj introduced the notion of mean labeling of graphs [4]. R. Ponraj et al. introduced the concept of Radio mean labeling of graphs [5]. The notion of degree splitting of graphs was introduced by R.Ponraj and S. Somasundaram [7].C. David Raj, A. Subramanian and

K. Sunitha determined the radio mean labeling of double triangular snake graph and quadrilateral snake graph along with some path and cycle related graphs [6,9]. C. David Raj, M. Deva Saroja and Brindha Mary V.T introduced the radio mean labeling on splitting anddegree splitting graphs [11]. C. David Raj and Brindha Mary.V.T determined radio mean graceful labeling on degree splitting of cycle related graphs [8] and even radio mean graceful labeling on degree splitting of snake related graphs [10]. S. Somasundaram, P.Vidhyarani and R. Ponraj introducedthe concept of geometric mean labeling of graphs [12]. The concept of geometric mean labeling on degree splitting graphs was found by S. Somasundaram et al. [13]. Radio geometric graceful labeling was introduced by K.N. Meera [14]. We refer Gallian for more comprehensive survey [2]. We follow Harary [3] for some standard words, expressions and symbols. The notations DS(G) is the degree splitting of G, $d(u,v)$ is the distance between the vertices u and v and diam(G) is the diameter of G.

Definition 1: A walk of a graph is an alternating sequence of vertices and edges $v_0x_1, v_1x_2, \dots, v_{n-1}, x_n v_n$beginning and ending with vertices such that each edge x_i incident with v_{i-1} and v_i.A walk with distinct vertices is called path. A path with n vertices is denoted byP_n.

Definition 2: The distance d(u,v) from a vertex u to a vertex v in a connected graph G is the minimum of the length of the u-v paths in G

Definition 3: The eccentricity, e(V), of a vertex v in a connected graph G is the distance between v and a vertex farthest from v in G.

Definition 4: The diameter, diam(G), of G is the greatest eccentricity among the vertices of G.

Definition 5: The sum of two graphs G and H, denoted by G+H, is the graph obtained by taking disjoint copies of G and H and then adding every edge xy, where x is a vertex in G and y is a vertex in H.

Definition 6: A closed path is called a cycle of G. A cycle on n vertices is denoted by C_n.

Definition 7: An alternate triangular cycle $A(C_{2n})$ is obtained from an even cycle C_{2n} with vertex set $\{x_1, y_1, x_2, y_2, \dots, x_n, y_n\}$ by joining x_i and y_i to a new vertex z_i, $1 \leq i \leq n$ i.e. every alternate edge of a cycle is replaced by C_3.

Definition 8: Let G=(V, E) be a graph with V = $S_1 \cup S_2 \cup \dots, S_i \cup T$, where each S_i is a set of vertices having at least two vertices and having the same degree and T= $V - \cup S_i$.The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices$w_1, w_2, \dots, w_i$ and joining w_i to each vertex of $S_i (1 \leq i \leq t)$.

Definition 9: A Radio geometric mean graceful labeling of a connected graph G is a bijection μ from the vertex set V(G) to {1,2,3,…,|V(G)|} such that for any two distinct vertices u and v of G, $d(u,v) + \lceil \sqrt{\mu(u)\mu(v)} \rceil \geq 1 + diam(G)$. A graph which admits radio geometric mean graceful labeling is called radio geometric mean graceful graph.

II. Main Result

Theorem 2.1: $DS(C_n)$, $n \geq 3$ is a radio geometric mean graceful graph.

Proof:

Let $x_1, x_2, x_3, \dots\dots x_n$ be the vertices of cycle C_n. Introduce a new vertex x and join it with the vertices of C_n to obtain $DS(C_n)$ whose vertex set is V= $\{x_i \ / \ 1 \leq i \leq n\} \cup \{x\}$.

Clearly the $diam(DS(C_n))$=$\begin{Bmatrix} 1 \ if n = 3 \\ 2 \ if n > 3 \end{Bmatrix}$.

Define a bijection $\mu: V(DS(C_n)) \to \{1,2,3,\dots.|V(DS(C_n))|\}$ by

$\mu(x_i) = i; \ 1 \leq i \leq n;$

$\mu(x) = n + 1.$

To check the radio geometric mean graceful condition for μ.

Case 1: n=3

Subcase (i): Examine the pair (x_i, x_j), $1 \leq i \leq n-1, i+1 \leq j \leq n$;

$$d(x_i, x_j) + \lceil \sqrt{\mu(x_i)\mu(x_j)} \rceil \geq 1 + \lceil \sqrt{(i)(j)} \rceil \geq 2 = 1 + diam(\mathrm{DS}(C_n)).$$

Subcase (ii): Examine the pair (x_i, x), $1 \leq i \leq n$;

$$d(x_i, x) + \lceil \sqrt{\mu(x_i)\mu(x)} \rceil \geq 1 + \lceil \sqrt{(i)(n+1)} \rceil \geq 2.$$

Case 2: $n > 3$

Subcase (i): Examine the pair (x_i, x_j), $1 \leq i \leq n-1, i+1 \leq j \leq n$;

$$d(x_i, x_j) + \lceil \sqrt{\mu(x_i)\mu(x_j)} \rceil = 1 + \lceil \sqrt{(i)(j)} \rceil \geq 3 = 1 + diam(\mathrm{DS}((C_n)).$$

Subcase (ii): Examine the pair $(x_i, x), 1 \leq i \leq n$;

$d(x_i, x) + \lceil\sqrt{\mu(x_i)\mu(x)}\rceil = 1 + \lceil\sqrt{(i)(n+1)}\rceil \geq 3.$

Thus all the pair of vertices satisfies the radio geometric mean graceful condition.

Hence $DS(C_n)$ is a radio geometric mean graceful graph.

Example 2.2:

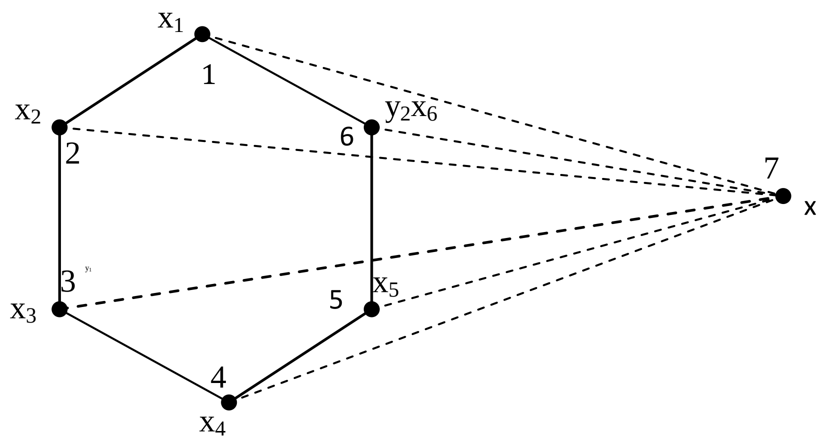

Fig 1. Radio Geometric mean graceful labeling of $DS(C_6)$.

Theorem 2.3: $DS(C_n + K_1), n \geq 3$ is a radio geometric mean graceful graph.

Proof:

Let $x_1, x_2, x_3, \ldots\ldots x_n$ be the vertices of cycle C_n. Let x be the vertex of K_1. Joining each vertexof the cycle C_n with the vertex x. The resulting graph is $C_n + K_1$. Introduce a new vertex y and join it with the vertices of $C_n + K_1$ graph of degree three. The new graph so obtained is DS($C_n + K_1$) whose vertex set is V= $\{x, x_i \ / \ 1 \leq i \leq n\} \cup \{y\}$.

The $diam(DS(C_n + K_1))$ = $\begin{Bmatrix} 1 \ if n = 3 \\ 2 \ if n \geq 4 \end{Bmatrix}$.

Define a bijection $\mu: V\big(DS(C_n + K_1)\big) \to \big\{1,2,3, \ldots. \big|V\big(DS(C_n + K_1)\big)\big|\big\}$ by

$\mu(x_i) = i \, , 1 \leq i \leq n;$

$\mu(x) = n + 1;$

$\mu(y) = n + 2$.

To check the radio geometric mean graceful condition for μ.

Case 1: $n = 3$

Subcase (i): Examine the pair (x_i, x_j), $1 \leq i \leq n-1, i+1 \leq j \leq n$;

$d(x_i, x_j) + \left\lceil\sqrt{\mu(x_i)\mu(x_j)}\right\rceil \geq 1 + \left\lceil\sqrt{(i)(j)}\right\rceil \geq 2 = 1 + diam(\mathrm{DS}(C_n + K_1))$.

Subcase (ii): Examine the pair (x_i, x), $1 \leq i \leq n$;

$d(x_i, x) + \left\lceil\sqrt{\mu(x_i)\mu(x)}\right\rceil \geq 1 + \left\lceil\sqrt{(i)(n+1)}\right\rceil \geq 2.$

Subcase (iii): Examine the pair (x_i, y), $1 \leq i \leq n$;

$d(x_i, y) + \left\lceil\sqrt{\mu(x_i)\mu(y)}\right\rceil \geq 1 + \left\lceil\sqrt{(i)(n+2)}\right\rceil \geq 2.$

Subcase (iv): Examine the pair (x, y);

$d(x, y) + \left\lceil\sqrt{\mu(x)\mu(y)}\right\rceil \geq 2 + \left\lceil\sqrt{(n+1)(n+2)}\right\rceil \geq 2.$

Case 2: $n \geq 4$

Subcase (i): Examine the pair (x_i, x_j), $1 \leq i \leq n-1, i+1 \leq j \leq n$;

$d(x_i, x_j) + \left\lceil\sqrt{\mu(x_i)\mu(x_j)}\right\rceil \geq 1 + \left\lceil\sqrt{(i)(j)}\right\rceil \geq 3 = 1 + diam(\mathrm{DS}(C_n + K_1))$.

Subcase (ii): Examine the pair (x_i, x), $1 \leq i \leq n$;

$d(x_i, x) + \left\lceil\sqrt{\mu(x_i)\mu(x)}\right\rceil \geq 1 + \left\lceil\sqrt{(i)(n+1)}\right\rceil \geq 3.$

Subcase (iii): Examine the pair (x_i, y), $1 \leq i \leq n$;

$d(x_i, y) + \left\lceil\sqrt{\mu(x_i)\mu(y)}\right\rceil \geq 1 + \left\lceil\sqrt{(i)(n+2)}\right\rceil \geq 3.$

Subcase (iv): Examine the pair (x, y);

$d(x, y) + \left\lceil\sqrt{\mu(x)\mu(y)}\right\rceil \geq 2 + \left\lceil\sqrt{(n+1)(n+2)}\right\rceil \geq 3.$

Thus all the pair of vertices satisfies the radio geometric mean graceful condition.

Hence $DS(C_n + K_1)$ is a radio geometric mean graceful graph.

Example 2.4:

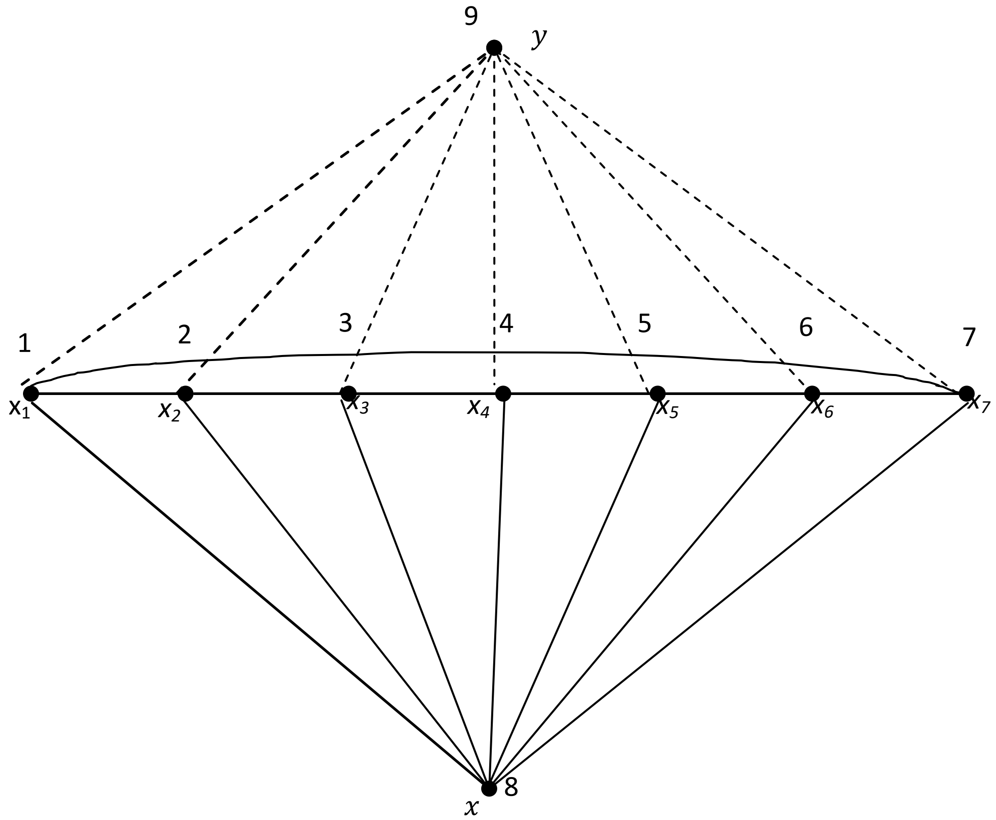

Fig 2. Radio Geometric Mean graceful labeling of $DS(C_n + k_1)$.

Theorem 2.5: DS $(A(C_{2n})), n \geq 3$is a radio geometric mean graceful graph.

Proof:

Let $x_1, x_2, x_3, \dots, x_{2n}$ be the vertices of even cycle C_{2n} . Joinx_{2i-1} ,x_{2i} with y_i,$1 \leq i \leq n$ to form $A(C_{2n})$. Introduce two new vertices $x\ and\ y$ and join them with the vertices of $A(C_{2n})$of degree three and two respectively. Then the resultant graph is

$DS(A(C_{2n}))$whose vertex set is V= $\{x_i, 1 \leq i \leq 2n,\ y_i, 1 \leq i \leq n\} \cup \{x, y\}$.

Clearly the $diam(DS(A(C_{2n}))) = 3$.

Define a bijection $\mu: V(DS(A(C_{2n}))) \rightarrow \{1,2,3, \dots . |V(DS(A(C_{\ 2n})))|\}$ by

$\mu(x_i) = n + i + 1\ ;\ 1 \leq i \leq 2n$;

$\mu(y_i) = i$;$1 \leq i \leq n$;

$\mu(x) = n + 1;$

$\mu(y) = 3n + 2.$

Now we verify the radio geometric mean graceful condition for μ.

Case (i): Verify the pair (x_i, x_j) , $1 \leq i \leq n-1,\ i+1 \leq j \leq 2n$;

$$d(x_i, x_j) + \left\lceil \sqrt{\mu(x_i)\mu(x_j)} \right\rceil \geq 1 + \left\lceil \sqrt{(n+i+1)(n+j+1)} \right\rceil \geq 4$$

$= 1 + diam(\mathrm{DS}(A(C_{2n})))$.

Case (ii): Verify the pair (x_i, y_j), $1 \leq i \leq 2n, 1 \leq j \leq n$;

$$d(x_i, y_j) + \left\lceil \sqrt{\mu(x_i)\mu(y_j)} \right\rceil \geq 1 + \left\lceil \sqrt{(n+i+1)(j)} \right\rceil \geq 4.$$

Case (iii): Verify the pair (x_i, x), $1 \leq i \leq n$;

$$d(x_i, x) + \left\lceil \sqrt{\mu(x_i)\mu(x)} \right\rceil \geq 1 + \left\lceil \sqrt{(n+i+1)(n+1)} \right\rceil \geq 4.$$

Case (iv): Verify the pair (x_i, y), $1 \leq i \leq 2n$;

$$d(x_i, y) + \left\lceil \sqrt{\mu(x_i)\mu(y)} \right\rceil \geq 1 + \left\lceil \sqrt{(n+i+1)(3n+2)} \right\rceil \geq 4.$$

Case (v): Verify the pair (y_i, y_j), $1 \leq i \leq 2n,\ i+1 \leq j \leq n$;

$$d(y_i, y_j) + \left\lceil \sqrt{\mu(y_i)\mu(y_j)} \right\rceil \geq 2 + \left\lceil \sqrt{(i)(i+1)} \right\rceil \geq 4.$$

Case (vi): Verify the pair (y_i, x) , $1 \leq i \leq n$;

$$d(y_i, x) + \left\lceil \sqrt{\mu(y_i)\mu(x)} \right\rceil \geq 2 + \left\lceil \sqrt{(i)(n+1)} \right\rceil \geq 4.$$

Case (vii): Verify the pair (y_i, y), $1 \leq i \leq n$;

$$d(y_i, y) + \left\lceil \sqrt{\mu(y_i)\mu(y)} \right\rceil \geq 2 + \left\lceil \sqrt{(i)(3n+2)} \right\rceil \geq 4.$$

Case (viii): Verify the pair (x, y);

$$d(x, y) + \left\lceil \sqrt{\mu(x)\mu(y)} \right\rceil \geq 3 + \left\lceil \sqrt{(n+1)(3n+2)} \right\rceil \geq 4.$$

Thus all the pair of vertices satisfies the radio geometric mean graceful condition.

Hence DS $A((C_{2n}))$ is a radio geometric mean graceful graph.

Example 2.6:

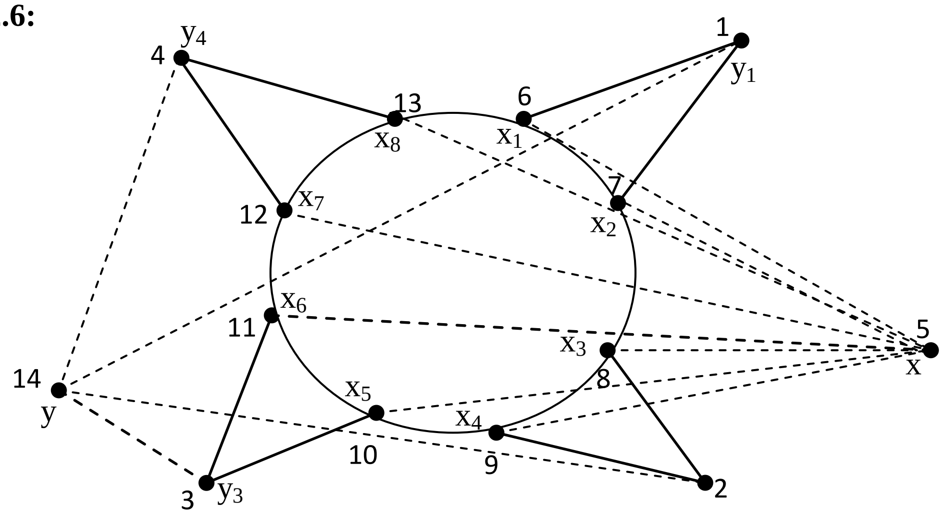

Fig 3. Radio Geometric Mean graceful Labeling of $DS(C_{2(4)})$.

3.Conclusion

The labeling of graphis an interesting and vast research area which is very useful and it is extended in various topics by several people. Radio geometric mean graceful labeling is discussed in this paper and some of the results are obtained. More results will be done in the further research article. This paper will help the beginners in research.

REFERENCES

[1] Gary Chartrand, David Erwin, P. Zhang and Harary. P,Radio labeling of graphs, Bulletin of the Institute of Combinatorics and its Applications.33 (2001), 77-85.

[2] J.A.Gallian, "A dynamic survey of graph labeling", The Electronic Journal of Combinatorics (2019),DS6.

[3] F. Harary, Graph Theory, Narosa publication House Reading, New Delhi, (1988).

[4] S. Somasundaram, R.Ponraj, " Mean labeling of Graphs", NationalAcademy of Science Letters, 26(2003),210-213.

[5] R.Ponraj, S. Sathish Narayanan and R.Kala, Radio Mean labeling of a graph, AKCE International Journal of graphs and combinatorics 12 (2015), 224-228.

[6] C. David Raj, A. Subramanian and K. Sunitha, Radio mean labeling of Double triangular snake graph and quadrilateral snake graph, International Journal of Mathematical Archive-8(11),2017, 109-115, ISSN 2229-5046.

[7]R.Ponraj and S. Somasundaram, On the degree splitting graph of a graph, NATL.ACAD.SCI. LETT: 27(7-8) (2004), 275-278.

[8] C. David Raj and Brindha Mary V.T, Radio Mean Graceful Labeling on degree splitting of cycle related Graphs, Advances and applications in Mathematical Sciences, Vol. 21, issue 6, App 22, pp: 3057-3064.

[9] K. Sunitha, C. David Raj, and A. Subramanian, Radio mean labeling of path and cycle related graphs, Global Journal of Mathematical Sciences: Theory and Practical, Vol.9, number 3(2017, pp.337-345, International Research publishing house.

[10] Brindha Mary V.T, C. David Raj and C. Jayasekaran, Even Radio Mean Graceful labeling on degree splitting of snake related graphs, South East Asian J. of Mathematics and Mathematical Sciences, Vol. 18, No.2(2022),pp. 197-204

[11] C. David Raj, M. Deva Saroja and Brindha Mary V.T, Radio Mean Labeling on degree splitting of Graphs,The International Journal of analytical and experimental modal analysis, Vol. XII, issue II, Feb(2020) , pp: 1078-1082.

[12] S. Somasundaram, P. Vidhyarani and R. Ponraj, 2011,"Geometric Mean Labeling of Graphs", Bulletin of Pure and Applied Sciences, 30E(2),153-160.

[13] S. Somasundaram, S.S. Sandhya and S.P. Viji, Geometric mean labelling on Degree Splitting graphs, Journal of Discrete Mathematical Sciences and cryptography,19(2), 305-320, DOI: 10.1080/09720529.2015.1084781.

[14] K.N. Meera, Radio Geometric graceful graphs, presented in Icon AMMA2018,2019 IOP Conf. ser.: Mater. Sci. Eng.577 012167.