Estimation of Least-Cost Testing Sequence of Continuous Sampling Plan (Csp-1) Using Gert Analysis

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Abstract
This paper explains the evaluation of least cost testing sequence of Continuous sampling plan of type CSP-1 using GERT. The analysis was done using Boothroyd's rule. Based on this rule, the mean and variance of the total cost including in the test can be easily calculated for the analysis. The optimum test sequence which minimizes the mean of the total cost and if the sequences have equal minimum cost, then the rule enables optimum sequence having smaller variance of the total cost included in the test.

Keywords: Contionous Sampling plan, GERT, AOQ, W-function, least cost.

1. INTRODUCTION:

Graphical Evaluation and Review Technique commonly known as GERT, is a network analysis technique used in project management that allows probabilistic treatment of both network logic and estimation of activity duration. The technique was first discovered in 1966 by Dr. Alan B. Pritsker of Purdue University. Compared to other techniques, GERT is only rarely used in complex systems. GERT allows loops between tasks. GERT has been applied to modeling of sampling plans and promises to be value in encouraging statistical quality control. GERT is a technique for the analysis of a class of networks which have the following characteristics: (1) a probability that a branch of the network is indeed part of a realization of the network; and (2) an elapsed time or time interval associated with the branch if the branch is part of the realization of the network. Such networks will be referred to as stochastic networks and consist of a set of branches and nodes. A realization of a network is a particular set of branches and nodes which describe the network for one experiment. If the time associated with a branch is a random variable, then a alization also implies that a fixed time has been selected for each branch. GERT will derive both probability that a node is realized and the conditional moment generating function (M.G.F) of the elapsed time required to tranverse betweenany two nodes [7].

1.1 STEPS IN APPLYING GERT

1. Convert a qualitative description of a system or problem to a model in network form;
2. Collect the necessary data to describe the branches of the network;
3. Obtain an equivalent one-branch function between two nodes of thr network;

4. Convert theequivalent function into the following two performance measures of the network;
   a. The probability that a specific node is realized; and
   b. The M.G.F of the time associated with an equivalent network;
5. Make inferences concerning the system under study from the information obtained in 4 above.

1.2 W-FUNCTION OF A SAMPLING PLAN

For designing a sampling plan using GERT, first the sampling plan can be represented in a network diagram. The network diagram consists of logical nodes and directed branches. The nodes are each state of the plan and a branch has probability that the activity associated with it will be performed. The sample size *n* associated with a branch is characterized by the moment generating function (*m g f*) of the form

$$M_n(\theta) = \sum_n \exp(n\theta)\, f(n)$$

(1)

where $f(n)$ denotes the density function of n and θ is any real variable. The probability φ that the branch is realized is multiplied by the *mgf to* yield the *W*-function such that

$$W(\theta) = \phi M_n(\theta) \qquad (2)$$

The *W*-function is used to obtain the information on the relationship which exists between the nodes [2].

1.3 OPERATING PROCEDURE OF OF CSP-1 PLAN

The operating procedure of the CSP-1 plan as stated by Dodge is as follows [4,8] :

(a) Inspect 100% of the units consecutively as produced and continue such inspection until *i* units in succession are found clear of defects.

(b) When *i* units in succession are found clear of defects, discontinue 100% inspection, and inspect only a fraction *f* of the units.

(c) If a sample unit is found defective, revert immediately to a 100% inspection of succeeding units and continue until again *i* units in succession are clear of defects.

(d) Correct or replace with good units, all defective units found.

2. GERT ANALYSIS FOR THE TESTING SEQUENCE OF CSP-1 PLAN

Suppose there are series of inspection tests $T_i (i = 1,2,3 \ldots\ldots)$ costing C_{T_i} per item tested with average acceptance rate q_i . If an item is found to be rejected at test T_i , costing C_{R_i} for the item to be reworked. Assume that the C's and q's are unaffected by the order in which the tests are executed and so mutually independent.

The possible states of the CSP-1 inspection system described can be defined as follows:

S_0 : Initial state of the plan.

$S_1(k)$: State in which k (= 1, 2..., i) preceding units are found clear of defects during 100% inspection.

S_P : Initial state of partial inspection.

S_A : State in which current unit is accepted.

S_R : State in which current unit is rejected.

The above states enable us to construct GERT network representation of the inspection of CSP-1 plan as shown in Fig. (1). Suppose that the process is in statistical control, so that the probability of any incoming unit being defective is (p) and the probability of any unit being non-defective is $q = 1 - p$ [6]. Now by applying Mason's (1953) rule for the representation of w-function from the initial node S_0 to the terminal nodes S_A and S_R [5,8] are respectively found as

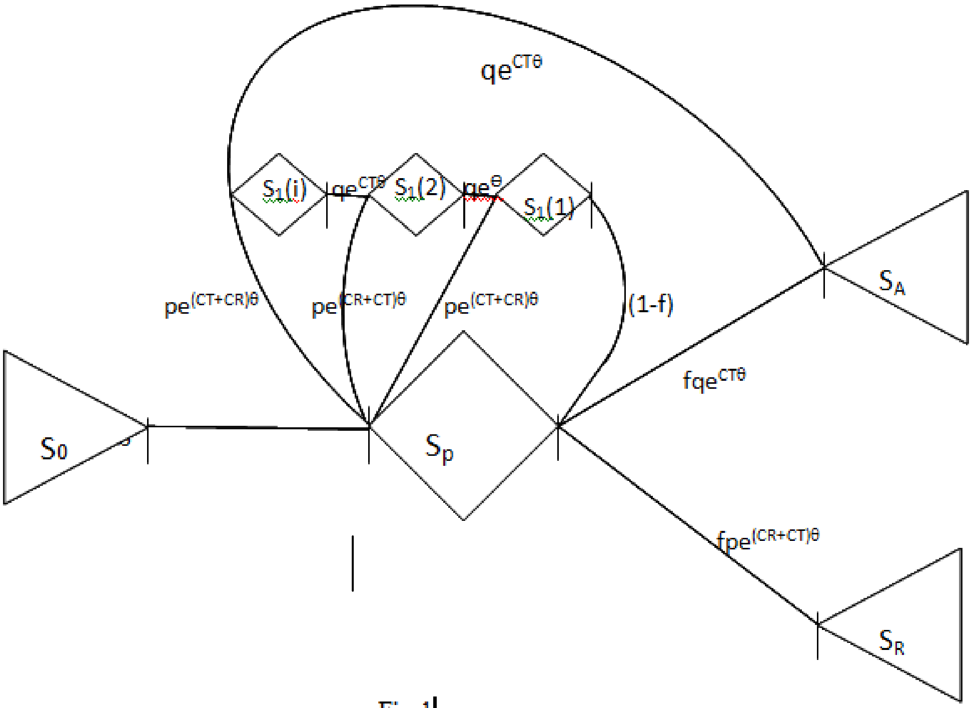

Fig.1

The w-function from the initial node S_0 to the terminal S_A and S_R are respectively given as

$$W_A(\theta) = \frac{fqe^{CT\theta}+(1-f)(qe^{CT\theta})^i}{1-[(1-f)pe^{(CT+CR)\theta}[(1-\left(qe^{CT\theta}\right)^i)/((1-qe^{CT\theta})]} \tag{3}$$

and

$$W_R(\theta) = \frac{fpe^{(CT+CR)\theta}}{1-[(1-f)pe^{(CT+CR)\theta}[(1-\left(qe^{CT\theta}\right)^i)/((1-qe^{CT\theta})]}$$

(4)

Therefore $P_A = [W_A(\theta)]_{\theta=0} = \frac{fq+(1-f)(q)^i}{1-[(1-f)p[(1-(q)^i)/((1-q)]} = \frac{fq+(1-f)q^i}{1+(1-f)q^i}$ (5)

$$P_R = [W_R(\theta)]_{\theta=0} = \frac{fp}{1-[(1-f)p[(1-(q)^i)/((1-q)]} = \frac{fp}{1+(1-f)q^i} \tag{6}$$

where P_A and P_R stands for probability of acceptance and rejection (of a unit) by CSP-1 plan respectively. Since P_A fraction of accepted units are defective with probability p and $(1 - P_A)$ fraction are non-defective with probability $q = 1 - p$.

From (5) and (6) the mean $E(C(T_i)$ and $V(C(T_i)$ of the total cost per item at test T_i [3] is

$$E(C(T_i) = P_A[\frac{d}{d\theta}M_A(\theta)]_{\theta=0} + P_R[\frac{d}{d\theta}M_R(\theta)]_{\theta=0} \tag{7}$$

and

$$V(C(T_i) = P_A\left|[\frac{d^2}{d\theta^2}M_A(\theta)]_{\theta=0} - \left\{[\frac{d}{d\theta}M_A(\theta)]_{\theta=0}\right\}^2\right|$$
$$+P_R\left|[\frac{d^2}{d\theta^2}M_R(\theta)]_{\theta=0} - \left\{[\frac{d}{d\theta}M_R(\theta)]_{\theta=0}\right\}^2\right| \tag{8}$$

$$\underset{A}{M}(\theta) = \frac{\underset{A}{W}(\theta)}{P_A} = \frac{fqe^{CT\theta}+(1-f)(qe^{CT\theta})^i}{1-[(1-f)pe^{(CT+CR)\theta}[(1-(qe^{CT\theta})^i)/((1-qe^{CT\theta})]} \text{ x } \frac{1+(1-f)q^i}{fq+(1-f)q^i} \tag{9}$$

$$\frac{d}{d\theta}M_A(\theta) = \frac{1+(1-f)q^i}{fq+(1-f)q^i} \text{ x } \frac{d}{d\theta}\left\{\frac{fqe^{CT\theta}+(1-f)(qe^{CT\theta})^i}{1-[(1-f)pe^{(CT+CR)\theta}[(1-(qe^{CT\theta})^i)/((1-qe^{CT\theta})]}\right\}$$

$$P_A[\frac{d}{d\theta}M_A(\theta)]_{\theta=0} = \frac{\left\{\left((1-q)-(1-f)p(1-q^i)\right)\left(-2f^2qC_T+fqC_T+i(1-f)q^i\,C_T-(1+i)(1-f)q^{i+1}\,C_T\right)\right\}}{\left((1-q)-(1-f)p(1-q^i)\right)^2} -$$
$$\frac{\{(fq+(1-f)q^i(1-q))((1-f)pq^i(iC_T+C_R+C_T)-qC_T-(1-f)p(C_R+C_T)\}}{\left((1-q)-(1-f)p(1-q^i)\right)^2} \tag{10}$$

$$\underset{R}{M}(\theta) = \frac{\underset{R}{W}(\theta)}{P_R} = \frac{fpe^{(CT+CR)\theta}((1-qe^{CT\theta})}{[(1-qe^{CT\theta})-[(1-f)pe^{(CT+CR)\theta}(1-(qe^{CT\theta})^i)]} \text{ x } \frac{1+(1-f)q^i}{fp} \tag{11}$$

$$\frac{d}{d\theta}M_R(\theta) = \frac{1+(1-f)q^i}{fp} \text{ x } \frac{d}{d\theta}\left\{\frac{fpe^{(CT+CR)\theta}((1-qe^{CT\theta})}{[(1-qe^{CT\theta})-[(1-f)pe^{(CT+CR)\theta}(1-(qe^{CT\theta})^i)]}\right\} P_R[\frac{d}{d\theta}M_R(\theta)]_{\theta=0} =$$
$$\frac{\left\{\left((1-q)-(1-f)p(1-q^i)\right)(-fpqC_T+fp(1-q)(C_T+C_R))\right\}}{\left((1-q)-(1-f)p(1-q^i)\right)^2} -$$
$$\frac{\{(fp(1-q))((1-f)pq^i(iC_T+C_R+C_T)-qC_T-(1-f)p(C_R+C_T)\}}{\left((1-q)-(1-f)p(1-q^i)\right)^2}$$

$$= \frac{\begin{gathered}\{-fp^2C_T+fp^3(C_T+C_R)+(1-f)fp^2qC_T-(1-f)fp^2q^{i+1}\,C_T-(1-f)fp^3(C_T+C_R)+f(1-f)q^{i+1}p^3(C_T+C_R)- \\ f(1-f)q^ip^3((i+1)C_T+C_R)+fp^2C_Tq+f(1-f)p^3(C_T+C_R)\}\end{gathered}}{\left((1-q)-(1-f)p(1-q^i)\right)^2} \tag{12}$$

Adding equation (10) & (12) gives the mean $E(C(T_i)$ of the total cost per item at test T_i .

3. DETERMINATION OF LEAST COST TESTING SEQUENCE

For finding the least cost testing, the following lemmas and theorem should be known.

Lemma I

Let test T_J follow test T_i and the mean $E[C(T_{ij})]$ and variance $V[C(T_{ij})]$ of the total cost per item throughout both tests be given by

$E[C(T_{ij})] = E[C(T_i)] + P_{A_i}E[C(T_j)]$ (13)
and $V[C(T_{ij})] = V[C(T_i)] + P_{A_i}V[C(T_j)]$ (14)

Lemma 2

The optimum test is the test which minimizes the mean of the total cost

i.e $E[C(T_{ij})] < E[C(T_{ji})]$ if and only if $E[C(T_i)]/P_{R_i} < E[C(T_j)]/P_{R_j}$ (15)
and $E[C(T_{ij})] = E[C(T_{ji})]$ if and only if $E[C(T_i)]/P_{R_i} = E[C(T_j)]/P_{R_j}$ (16)

Theorem

Testing sequence $T_{123\ldots.n}$ $(T_1 \to T_2 \to T_3 \to T_4 \to \ldots\ldots..T_n)$ is a least – cost sequence of tests if and only if

$$\frac{E[C(T_1)]}{P_{R_1}} < \frac{E[C(T_2)]}{P_{R_2}} < \frac{E[C(T_3)]}{P_{R_3}} < \frac{E[C(T_4)]}{P_{R_4}} \ldots\ldots..\frac{E[C(T_n)]}{P_{R_n}} \quad (17)$$

If some sequences have the equal minimum mean cost then the optimal testing sequence is the sequence which has the smaller variance of the total cost [1,3].

4. ILLUSTRATIONS

The expected cost of testing sequences corresponding to different values of 'i' and 'f' are to be calculated and given in Table 1.

Test	AOQL	i	f	C_{T_i}	C_{R_i}	P_{R_i}	E[(C_{T_i})]	E[(C_{T_i})]/P_{R_i}
T_1	0.01	2	0.2341	20	200	0.0645	175.4611	2714.274
T_2		6	0.1373	17	170	0.00474	591.4959	124829.5
T_3		8	0.5515	13	199	0.23118	244.0284	1055.584
T_4		10	0.2987	10	100	0.12461	263.0988	2111.433
T_5		12	0.0924	20	200	0.220416	2233.32	10132.99
T_6		10	0.1158	17	170	0.217752	1381.232	6343.153
T_7		9	0.1409	13	199	0.226642	1172.471	5173.219
T_8		8	0.1661	10	100	0.232663	525.9709	2260.658

Table1. Expected cost of testing sequences

From the table, we can see that the expected cost of testing sequence is minimum is for T_3 . Then the minimum cost is for T_4, T_8, T_1, T_7, T_6, T_5 and T_2. The optimum test sequence is the sequence which minimizes the mean of the total cost. Therefore the least cost testing sequence becomes $T_{34817652}$ $(\mathit{that\ is}, T_3 \to T_4 \to T_8 \to T_1 \to T_7 \to T_6 \to T_5 \to T_2)$. If the sequences have equal minimum cost, then the rule enables optimum sequence having smaller variance of the total cost included in the test. For finding the variance of the total cost, the values of $\frac{d^2}{d\theta^2}M_A(\theta)$ and $\frac{d^2}{d\theta^2}M_R(\theta)$ are to be calculated.

$$\frac{d^2}{d\theta^2}M_A(\theta)=\frac{d}{d\theta}\frac{\left\{\left(\left(1-qe^{CT\theta}\right)-(1-f)pe^{(CT+CR)\theta}\left(1-q^ie^{iCT\theta}\right)\right)\left(\begin{matrix}-2f^2qe^{CT\theta}C_T+fqe^{CT\theta}C_T+\\ i(1-f)q^{\,i}e^{iCT\theta}C_T-(1+i)(1-f)q^{i+1}e^{CT(i+1)\theta}C_T\end{matrix}\right)\right\}}{\left(\left(1-qe^{CT\theta}\right)-(1-f)pe^{(CR+CT)\theta}\left(1-q^i e^{iCT\theta}\right)\right)^2}-$$

$$\left\{\frac{\{(fqe^{CT\theta}+(1-f)q^i e^{iCT\theta}(1-qe^{CT\theta}))((1-f)pe^{((i+1)CT+CR)\theta}q^i(iC_T+C_R+C_T)-qe^{CT\theta}C_T \quad (1-f)pe^{((CT+CR)\theta}(C_R+C_T))\}}{\left(\left(1-qe^{CT\theta}\right)-(1-f)pe^{(CT+CR)\theta}\left(1-q^ie^{iCT\theta}\right)\right)^2}\, x\,\frac{1+(1-f)q^i}{fq+(1-f)q^i}\right.$$

$$\frac{d}{d\theta}\{\left(\left(1-qe^{CT\theta}\right)-(1-f)pe^{(CT+CR)\theta}\left(1-q^ie^{iCT\theta}\right)\right)(-2f^2qe^{CT\theta}C_T+fqe^{CT\theta}C_T+i(1-f)$$
$$\left(q^{\,i}e^{iCT\theta}C_T\right)-((1+i)(1-f)q^{i+1}e^{CT(i+1)\theta}C_T))-\left(fqe^{CT\theta}+(1-f)q^i e^{iCT\theta}\left(1-qe^{CT\theta}\right)\right)$$
$$\left((1-f)pe^{((i+1)CT+CR)\theta}q^i(iC_T+C_R+C_T)-qe^{CT\theta}C_T-(1-f)pe(^{(CT+CR)\theta}(C_R+C_T)\right)\}$$

$$=-2fq^2{C_T}^2e^{2C_T\theta}+3fq^2{C_T}^2e^{3C_T\theta}-(1-f)(i+1)q^{i+1}{C_T}^2e^{(i+1)C_T\theta}-(1-f)(i+2)$$
$$q^{i+2}{C_T}^2e^{(i+2)C_T\theta}+fC_Tqe^{C_T\theta}-2fq^2{C_T}^2e^{2C_T\theta}-2fq^2{C_T}^2e^{2C_T\theta}+3fq^3{C_T}^2e^{3C_T\theta}-(1-f)$$
$$q^i{C_T}^2e^{iC_T\theta}-(1-f)(i+1)q^{i+1}{C_T}^2e^{(i+1)C_T\theta}-(1-f)(i+1)q^{i+1}{C_T}^2e^{(i+1)C_T\theta}+(i+2)$$
$$(1-f)q^{i+2}{C_T}^2e^{(i+2)C_T\theta}+f(1-f)pC_T(3C_T+C_R)e^{(3C_T+C_R)\theta}-f(1-f)p((3+i)C_T+C_R)$$
$$q^iC_Te^{((3+i)C_T+C_R)\theta}+(1-f)^2p((i+2)C_T+C_R)q^{i+1}C_Te^{((i+2)C_T+C_R)\theta}-(1-f)^2p((2i+2)$$
$$((2i+2)C_T+C_R)q^{2i+1}C_Te^{((2i+2)C_T+C_R)\theta}-f(1-f)pqC_T(2C_T+C_R)e^{(2C_T+C_R)\theta}+f(1-f)$$
$$pq^{i+1}C_T((i+2)C_T+C_R)e^{((i+2)C_T+C_R)\theta}+f(1-f)pq^2C_T(3C_T+C_R)e^{(3C_T+C_R)\theta}-f(1-f)$$
$$((i+3)C_T+C_R)pq^{i+2}C_Te^{((i+3)C_T+C_R)\theta}-i(1-f)^2pq^iC_T((i+1)C_T+C_R)e^{((i+1)C_T+C_R)\theta}+i$$
$$i(1-f)^2pq^{2i}C_T((2i+1)C_T+C_R)e^{((2i+1)C_T+C_R)\theta}+i(1-f)^2pq^{i+1}C_T((i+2)(C_T+C_R)$$
$$e^{((i+2)C_T+C_R)\theta}-i(1-f)^2pq^{2i+1}C_T((2i+2)C_T+C_R)e^{((2i+2)C_T+C_R)\theta}+2fq^2{C_T}^2e^{(2C_T)\theta}-f$$
$$(1-f)piq^{i+1}C_T((i+2)C_T+C_R)e^{((i+2)C_T+C_R)\theta}+f(1-f)pq(C_T+C_R)(2C_T+C_R)e^{(2C_T+C_R)\theta}$$
$$-f(1-f)pq^{i+1}(C_T+C_R)((i+2)C_T+C_R)e^{((i+2)C_T+C_R)\theta}-3fq^3{C_T}^2e^{(3C_T)\theta}+f(1-f)pq^{i+2}$$
$$(C_T+C_R)((i+3)C_T+C_R)e^{((i+3)C_T+C_R)\theta}-f(1-f)pq^2(C_T+C_R)(3C_T+C_R)e^{(3C_T+C_R)\theta}+f$$
$$(1-f)pq^{i+2}(C_T+C_R)((i+3)C_T+C_R)e^{((i+3)C_T+C_R)\theta}+(1-f)q^{i+1}C_T((i+1)(C_T+C_R)$$
$$e^{((i+1)C_T+C_R)\theta}-(1-f)^2pq^{2i+1}C_T((2i+1)C_T+C_R)e^{((2i+1)C_T+C_R)\theta}-(1-f)^2pq^{i+1}(C_T$$
$$+C_R)((i+2)C_T+C_R)e^{((i+2)C_T+C_R)\theta}+(1-f)^2pq^{2i+1}C_T((2i+2)C_T+C_R)e^{((2i+2)C_T+C_R)\theta}$$

(18)

$$\frac{d}{d\theta}\left(\left(1-qe^{C_T\theta}\right)-(1-f)pe^{(C_T+C_R)\theta}\left(1-q^ie^{iC_T\theta}\right)^2\right)=$$

$$2\left(\left(1-qe^{C_T\theta}\right)-(1-f)pe^{(C_T+C_R)\theta}\left(1-q^ie^{iC_T\theta}\right)\right)\left(-qC_Te^{CT\theta}\right)-(1-f)p(-iq^iC_T$$

$$e^{((i+1)C_T+C_R)\theta}+\left(1-q^ie^{iC_T\theta}\right)(C_T+C_R)e^{(C_T+C_R)\theta}))$$

$$\frac{d}{d\theta}\left(\left(1-qe^{C_T\theta}\right)-(1-f)pe^{(C_T+C_R)\theta}\left(1-q^ie^{iC_T\theta}\right)^2\right)/(\theta=0)=$$

$$2\{-qC_T+(1-f)piC_Tq^i-(1-f)p(C_T+C_R)+(1-f)pq^i(C_T+C_R)+q^2C_T(1-f)\}$$

$$2\{piq^{i+1}C_T+(1-f)pq(C_T+C_R)-(1-f)pq^{i+1}+(1-f)pqC_T-(1-f)^2p^2iC_Tq^i+\}$$

$$2\{(1-f)^2p^2i(C_T+C_R)-(1-f)^2p^2q^i-(1-f)pq^{i+1}C_T+(1-f)^2p^2iC_Tq^{2i}-(1-f)^2\}$$

$$2\{p^2q^i(C_T+C_R)+(1-f)^2p^2q^{2i}\} \qquad (19)$$

Using equation (18) and (19) , we can find the value of $\frac{d^2}{d\theta^2}M_A(\theta)$.

$$[\frac{d}{d\theta}M_R(\theta)] = \frac{\begin{array}{c}\left\{\left(1-qe^{CT\theta}\right)-(1-f)pe^{(CT+CR)\theta}\left(1-\left(qe^{CT\theta}\right)^i\right)\left(-fpe^{(2CT+CR)\theta}qC_T+fpe^{(CT+CR)\theta}\left(1-qe^{CT\theta}\right)(C_T+C_R)\right)\right\}\\ -\{(fpe^{(CT+CR)\theta}\left(1-qe^{CT\theta}\right))((1-f)pe^{(iCT+CR)\theta}q^i(\underline{i}C_T+C_R+C_T)-qe^{CT\theta}C_T-(1-f)pe^{(CT+CR)\theta}(C_R+C_T)\}\end{array}}{\left(\left(1-qe^{CT\theta}\right)-[(1-f)pe^{(CT+CR)\theta}(1-\left(qe^{CT\theta}\right)^i)\right)^2}\ x\,\frac{1+(1-f)q^i}{fp}$$

$$\frac{d}{d\theta}\left\{(1-qe^{CT\theta})-(1-f)pe^{(CT+CR)\theta}(1-\left(qe^{CT\theta}\right)^i)\left(-fpe^{(2CT+CR)\theta}qC_T+fpe^{(CT\theta)}\right)\right\}$$

$$\left\{\left(fpe^{(CR)\theta}\left(1-qe^{CT\theta}\right)(C_T+C_R)\right)\right\}\{(fpe^{(CT+CR)\theta}\left(1-qe^{CT\theta}\right))((1-f)pe^{(iCT+CR)\theta}q^i(iC_T)\} \quad -$$

$$(C_R+C_T)-qe^{CT\theta}C_T-\ (1-f)pe^{(CT+CR)\theta}(C_R+C_T)\}=-2fq^2{C_T}^{2^{2\theta}}+3fq^3{C_T}^2e^{3C_T\theta}-$$

$$(1-f)(i+1)q^{i+1}{C_T}^2e^{(i+1)C_T\theta}-(1-f)(i+2)q^{i+2}{C_T}^2e^{(i+2)C_T\theta}+f{C_T}^2qe^{C_T\theta}-2fq^2{C_T}^2$$

$$e^{2C_T\theta}-(1-f)q^i{C_T}^2e^{iC_T\theta}-i(1-f)(i+1)q^{i+1}{C_T}^2e^{(i+1)C_T\theta}-2fq^2{C_T}^2e^{2C_T\theta}+3fq^3{C_T}^2$$

$$e^{3C_T\theta}-i(1-f)(i+1)q^{i+1}{C_T}^2e^{(i+1)C_T\theta}+i(i+2)(1-f)q^{i+2}{C_T}^2e^{(i+2)C_T\theta}-3f(1-f)p$$

$${C_T}^2q^2e^{(3C_T+C_R)\theta}-f(1-f)pq^2C_TC_Re^{(3C_T+C_R)\theta}f+(1-f)f\,p(i+1){C_T}^2q^{i+2}C_Te^{((i+1)C_T+C_R)\theta}$$

$$+(1-f)^2pq^{i+1}C_TC_Re^{((i+2)C_T+C_R)\theta}-(1-f)^2p(2i+2)q^{2i+1}{C_T}^2e^{((2i+2)C_T+C_R)\theta}-(1-f)^2p$$

$$pq^{2i+1}C_TC_Re^{((2i+2)C_T+C_R)\theta}-2f(1-f)pq{C_T}^2e^{(2C_T+C_R)\theta}-f(1-f)pqC_TC_Re^{(2C_T+C_R)\theta}+f$$

$$(1-f)pq^{i+1}C_T((i+2)C_T+C_R)e^{((i+2)C_T+C_R)\theta}-i(1-f)^2pq^iC_T((i+1)C_T+C_R)e^{((i+1)C_T)\theta}$$

$$e^{(C_R)\theta}+i(1-f)^2pq^{2i}C_T((2i+1)C_T+C_R)e^{((2i+1)C_T+C_R)\theta}+f(1-f)pq^2C_T(3C_T+C_R)e^{3C_T\theta}$$
$$e^{C_R\theta}-f(1-f)pq^{i+2}C_T((3+i)C_T+C_R)e^{((i+3)C_T+C_R)\theta}-i(1-f)^2pq^{i+1}C_T((i+2)C_T+C_R)$$
$$e^{((i+2)C_T+C_R)\theta}-i(1-f)^2pq^{2i+1}C_T((2i+2)C_T+C_R)e^{((2i+2)C_T+C_R)\theta}-2fq^2{C_T}^2e^{(2C_T)\theta}-3f$$
$$q^3{C_T}^2e^{(3C_T)\theta}+if(1-f)pq^{i+1}C_T((i+2)C_T+C_R)e^{((i+2)C_T+C_R)\theta}-if(1-f)pq^{i+2}C_T(C_R+$$
$$(i+3)C_T)e^{((i+3)C_T+C_R)\theta}+f(1-f)pq(2C_TC_R+2{C_T}^2+{C_R}^2+C_TC_R)e^{(2C_T+C_R)\theta}+f(1-f)$$
$$pq(2C_TC_R+2{C_T}^2+{C_R}^2+C_TC_R)e^{(2C_T+C_R)\theta}+f(1-f)pq^{i+1}((2+i)C_TC_R+(2+i){C_T}^2$$
$$+{C_R}^2+C_TC_R)e^{((i+2)C_T+C_R)\theta}+f(1-f)pq^2(3C_TC_R+3{C_T}^2+{C_R}^2+C_TC_R)e^{(3C_T+C_R)\theta}+f$$
$$(1-f)pq^{i+2}((3+i)C_TC_R+(3+i){C_T}^2+{C_R}^2+C_TC_R)e^{((i+3)C_T+C_R)\theta}-2(1-f){C_T}^2q^23$$
$$3(1-f){C_T}^2q^3e^{3C_T\theta}-(1-f)^2pq(C_T+C_R)(C_T+2C_R)e^{(2C_T+C_R)\theta}(1-f)^2pq^{i+1}+(C_T+C_R)$$
$$(C_R+(2+i)C_T)e^{((2+i)C_T+C_R)\theta}+(1-f)^2pq(C_T+C_R)(C_T+3C_R)e^{(3C_T+C_R)\theta}-(1-f)^2pq^{i+2}$$
$$(C_T+C_R)(C_T+(i+3)C_R)e^{((i+3)C_T+C_R)\theta} \quad (20)$$

$$\frac{d}{d\theta}\Big\{(1-qe^{CT\theta})-(1-f)pe^{(CT+CR)\theta}(1-\left(qe^{CT\theta}\right)^i)\left(-fpe^{(2CT+CR)\theta}qC_T+fpe^{(CT+CR)\theta}\right)\Big\}$$
$$\left(1-qe^{CT\theta}\right)(C_T+C_R))\}-\ \{(fpe^{(CT+CR)\theta}\left(1-qe^{CT\theta}\right)((1-f)(iC_T+C_R+C_T)pe^{(iCT+CR)\theta}$$
$$q^i-qe^{CT\theta}C_T-\ (1-f)pe^{(CT+CR)\theta}(C_R+C_T)\}\,/(\theta=0)=-2fq^2{C_T}^2+3fq^3{C_T}^2-(1-f)$$
$$(i+1)q^{i+1}{C_T}^2+-(1-f)(i+2)q^{i+2}{C_T}^2+f{C_T}^2q-2fq^2{C_T}^2-(1-f)q^i{C_T}^2-i(1-f)$$
$$(i+1)q^{i+1}{C_T}^2--2fq^2{C_T}^2+3fq^3{C_T}^2-i(1-f)(i+1)q^{i+1}{C_T}^2\ +i(i+2)(1-f)q^{i+2}{C_T}^2$$
$$-3f(1-f)p{C_T}^2q^2-f(1-f)pq^2C_TC_R\,f(1-f)f\,p(i+1){C_T}^2q^{i+2}C_T+(1-f)f\,pC_TC_R$$
$$q^{i+2}C_T+(1-f)^2p(i+2)q^{i+1}{C_T}^2+(1-f)^2pq^{i+1}C_TC_R-(1-f)^2pq^{i+1}C_TC_R-(1-f)^2$$
$$p(2i+2)q^{2i+1}{C_T}^2-(1-f)^2pq^{2i+1}C_TC_R-2f(1-f)pq{C_T}^2-f(1-f)pqC_TC_R+f(1-f)$$
$$pq^{i+1}C_T((i+2)C_T+C_R)-i(1-f)^2pq^iC_T((i+1)C_T+C_R)+i(1-f)^2pq^{2i}C_T((2i+1)C_T$$
$$+C_R)\ +f(1-f)pq^2C_T(3C_T+C_R)-f(1-f)pq^{i+2}C_T((3+i)C_T+C_R)-i(1-f)^2pq^{i+1}$$
$$C_T((i+2)C_T+C_R)-i(1-f)^2pq^{2i+1}C_T((2i+2)C_T+C_R)-2fq^2{C_T}^2-3fq^3{C_T}^2+if$$
$$(1-f)pq^{i+1}C_T((i+2)C_T+C_R)-if(1-f)pq^{i+2}C_T((i+3)C_T+C_R)+f(1-f)pq(2C_TC_R$$

$+2{C_T}^2 + {C_R}^2 + C_T C_R) + f(1-f)pq^{i+1}((2+i)C_T C_R + (2+i){C_T}^2 + {C_R}^2 + f(1-f)pq^2$

$(3C_T C_R + 3{C_T}^2 + {C_R}^2 + C_T C_R) + f(1-f)pq^{i+2}((3+i)C_T C_R + +C_T C_R)(3+i){C_T}^2 + {C_R}^2 +$

$C_T C_R) - 2(1-f){C_T}^2 + q^2 + 3(1-f){C_T}^2 - (1-f)^2 pq(C_T + C_R)(C_T + 2C_R) + (1-f)^2 p$

$q^{i+1}(C_T + C_R)(C_R + (2+i)C_T) + (1-f)^2 pq(C_T + C_R)(C_T + 3C_R) - (1-f)^2 pq^{i+2}(C_T + C_R)$
$(C_T + (i+3)C_R)$ (21)

Using equation (18) and (19), we can find the value of $\frac{d^2}{d\theta^2} M_R(\theta)$. Therefore the variance of the testing cost , $V(C(T_i)$ is easily calculated using the equation (8).

CONCLUSION

This paper explains a new approach for evaluating the efficiency of Continuous sampling plan of type CSP-1. This is done by minimizing the total cost of inspection included in the testing sequence. This approach can be applied to any sampling plans. For the analysis, an optimization technique GERT is introduced for finding the expected cost of inspection of testing sequence. The effectiveness of a sampling plan can be measured by different methods and based on different performance measures like AOQ, MAAOQ, and AOQL e.tc. But this paper explains the way of finding the effectiveness of a sampling plan based on the cost of inspection included in the testing sequence. For the study, a series of 8 test sequences $T_1, T_2, \ldots\ldots T_8$ is taken corresponding to a fixed AOQL 0.01. Based on this AOQL, the expected cost of testing sequence E $[(C_{T_i})]$ and E $[(C_{T_i})]/P_{R_i}$ for a given clearance number 'i' and sampling fraction 'f' are calculated. The optimum test sequence is the sequence which minimizes the expected cost of the sequence. Therefore the least cost testing sequence based on the above illustration is obtained and is $T_{34817652}$ ($that\ is$, $T_3 \to T_4 \to T_8 \to T_1 \to T_7 \to T_6 \to T_5 \to T_2$). If the sequences have equal minimum cost, then the rule enables optimum sequence having smaller variance of the total cost included in the test. In this paper the variance of the total cost included in the testing sequence is also evaluated. The formula evaluated for variance is too lengthy so it will be easier if a software program can be developed using MATLAB. This approach can also be applied to other Continuous sampling plans of the type CSP-2, CSP-3, CSP-5 and also for skip lot sampling plan. The speciallity of this paper is the introduction of a new optimization technique GERT in the field of acceptance sampling and explains the simplicity of evaluating the performance measures compared to other methods. The GERT approach explains the problem in an algorithmic way and therefore developing programming is very simple.

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