

# Stability Study of SATER Models in a Dynamic Approach with Application

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## Abstract

This paper related to provide stability conditions of Self-Exciting Threshold Autoregressive model. This stability will be presented by the method developed by developer of this model, as well as finding these conditions by dynamical technique method.

Finally, we will apply these conditions in a dynamical technique to the historical weekly data when the market is closed of silver prices per ounce in dollars for the period from (1-january 2015 to 18 September 2022) with 401 observation. It was applied using MATLAB (R2022a) put at the end of the paper in the form (m.file).

**Keywords:-** Threshold model, SETAR model, Local linearization approximation, singular point, Non-linear.

## 1-Introduction

The 2-nd stationarity stochastic process the most crucial requirement for the random error in every time series model must be a white noise process with zero mean, constant variance, and uncorrelated values if the mean, and variance of the process do not depend on time  $t$ .

The threshold model is the breakthrough in univariate time series. In the past three decades, non-linear have received a great deal more attention. The threshold autoregressive (TAR) model developed by is one particular model that frequently appears in economic research [14]. In order to define the regime, the TAR model compares a threshold value to a state-determining variable  $\{x_t\}$ . It should be noted that in different economic contexts, the empirical presence of a threshold seems to be crucial. The TAR model has proved a useful tool for modeling non-linear time series [15]. For example, Chan 2009 provided a useful technique for academics of non-linear time series analysis [1], while Li 2009 looks at the threshold approaches used in volatility model [4]. The threshold unit-root test and threshold co-integration are both part of the extension of the notion on non-linearity [3]. The shifting autoregressive model, which was first put out by Tong and Lim 1980 [15], codified by Hamilton 1989 and subsequently used by McCulloch and Tsay, is another notable use of the TAR model 1994 [5]. The threshold variable in this model is a random implicit (unobservable) indicator. This also developed by Wong and Li 2000 [16]. The piecewise class

linear threshold autoregressive TAR designs known as the (Self-Exciting threshold autoregressive model) are of particular interest in this study. The state-determining variable in the SETAR model created endogenously which is an essential feature. Emarh 2017 argues that the SETAR model, a TAR group modeling performs better than many other linear and non-linear modeling [2]

In this research, we analyze the SETAR model's stationarity requirement using a dynamical technique that approximates the model to a linear difference equation, T. Ozaki (1985) developed the local linearization approximation approach after he discovered the exponential autoregressive model's stability requirements (EXPAR) [13]. This model was used by Mohammad and Salim in 2007 to establish the logistic autoregressive model's stability condition [11]. In 2010, Mohammad and Ghannam done research on the Cauchy model's stability condition [7]. In 2016 Mohammad and Ghaffar studying the stationarity of GARCH(Q,P) model [8]. In 2018 Mohammad and Mudhir studying the EGARCH(Q,P) model [10], and Noori and Mohammad studying the GJR-GARCH(Q,P) model [12]

## 2-Preliminaries

By taking the system into account, a non-linear time series model in discrete time can be represented as a discrete time dynamical system.

$$x_t = f(x_{t-1}, x_{t-2}, \dots, x_{t-Q}, \epsilon_t) \quad \dots(2.1)$$

Where  $f$  is a non-linear function and  $\epsilon_t$  is a system random error that frequently behaves like white noise ( $\epsilon_t \sim iid N(0, \sigma_z^2)$ ).

The model SETAR expression by form

$$x_t = \sum_{i=1}^Q (\alpha_i + \beta_i x_{t-i} + \epsilon_t) I_{x_{t-d} \leq r} + (\gamma_i + \delta_i x_{t-i} + \varphi_i \epsilon_t) (1 - I_{x_{t-d} \leq r}) \quad \dots(2.2)$$

Where  $\epsilon_t \sim iid N(0,1)$

And  $\alpha_i, \beta_i, \gamma_i, \delta_i$  and  $\varphi_i$  are real constants, and where  $I_{x_{t-i}}$  is indicator function defined as

$$I_{x_{t-i}} = \begin{cases} 0 & \text{if } x_{t-i} > r \\ 1 & \text{if } x_{t-i} \leq r \end{cases} \quad \forall i$$

In order to analyze the stability requirement of the non-zero fixed point of the original dynamical system, the local linearization approach deals with approximating a non-linear dynamical system to a linear dynamical system. A function's non-zero fixed point (or singular point) is also non-zero. if their immediate area lacks any other fixed points  $\xi$ . is satisfied if and only if the following conditions are met:

$$\xi = f(\xi) \quad \dots(2.3)$$

This method entails placing minor perturbations in the area of the non-zero singular point  $\xi$  with a radius  $\xi_t$  small enough such that  $\xi_t^n \rightarrow 0$  for  $n \geq 2$ . The result of this minor adjustment made by substituting  $\xi + \xi_{t-i}$  instead of  $x_{t-i}$  for  $1 \leq i \leq Q$  hence, we employ a variational equation.

$$x_{t-i} = \xi + \xi_{t-i} \quad \text{for } 1 \leq i \leq Q. \quad \dots(2.4)$$

We obtain a linear difference equation of order  $Q$  in terms of  $\xi_t, \xi_{t-1}, \xi_{t-2}, \dots, \xi_{t-Q}$  after substituting this variational equation in (2.1) as an example, and we may examine the stability of this linear difference equation via the roots of its characteristic equation.

**Lemma 2.1** [7],[9]

Let  $c_1, \delta_2, \dots, \delta_r$  be a non-negative real numbers, the following polynomial :

$$P(z) = 1 - \sum_{i=1}^r \delta_i z^i$$

does not have a roots inside and on the unit circle if and only if  $P(z) > 1$ .

The idea of this models laid by Tong (1977a) and Tong (1978) are the first works to discuss the notion of modeling discrete time series data using piecewise linear models in a systematic manner (1978a,1978b,1980). Available in Tong and Lim is a thorough explanation, several applications, and debate (1980).

Let  $J_t$  be an observable (indicator) random variable with integer values  $\{1, 2, \dots, \ell\}$  and let  $\{x_t\}$  be a  $k$ -dimensional time series for each  $t$ .  $A^{(j)}, B^{(j)}$ , and  $H^{(j)}$  are  $k \times k$  (non-random) matrix coefficients,  $C^{(j)}$  is a  $k \times 1$  vector of constants, and  $\epsilon_t \sim iid$  of  $k$ -dimensional random vectors with zero mean and a covariance matrix. This is a classic version of a threshold mode 1 in time series analysis.

$$x_t = B^{(j_t)} x_t + A^{(j_t)} x_{t-1} + H^{(j_t)} \epsilon_t + C^{(j_t)} \quad \dots(2.5)$$

$J_t$  denotes the mode of the dynamic mechanism in the aforementioned canonical form.  $J_t$  is selection is purposefully left open-ended to accommodate a significant variety of possible circumstances.

Of course, the decision  $J_t$  makes, which depends on  $t$ , will be of urgent importance. because the ideas of threshold and time delay are directly tied to this decision.

We now identify a few intriguing threshold model specific situations for additional research.[tong 1983]

The developed on the equation (2.5) by Hansen's approach and the SETAR model of inference .The definition of a SETAR with two regimes is:

$$x_t = (\alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p})I(x_t \leq r) + \dots(2.6)$$

$$(\beta_0 + \beta_1 y_{t-1} + \dots + \beta_p y_{t-p})I(x_t > r) + \epsilon_t$$

where the error  $\epsilon_t$  is taken to be  $iid(0, \sigma^2)$  and  $x_{t-d}$  is the threshold variable that determines which regime is in operation at time  $t$ . The indicator function  $I(\cdot)$ , where  $p$  is the model's autoregressive order,  $r$  is the threshold parameter, and  $d$  is referred to as the delay parameter, predicts when one regime will move into another. The parameters  $\beta_j$  are the upper regime coefficients (observations in which  $x_{t-d} > r$ ) and  $\alpha_j$  are the lower regime coefficients (observations in which  $x_{t-d} \leq r$ ) of the autoregressive function. The equation (1) may be expressed more succinctly as follows:

$$x_t = \theta y_t(r)' + \epsilon_t \dots(2.7)$$

where  $\theta = (\alpha' \beta')' = ((\alpha_0 \alpha_1 \dots \alpha_p)(\beta_0 \beta_1 \dots \beta_p))'$  and  $y_t(r)' = [y_t' I(x_{t-d} \leq r) y_t' I(x_{t-d} > r)]$ . For a certain value of the SETAR model is simplified to equation (2.7), and the conditional least square estimator of  $\theta(r)$  is:

$$\hat{\theta}(r) = \left[ \sum_{t=1}^n y_t(r) x_t(r)' \right]^{-1} \left[ \sum_{t=1}^n y_t(r) x_t \right] \dots(2.8)$$

The residuals of this model are  $\hat{\epsilon} = x_t - y_t(r)' \hat{\theta}(r)$ , and we can use the following formula to estimate their variance:  $\hat{\sigma}^2(r) = \frac{1}{n} \sum_{t=1}^n \hat{\epsilon}_t^2(r)$  When is unknown, it must be estimated while minimizing the residual variance; this yields the estimators for  $\hat{\theta} = \hat{\theta}(\hat{y})$  and  $\hat{\sigma}^2 = \hat{\sigma}^2(\hat{y})$ .

We will prove that stability conditions in special case when  $i = 1$ . That is mean the model SETAR become by form :

$$x_t = (\alpha + \beta x_{t-1} + \epsilon_t)I_{x_{t-d} \leq r} + (\gamma + \delta x_{t-1} + \varphi \epsilon_t)(1 - I_{x_{t-d} \leq r})$$

$$\epsilon_t \sim iid N(0,1)$$

To find the stability conditions by local linearization approximation . We first find the non-zero singular point  $\xi$  as following :

$$\begin{aligned}
 x_{t-s} &= \xi \quad \text{for } s = 0, 1, 2, \dots, p \\
 \xi &= (\alpha + \beta\xi + \epsilon_t)I_{x_{t-d} \leq r} + (\gamma + \delta\xi)(1 - I_{x_{t-d} > r}) \\
 \xi &= (\alpha + \gamma + \beta\xi + \delta\xi) \cdot \frac{1}{2} \\
 \xi &= \frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} \cdot \xi \\
 \xi \left(1 - \frac{\beta + \delta}{2}\right) &= \frac{\alpha + \gamma}{2} \\
 \xi &= \frac{\frac{\alpha + \gamma}{2}}{\frac{2 - (\beta + \delta)}{2}} \\
 \therefore \xi &= \frac{\alpha + \gamma}{2 - (\beta + \delta)} \quad \dots(1)
 \end{aligned}$$

Then the non-zero singular point exist if :

$$\frac{\alpha + \gamma}{2 - (\beta + \delta)} > 1$$

To find a stability condition of the non-zero singular point we will use a local linearization technique as following :

$$\begin{aligned}
 \text{Put } x_s &= \xi + \xi_s \quad \text{for } s = t, t-1 \\
 \xi + \xi_s &= (\alpha + \beta\xi + \xi_{s-1})I_{x_{t-d} \leq r} + (\gamma + \delta\xi + \xi_{s-1})(1 - I_{x_{t-d} \leq r}) \\
 \xi + \xi_s &= [\alpha + \gamma + \beta(\xi + \xi_{s-1}) + \delta(\xi + \xi_{s-1})] \cdot \frac{1}{2} \\
 \xi &= \frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} \cdot (\xi + \xi_{s-1}) \\
 \xi + \xi_s &= \frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} \xi + \frac{\beta + \delta}{2} \xi_{s-1} \\
 \xi \left[1 - \frac{\beta + \delta}{2}\right] + \xi_s - \frac{\alpha + \gamma}{2} &= \frac{\beta + \delta}{2} \xi_{s-1}
 \end{aligned}$$

From equation (1) we get :

$$\therefore \xi_s = \frac{\beta + \delta}{2} \xi_{s-1} \quad \dots(2)$$

The equation (2) is difference linear equation with constant coefficient and the characteristic equation for it has a form :

$$\lambda^r - \frac{\beta + \delta}{2} \lambda^{r-i} = 0 \quad \dots(3)$$

Then the non-zero singular point is stable if the roots of (3) lies inside the unite circle, i.e.

$|\varphi_i| < 1$  for  $i = 0, 1, 2, \dots, r$ . Where  $\varphi_i$  is the root of characteristic equation for (3)

from (3)

$$\begin{aligned} \lambda^r - \frac{\beta + \delta}{2} \lambda^{r-i} &= 0 \\ P\left(\frac{1}{\lambda}\right) &= 1 - \left(\frac{\beta + \delta}{2}\right) \left(\frac{1}{\lambda}\right)^i = 0 \end{aligned} \quad \dots(4)$$

since  $\lambda^r \neq 0$  and then by Lemma(2.1) the polynomial (4) does not have a roots inside and on the unit cycle if and only if  $P\left(\frac{1}{\lambda}\right) > 0$

$$\therefore \left|\frac{1}{\lambda_i}\right| > 1 \text{ for } i = 0, 1, 2, \dots, r$$

$$\therefore |\lambda_i| < 1 \text{ for } i = 0, 1, 2, \dots, r$$

and since  $P(1) > 0$  then

$$\left[1 - \frac{\beta + \delta}{2}\right] > 0$$

which is implies that  $\frac{\beta + \delta}{2} < 1$

$$\Rightarrow \beta + \delta < 2 \quad \blacksquare.$$

### Proposition:

If the SETAR(1) model possess the stability condition of a limit cycle of period  $k > 1$  then this limit cycle is orbitally stable if

$$\left|\frac{\beta + \delta}{2}\right|^k < 1$$

### Proof:

Suppose that the model possess a limit cycle of period  $k$  namely

$$x_t, x_{t-1}, x_{t-2}, \dots, x_{t-k} = x_t \quad k > 1$$

Near the neighborhood of each point of a limit cycle with sufficient small radius  $x_t$  such that  $\xi_t^n \rightarrow 0$  for  $n \geq 2$  placed  $x_t = x_t + \xi_t$ ,  $x_{t-1} = x_{t-1} + \xi_{t-1}$

The SETAR model given by :

$$\begin{aligned}
 x_t &= \frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} \cdot x_{t-1} \\
 x_t + \xi_t &= \frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} \cdot (x_{t-1} + \xi_{t-1}) \\
 x_t + \xi_t &= \frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} \cdot x_{t-1} + \frac{\beta + \delta}{2} \cdot \xi_{t-1} \\
 \therefore x_t &= \frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} \cdot x_{t-1} \\
 \Rightarrow \frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} \cdot x_{t-1} + \xi_t &= \frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} \cdot x_{t-1} + \frac{\beta + \delta}{2} \cdot \xi_{t-1} \\
 \Rightarrow \xi_t &= \frac{\beta + \delta}{2} \cdot \xi_{t-1} \quad \dots(5)
 \end{aligned}$$

The equation (5) is linear difference equation of order one, and to solution it (find the value of  $\xi_t$ ) is difficult but the important is converge solution of equation (5) to zero, and for all  $t \rightarrow \infty$ .

If the solution is convergent then the limit cycle is tropically stable, and from this convergence we get :

$$\left| \frac{\xi_{t+k}}{\xi_t} \right| < 1 \quad \dots(6)$$

For  $k$  steps for equation (5) we get :

$$\xi_t = \frac{\beta + \delta}{2} \cdot \xi_{t-k}$$

But

$$\begin{aligned}
 \xi_{t+k} &= \frac{\beta + \delta}{2} \cdot \xi_{t-(k+1)} \\
 \xi_{t+k} &= \prod_{j=1}^k \left[ \frac{\beta + \delta}{2} \right] \xi_t \\
 \frac{\xi_{t+k}}{\xi_t} &= \prod_{j=1}^k \left[ \frac{\beta + \delta}{2} \right]
 \end{aligned}$$

From condition (6) then the limit cycle for  $k$  period is stable if satisfy the last condition, and the SETAR model become tropically stable when :

$$\left| \frac{\xi_{t+k}}{\xi_t} \right| = \left| \prod_{j=1}^k \left[ \frac{\beta + \delta_j}{2} \right] \right| < 1$$

$$\therefore \left| \frac{\beta + \delta}{2} \right|^k < 1. \blacksquare$$

### 3-Application

#### 3.1- Data disruption

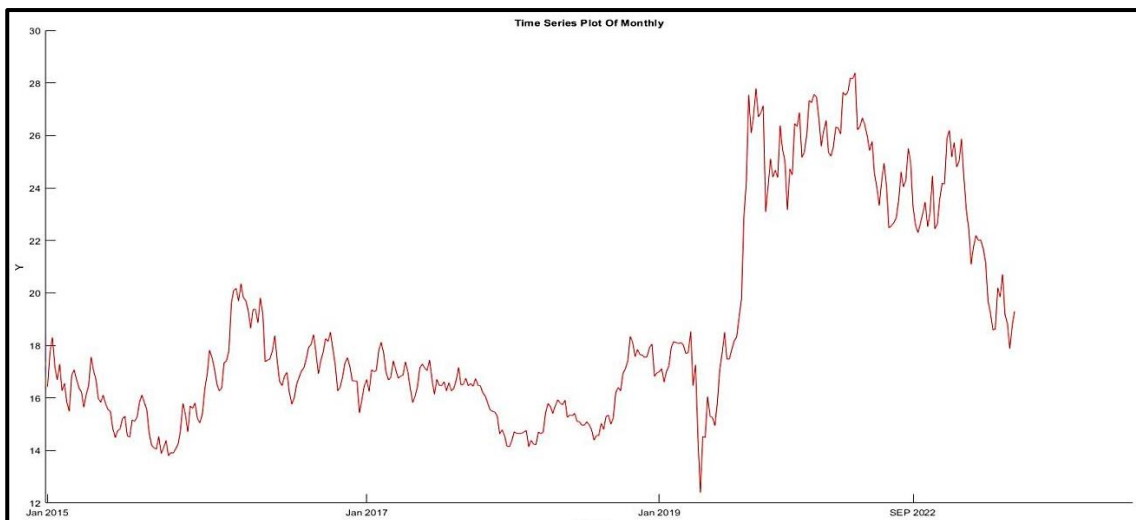
The data used in the applied side is represented by the weekly silver data per ounce in dollars at the weekly market close for the period from the first of January 2015 to the eighteenth of September 2022 by 401 observations. This data was obtained from the website(<https://sa.investing.com/commodities/silver>)

The software used for data analysis is MATLAB (R2022a)

#### 3.2- Data Analysis

- 1- The first stage is always to analyze any data. It is the most important stage of this analysis, which is drawing the chosen time series to see the shape and nature of the time series and its general trend.

Where figure (3.1) is the drawing of the time series for the weekly silver contracts when the market closed



**figure (3.1) is the drawing of the time series for the weekly silver contracts when the market closed**

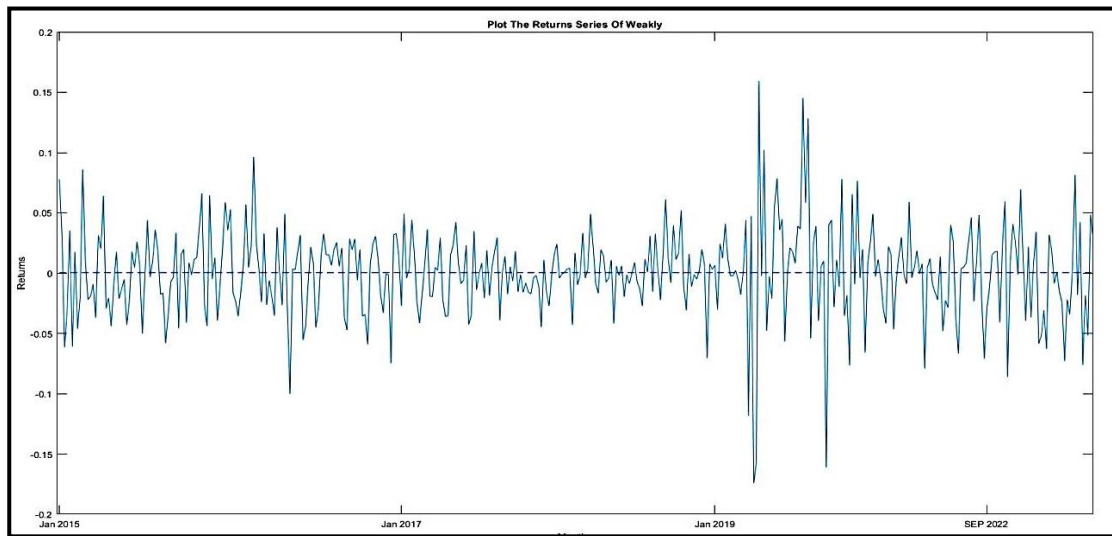
The first view of the form of the time series is that the series has no shape or nature for the direction of the series.



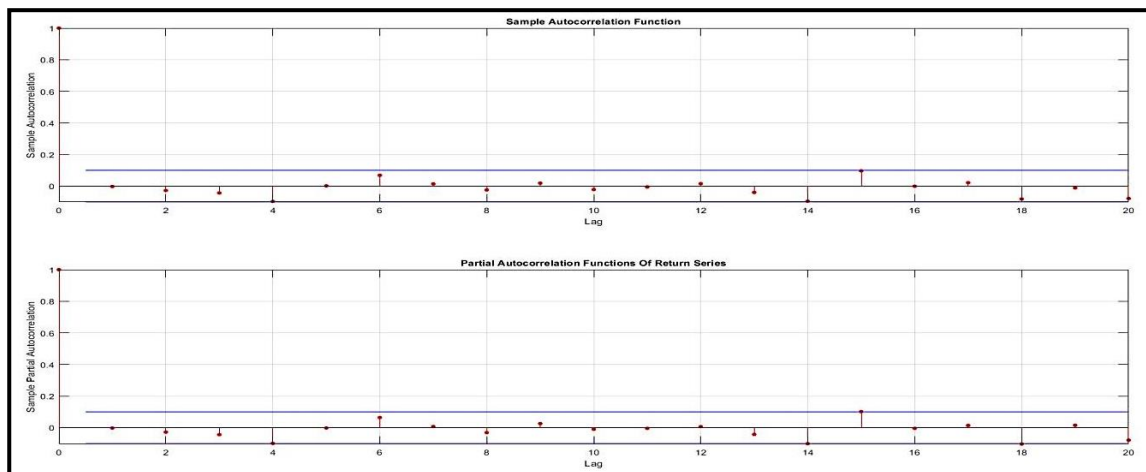
We transform the time series plotted above into a Return series using the mathematical relationship

$$r_t = \log \frac{p_x}{p_{x-1}}$$

Where  $p_t, p_{t-1}$  represent the observed data at  $t, t-1$  respectively and  $r_t$  represent the Returns series. Where Figure (3.2) represents the plot of the Returns series, while Figure (3.3) represents the plot of the total and partial autocorrelation functions of the Returns series.

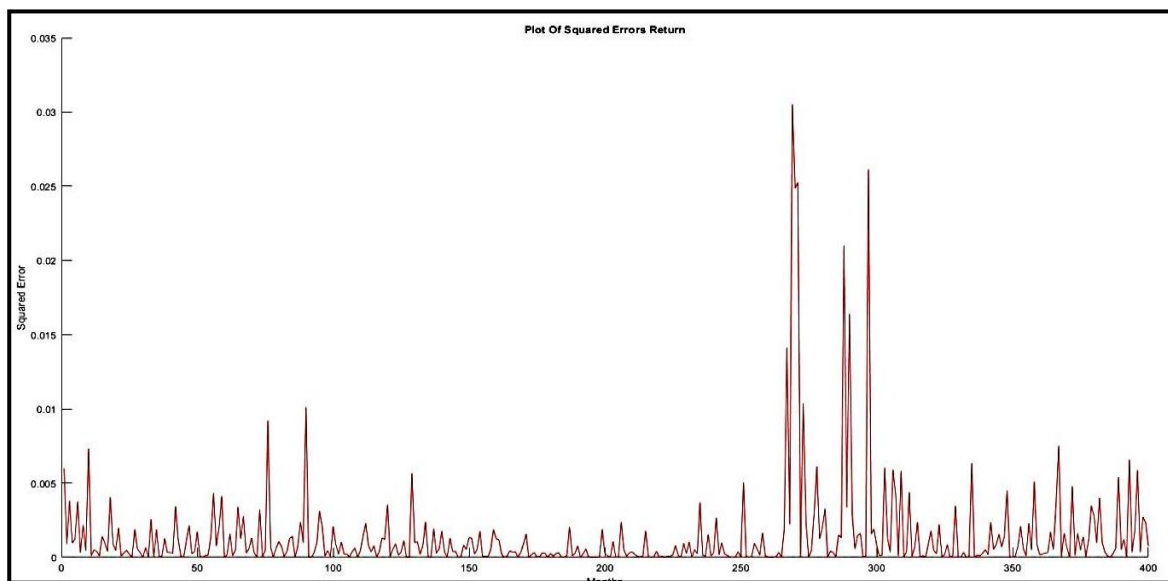


**figure (3.2) is the plot of the Reruns series**

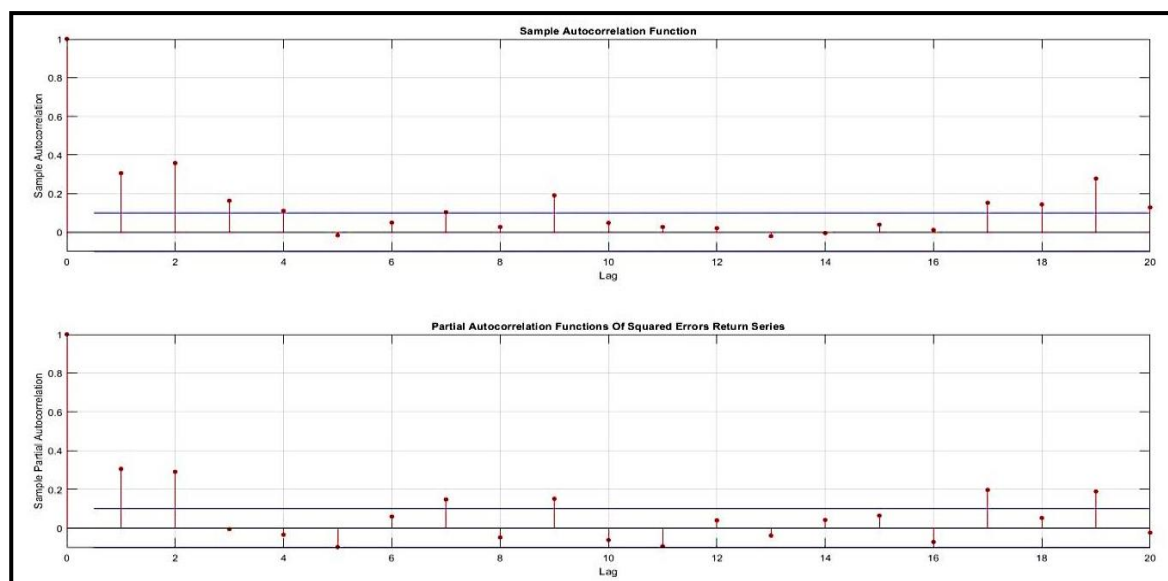


**figure (3.3) is the total and partial autocorrelation functions of the series**

Then we convert the returns series into the error series, then the error series, as Figure (3.4) represents the error series, while Figure (3.5) represents the plot of the total and partial autocorrelation functions of the error series.



**figure (3.4) is the plot of the Errors series**



**figure (3.5) is the total and partial autocorrelation functions of the Errors series**

Figure (3.5) represented by drawing the total and partial autocorrelation functions of the error series and for twenty lags, we note that there are a number of lags outside the limits of the confidence interval at  $Lags = 1, 2, 3, 4, 7, 9, 17, 18, 19, 20$  for ACF and  $Lags = 1, 2, 7, 9, 17, 19$  for PACF.

Now we convert the error series into a standard error series, then convert it into a standard Residuals series, and then again we draw the resulting series as well as draw the total and partial autocorrelation functions for it.

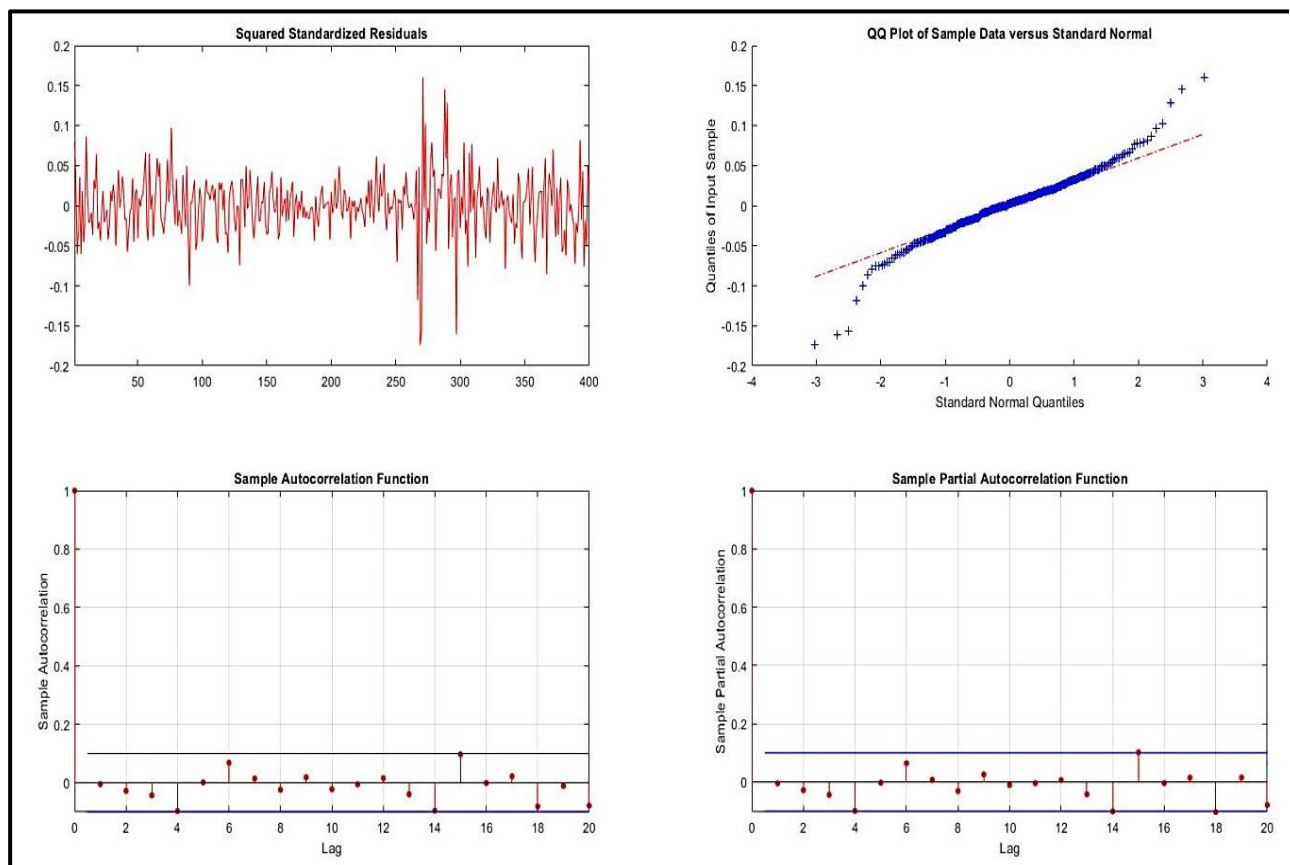


figure (3.6) is the drawing of the standard Residuals series, ACF and PACF

Most of lags of the ACF and PACF functions of the above series are within the limits of the confidence interval, which means that the series has gained numerical stability.

we fitting and estimating the parameters of the SETAR model, and the best way to estimate the parameters for the SETAR model is by using the maximum likelihood estimation function, where this step is to choose a best order for the model by using the AIC and BIC information criteria.

Whereas, Table (3.1) represents the log maximum likelihood estimation function, AIC and BIC information criteria while Table (3.2) represents the estimated parameter values of the models

SETAR(P)	Log.L	AIC	BIC
<b>SETAR(1)</b>	743.1233	-1.4802e+03	-1.4683e+03
<b>SETAR(2)</b>	743.2810	-1.4786e+03	-1.4626e+03
<b>SETAR(3)</b>	743.6746	-1.4773e+03	-1.4574e+03
<b>SETAR(4)</b>	745.6995	-1.4794e+03	-1.4555e+03
<b>SETAR(5)</b>	745.6997	-1.4774e+03	-1.4495e+03
<b>SETAR(6)</b>	746.5894	-1.4772e+03	-1.4452e+03
<b>SETAR(7)</b>	746.5985	-1.4752e+03	-1.4393e+03
<b>SETAR(8)</b>	746.8059	-1.4736e+03	-1.4337e+03
<b>SETAR(9)</b>	746.9504	-1.4719e+03	-1.4280e+03
<b>SETAR(10)</b>	746.9669	-1.4699e+03	-1.4220e+03

Table (3.1) the log maximum likelihood estimation function, AIC and BIC

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**information criteria**

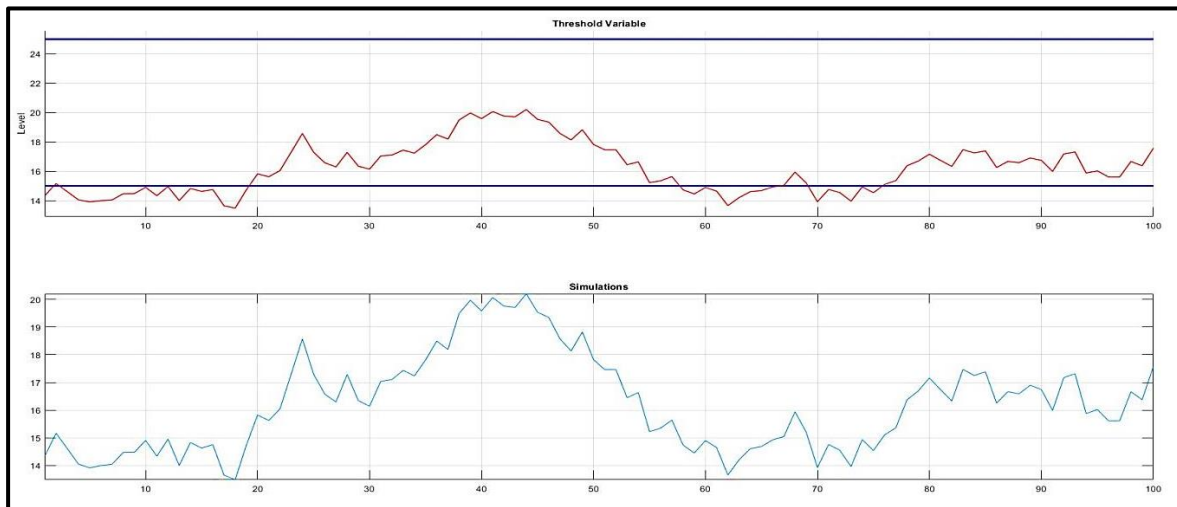
Table (3-2) shows that the SETAR(1) model is the best model with the lowest AIC and BIC values. The model's formula is

	Value	StandardError	TStatistic	PValue
Constant	0.00040435	0.0019073	0.212	0.83211
AR{1}	-0.0024273	0.030906	-0.078538	0.9374
Variance	0.0014251	6.168e-05	23.105	4.1304e-118

SETAR(P)	constant	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$	$a_{10}$
SETAR(1)	0.0004	-0.0024	-	-	-	-	-	-	-	-	-
SETAR(2)	0.0004	-0.0285	0.0028	-	-	-	-	-	-	-	-
SETAR(3)	0.00041	-0.0454	0.0286	0.0032	-	-	-	-	-	-	-
SETAR(4)	0.00045	-0.103	0.0465	0.029	0.0068	-	-	-	-	-	-
SETAR(5)	0.00045	-0.0012	0.103	0.0466	0.0291	0.0069	-	-	-	-	-
SETAR(6)	0.00041	-0.0079	0.022	0.0436	0.1023	0.0014	0.06951	-	-	-	-
SETAR(7)	0.00041	-0.0084	0.0221	0.0428	0.102	0.0014	0.06947	0.00705	-	-	-
SETAR(8)	0.00043	-0.0078	-0.02	0.0424	0.1055	0.0021	0.06855	0.00741	0.034	-	-
SETAR(9)	0.00041	-0.007	0.0206	0.044	0.1058	0.00095	0.0692	0.0081	0.0342	0.02837	-
SETAR(10)	0.00041	-0.007	0.0209	0.0437	0.1051	0.00092	0.06839	0.00782	0.0343	0.0285	-0.0097

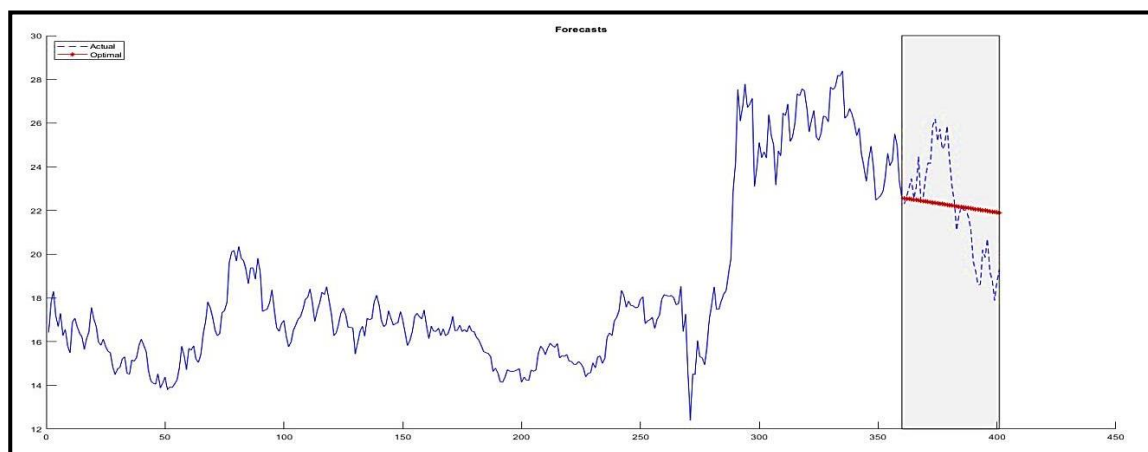
Table (3.2) the value of parameters estimation

As for the values of Threshold and simulation for the selected model, it is clamped as follows :

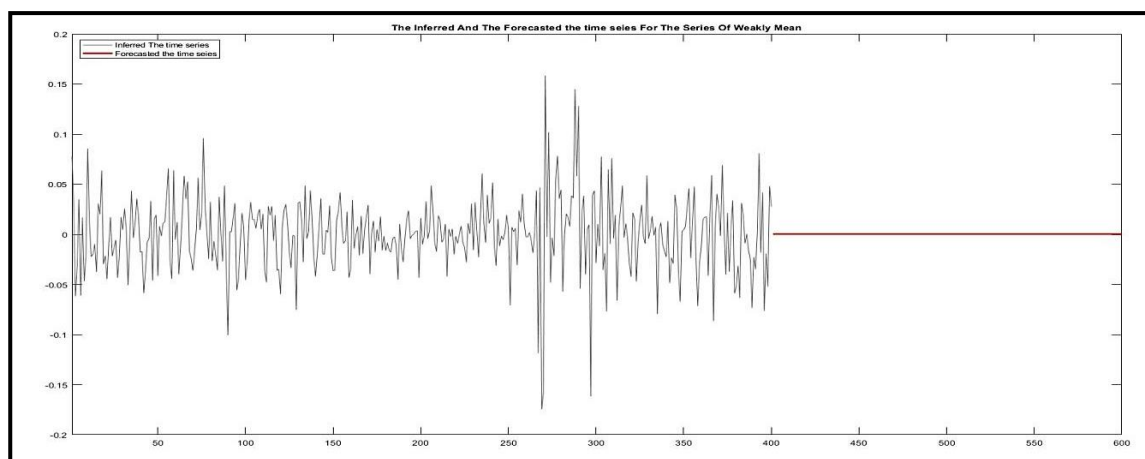


**figure (3.7) is the drawing of the Threshold and simulation**

Finally, we give a prediction for the stable series and inferred the time series



**figure (3.7) is the forecast of the tine series**



**figure (3.8) is the inferred the time series**

**M.file**

```

%% SETAR Model
clear,clc
close all
La=input('Lags number=')
cov=1;%input('covariave value=')
data=load('data4');
Data=data.ys2;
figure(1)
hold on
xlabel('Weeks');
h = gca;
h.XTick = [1 133 254 359];
h.XTickLabel = { 'Jan 2015','Jan 2017','Jan 2019',...
    'SEP 2022'};
ylabel('Y');
title('Time Series Plot Of Weekly')
plot(Data,'r');
hold off

r=price2ret(Data);
N=length(r);
meanR = mean(r);
error = r - mean(r);
squerror = error.^2;
figure(2)
plot(r)
hold on
plot(meanR*ones(N,1),'--b')
xlim([0,N])
xlabel('Weeks');
h = gca;
h.XTick = [1 133 254 359];
h.XTickLabel = { 'Jan 2015','Jan 2017','Jan 2019',...
    'SEP 2022'};
ylabel('Returns');
title('Plot The Returns Series Of Weekly')
hold off

figure(3)
hold on
xlabel('Weeks');ylabel('Squared Error');
title('Plot Of Squared Errors Return')
plot(squerror,'r');
hold off

figure(4)

```

```

subplot(2,1,1)
autocorr(r)
subplot(2,1,2)
parcorr(r)
title('Partial Autocorrelation Functions Of Return Series')

figure(5)
subplot(2,1,1)
autocorr(squerror)
subplot(2,1,2)
parcorr(squerror)
title('Partial Autocorrelation Functions Of Squared Errors Return Series')
[h,pValue,Qstat,cValue] = lbqtest(r,'Lags',[1:20]) ;
%% create SETAR Model
numObs = length(Data);
%AR=ar(Data,La)
ttL = threshold([15 25]);
mdl1 = arima(La,0,0);
mdl2 = arima(La,0,0);
mdl3 = arima(La,0,0);
Q=0;
P=La;
%Mdl= [mdl1; mdl2];
Mdl12 = tsVAR(ttL,[mdl1 mdl2 mdl3],SeriesNames="y")
% rng(0) % For reproducibility
% yb = simulate(Mdl12,numObs,NumPaths=1,Type="exogenous",Z=Data);
% figure
% tiledlayout(2,1)
% nexttile
% ttplot(ttL,'Data',Data)
% title('Threshold Variable')
% colorbar('off')
% nexttile
% plot(y)
% title("Simulations")
% grid on
% axis tight
MD1 = arima(P,0,Q);
[EstMdl2,EstParamCov2,LogL2,info2] = estimate(Mdl12,ttL,Data);
[EstMdl,EstParamCov,LogL,info] = estimate(MD1,r)
numParams = sum(any(EstParamCov));
[AIC,BIC] = aicbic(LogL,numParams,N)
rng(1) % For reproducibility
yf = simulate(EstMdl2,100);
figure(6)
tiledlayout(2,1)
nexttile
ttplot(ttL,'Data',yf)

```

```

title('Threshold Variable')
colorbar('off')
nexttile
plot(yf)
title("Simulations")
grid on
axis tight

rng default;

V=infer(EstMdl,r);
figure(7)
plot(V,'r')
xlim([1,N])
h = gca;
h.XTick = [1 133 254 359];
h.XTickLabel = { 'Jan 2015','Jan 2017','Jan 2019',...
    'SEP 2022'};
title('Infered time series')

StdRes=r./sqrt(V);
SquStdRes=StdRes.^2;

figure(8)
subplot(2,2,1)
plot(SquStdRes,'r')
xlim([1,N])
title('Squared Standardized Residuals')
subplot(2,2,2)
qqplot(SquStdRes)
subplot(2,2,3)
autocorr(SquStdRes)
subplot(2,2,4)
parcorr(SquStdRes)

[h,pValue,Qstat,cValue]=lbqtest(SquStdRes,'lags',[1:20]);

Vf = forecast(EstMdl,200,'Y0',r,'V0',V);
figure(9)
plot(V,'Color',[.4,.4,.4])
hold on
plot(N+1:N+200,Vf,'r','LineWidth',2)
xlim([1,N+200]);
legend('Infered The time series','Forecasted the time seies','Location','Northwest')
title('The Infered And The Forecasted the time seies For The Series Of Weekly Mean')
hold off

```



```

% UnConVar=EstMdl.Constant/(1-EstMdl.AR{1}-EstMdl.MA{1})
% rng default;
% Vsim = simulate(EstMdl,200,'NumPaths',1000,'E0',r,'V0',V);
% sim = mean(Vsim,2);
% figure(10)
% plot(Vf,'r','LineWidth',1.5)
% hold on
% plot(ones(200,1)*UnConVar,'k--','LineWidth',2)
% xlim([1 200]);
% title('The Forecasted The time series Of SEATR( , ) Model Compared With The Theoretical Variance')
% legend('Forecasted The time series','Un-Conditional Variance','Location','southEast')
% hold off
% figure(10)
% plot(sim,':','LineWidth',2)
% hold on
% plot(Vf,'r','LineWidth',2)
% plot(ones(200,1)*UnConVar,'k--','LineWidth',1.5)
% legend('Monte Carlo Forecast','MMSE','Theoretical')
% title('The bias in MMSE forecasting of SETAR')
% hold off
% logsim = mean(log(Vsim),2);
% logUnConVar = log(UnConVar)
%
% figure(11)
% plot(logsim,':','LineWidth',2)
% hold on
% plot(log(Vf),'r','LineWidth',2)
% plot(ones(200,1)*logUnConVar,'k--','LineWidth',1.5)
% legend('Simulated','MMSE','Theoretical')
% title('Unbias in MMSE forecasting of SETAR')
% hold off

```

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