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On (u,v) Distinct Congruent Spectrum of Graphs

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Abstract
An (u,v) distinct congruent spectrum of graph G with u vertices and v – edges is an injective mapping from the set of edges to the set of nonnegative integers {1,2,…v} in such a way that the set of values of vertices obtained by the sums mod |E(G)| +1 .In this paper certain class of graphs are discussed.

Keywords: distinct, congruent, graphs

1. Introduction

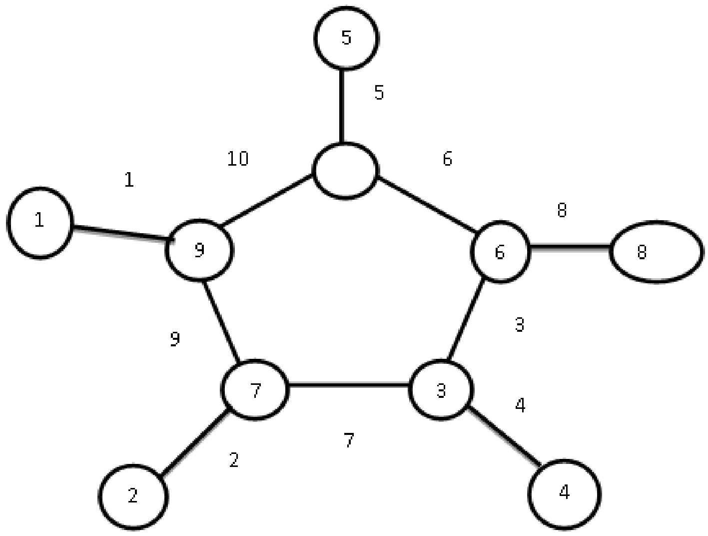
Let G=(V,E) be a connected graph, without multiple edges or loops. For any abelian group A(written additively) .Let A^*=A-{0}.A function f:E(G)→A^* is called a labeling of G. Any labeling induces a map f^+ : V(G)→A, defined by $f^+(v)=\sum f(u,v)$ where u,v ϵ E(G).If there exists a labeling f which induces a distinct label c on V(G),we say that f is an spectrum of distinct magic labeling and that G is an distinct congruence magic graph.We denote by Z_n the group of integers (mod n).In this paper ,we are interested in determining for which values of k ≥3 a graph is DC-magic. The set{k: G is Z_n-magic,n≥3}is called the Distinct-Congruent spectrum of a graph G and is denoted by DC (G).In this paper,weexamine,the DC-magic spectra of class of graphs .

2. Preliminaries

Definition 2.1

An (u,v) distinct congruent spectrum of graph G with u vertices and v-edges is an injective function from the edges of G to the set {1,2,3,4,……v} such that the vertices assigned to the labels (f(x)+f(y)) mod |E(G)|+1 where u=(x,y) then the vertex labels are distinct.

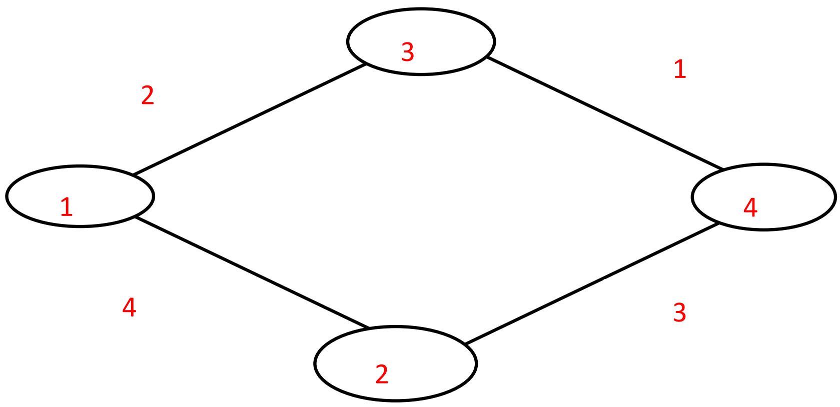
Example 2.2



Definition 2.3

Let G=(V,E) be a connected graph, without multiple edges or loops. For any abelian groupA(written additively) .Let A^*=A-{0}.A function f:E(G)→A^* is called a labeling of G. Any labeling induces amapf^+ :V(G)→A,definedbyf^+(v)=$\sum f(u, v)$mod|E(G)|+1where v is adjacent to u ,(u,v) ϵE(G).If there exists a labeling f which induces a distinct label c on V(G), we say that f is an (u,v) distinct congruent spectrum of graph G.

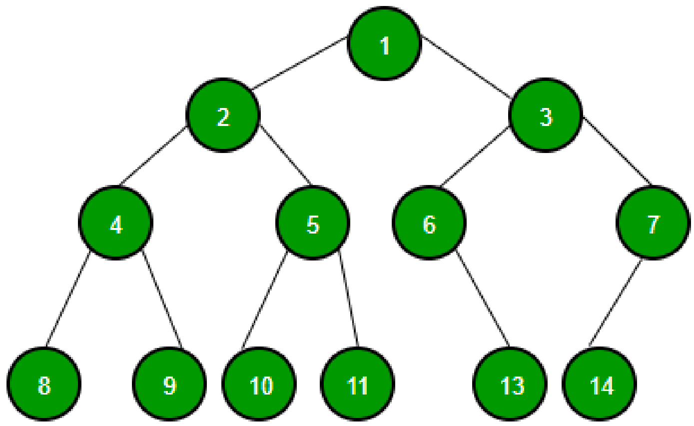
Example 2.4



Definition 2.5

A tree in which every node can have a maximum of two children is called Binary Tree. In a binary tree, every node can have either 0 children or 1 child or 2 children but not more than 2 children.At each level of i, the maximum number of nodes is 2^i.The height of the tree is defined as the longest path from the root node to the leaf node. The tree which is shown above has a height equal to 3. Therefore, the maximum number of nodes at height 3 is equal to (1+2+4+8) = 15. In general, the maximum number of nodes possible at height h is (2^0 + 2^1 + 2^2+….2^h) = 2^{h+1} -1.The minimum number of nodes possible at height h is equal to h+1.If the number of nodes is minimum, then the height of the tree would be maximum. Conversely, if the number of nodes is maximum, then the height of the tree would be minimum. If there are 'n' number of nodes in the binary tree.

Example 2.6



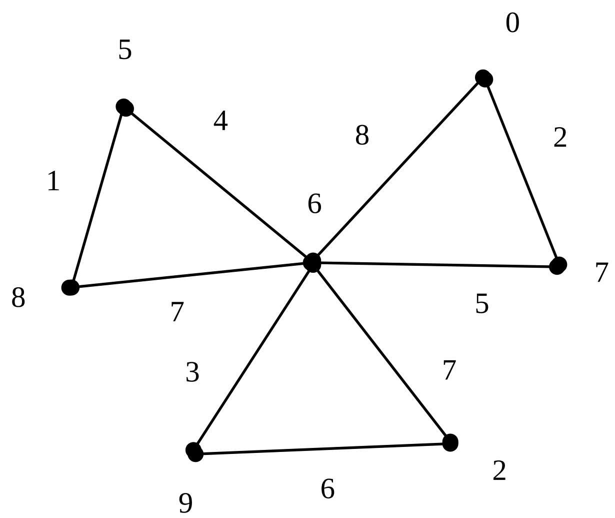
Definition 2.7

The friendship graph (or Dutch windmill graph or n-fan) F_n is a planar, undirected graph with $2n + 1$ vertices and $3n$ edges.

The friendship graph F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex, which becomes a universal vertex for the graph.

By construction, the friendship graph F_n is isomorphic to the windmill graph Wd(3, n). It is unit distance with girth 3, diameter 2 and radius 1. The graph F_2 is isomorphic to the butterfly graph.

Example 2.8



3. Main Results

Theorem 3.1

ThetreegraphT_2isan(u,v)distinctcongruentspectrum$Z_n$ofG.

Proof

LetV(G)=$\{v_1,v_2,v_3,\ldots..v_n\}$

E(G)=$\{e_1,e_2,e_3,\ldots..e_n\}$

Claim T_2isan(u,v)distinctcongruentspectrum.

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An injective function is defined by f:E (G)→{1,2,3,…..q}such that $f(e_i)=i, 1\leq i\leq n$ Then the induced vertex labeling $f^{+}(v)=\sum f(u,v)$mod|E(G)|+1where v is adjacent to u arealldistinct.

Construction 3.2

Since by using definition of labeling we label the edges by 1,2 from Z_3. We considered the vertices v_1, v_2,v_3,…, v_n in clock wise direction.

Labeling

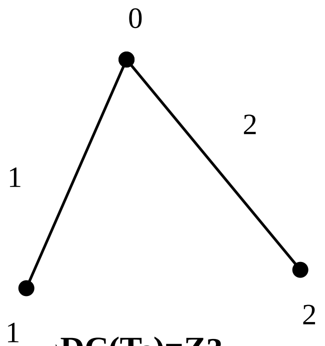

Fig1:DC(T_2)=Z3

Verification

v_1,v_2,v_3Vertices

$f(v_1v_2)=1, f(v_1v_3)=2$.

$f^{+}(v_1)=3, f^{+}(v_2)=1, f^{+}(v_3)=2$.

Therefore all are distinct

Congruency

3≡**0**(mod**3**)

1≡**1**(mod**3**)

2≡**2**(mod**3**)

Therefore T_2 is a distinct congruence magic **graph. T_3:-**

Let the integer set be Z_7={0,1,2,3,4,5,6}Construction:-

Since by using the definition of labeling, we label the edges by 1,2,3,4,5,6 fromZ_7.

Labeling

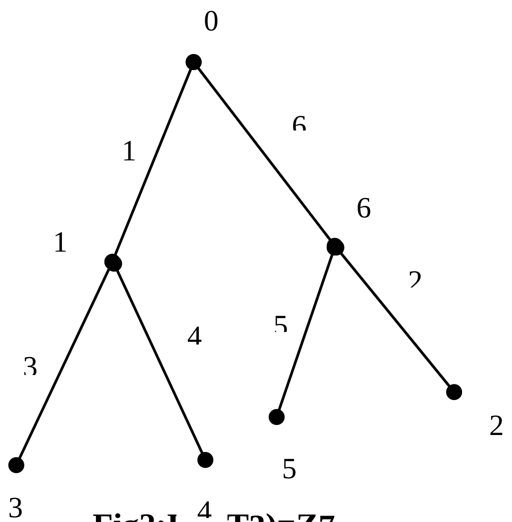

Fig2:DC(T3)=Z7

Verification 3.3

$v_1,v_2,v_3,v_4,v_5,v_6,v_7$,-Vertices

$f(v_1v_2)=1, f(v_2v_3)=3, f(v_2v_4)=4, f(v_1v_5)=6,$

$f(v_5v_6)=5, f(v_5v_7)=2,$

$f^+(v_1)=7, f^+(v_2)=8, f^+(v_3)=3, f^+(v_4)=4,$

$f^+(v_5)=13$, $f^+(v_6)=5, f^+(v_7)=2$

Thereforeallaredistinct

Congruency

7≡**<u>0</u>**(mod**7**)

8≡**<u>1</u>**(mod**7**)

≡**<u>3</u>**(mod**7**)

4≡**<u>4</u>**(mod**7**)

13≡**<u>6</u>**(mod**7**)

5≡**<u>5</u>**(mod**7**)

2≡**<u>2</u>**(mod**7**)

ThereforeT_3isadistinctcongruencemagicgraph.**F(3,7)**

LettheintegersetbeZ_{22}={0,1,2,3,4,5,….21}

Construction3.4:-

Since by using the definition of labeling, we label the edges by1,2,3,4,5,…21fromZ_{22}.

Labeling

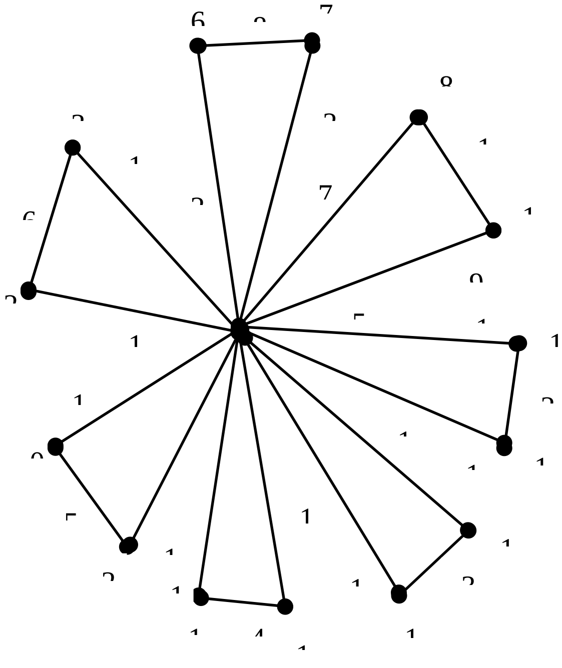

Fig3:DC(F(3,7))=Z22

Verification 3.5

$v_1,v_2,v_3,v_4,v_5,v_6, v_7,v_8,v_9,v_{10},v_{11},v_{12},v_{13},v_{14},v_{15}$,-Vertices

$f(v_1v_2)=1, f(v_3v_4)=2, f(v_5v_6)=3, f(v_7v_8)=4, f(v_9v_{10})=5, f(v_{11}v_{12})=6, f(v_{13}v_{14})=8, f(v_{15}v_1)=7, f(v_2v_{15})=9, f(v_3v_{15})=10, f(v_4v_{15})=11, f(v_5v_{15})=12, f(v_6v_{15})=13, f(v_7v_{15})=14, f(v_8v_{15})=15, f(v_9v_{15})=16, f(v_{10}v_{15})=17, f(v_{11}v_{15})=18,$,$f(v_{12}v_{15})=19, f(v_{13}v_{15})=20,$,$f(v_{14}v_{15})=21$, $f^+(v_i)$=distinct

,for all
i=1,2,…15Thereforeallaredistinct.ThereforeF(3,7)isadistinctcongruencemagicgraph.
F(3,4)
LettheintegersetbeZ$_{13}$={0,1,2,3,4,5,….12}Construction:-
Sincebyusingthedefinitionoflabeling,welabeltheedgesby1,2,3,4,5,…12fromZ$_{13}$.

Labeling 3.6

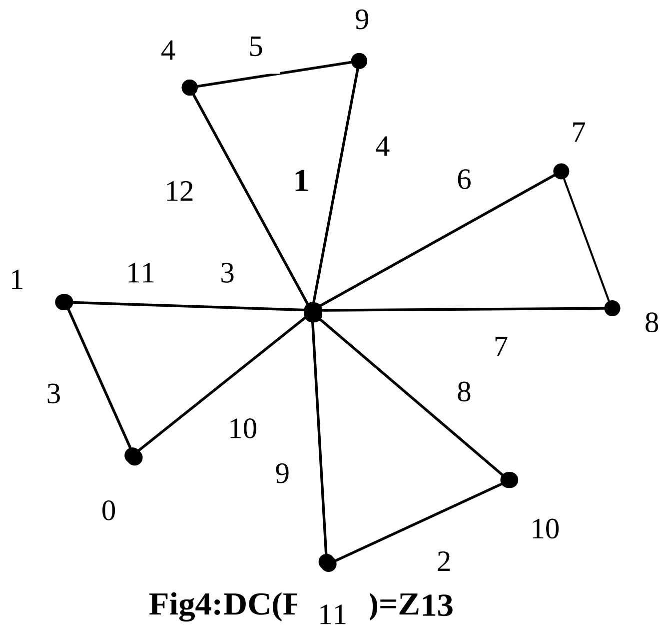

Fig4:DC(F)=Z13

Verification 3.7

v_1, v_2, v_3, v_4, v_5,v_6,v_7, v_8,v_9-Vertices
$f(v_1v_2)=1$,$f(v_3v_4)=2$,$f(v_5v_6)=3$,$f(v_7v_8)=5$,$f(v_1v_9)=6$,$f(v_2v_9)=7$,$f(v_3v_9)=8$,$f(v_4v_9)=9$,$f(v_5 v_9)=10$, $f(v_6 v_9)=11$, $f(v_7 v_9)=12$,$f(v_8 v_9)=4$,$f^+(v_1)=7$, $f^+(v_2)= 8$,$f^+(v_3)=10$, $f^+(v_4)=11$,$f^+(v_5)=13$, $f^+(v_6)=14$, $f^+(v_7)=17$, $f^+(v_8)=9$
Therefore all are distinct

Congruency 3.8

7≡**7**(mod**13**)
8≡**8**(mod**13**)
10≡**10**(mod**13**)
11≡**11**(mod**13**)
13≡**0**(mod**13**)
14≡**1**(mod**13**)
17≡**4**(mod**13**)
9≡**9**(mod**13**)

Therefore F (3,4) is a distinct congruence magic graph.

F(3,6):-

Let the integer set be Z$_{19}$={0,1,2,3,4,5,….18}Construction:-
Since by using the definition of labeling, we label the edges by1,2,3,4,5,…18fromZ$_{19}$.

Labeling 3.9 :-

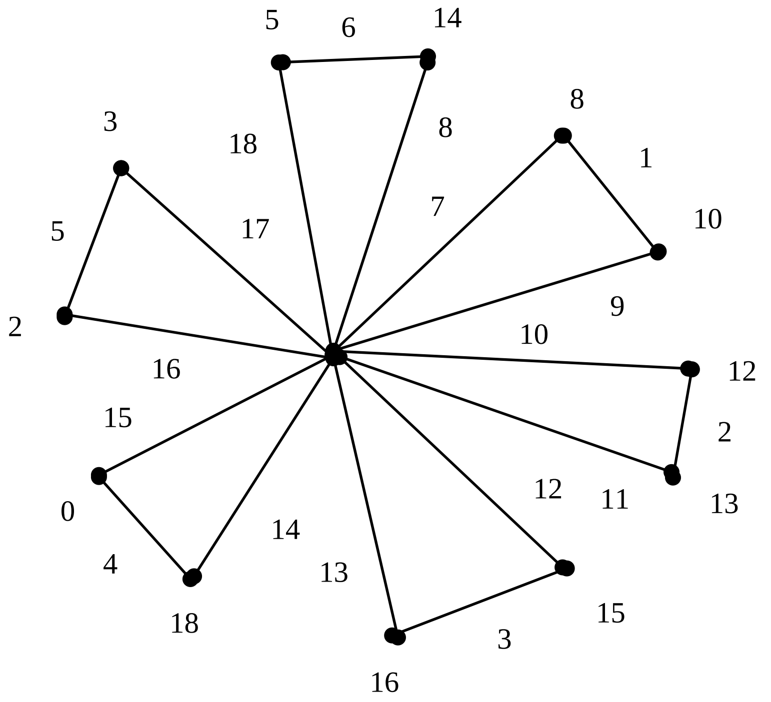

Fig4:DC(F(3,6))=Z19

Verification 3.10

v_1, v_2, v_3, v_4, v_5,v_6,v_7 v_8, v_9, v_{10},v_{11},v_{12}, v_{13}-Vertices

$f(v_1 v_2)=1$, $f(v_3 v_4)=2$, $f(v_5 v_6)=3$, $f(v_7 v_8)=4$,$f(v_9 v_{10})=5$, $f(v_{11} v_{12})=6$, $f(v_1 v_{13})=7$, $f(v_2 v_{13})=9$,$f(v_3 v_{13})=10$,$f(v_4v_{13})=11$,$f(v_5v_{13})=12$,$f(v_6 v_{13})=13$,

$f(v_7v_{13})=14$,$f(v_8v_{13})=15$,$f(v_9v_{13})=16$,$f(v_{10}v_{13})=17$,$f(v_{11} v_{13})=18$,$f(v_{12} v_{13})=8$

$f^+(v_1)=8$,$f^+(v_2)=10$, $f^+(v_3)=12$,$f^+(v_4)=13$,

$f^+(v_5)=15$,

$f^+(v_6)=16$,$f^+(v_7)=18$,$f^+(v_8)=19$, $f^+(v_9)=21$,$f^+(v_{10})=22$,$f^+(v_{11})=24$

$f^+(v_{12})=14$

Therefore all are distinct

Congruency

8≡**8**(mod**19**)

10≡**10**(mod**19**)

12≡**12**(mod**19**)

13≡**13**(mod**19**)

15≡**15**(mod**19**)

16≡**16**(mod**19**)

18≡**18**(mod**19**)

19≡**0**(mod**19**)

21≡**2**(mod**19**)

22≡**3**(mod**19**)

24≡**5**(mod**19**)

$14 \equiv \underline{\mathbf{14}} (\bmod \mathbf{19})$

Therefore **F (3,6)** is a distinct congruence magic graph.

Theorem 3.11

The conjecture that the Friendship graph **F(3,n)** is an (u,v) distinct congruent spectrum Z_n Of G.

Theorem 3.12

The conjecture that the Tree graph $\mathbf{T_n}$ is an (u,v) distinct congruent spectrum Z_n Of G.

4. Conclusion

The main aim of this paper is to explore role of Graph Labeling in Communication field. Graph Labeling is powerful tool that makes things ease in various fields of networking A overview is presented especially to project the idea of Graph Labeling. Researches may get some information related to graph labeling and its applications in communication field and can get some ideas related to their field of research.

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