Long Run Performance of a Cold Standby Repairable System with Standby Failure subject to Maximum Operation and Repair Time

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Abstract
The failure of cold standby units in standby mode poses huge challenges to the system effectiveness. So the main goal of this contribution is to improve the system effectiveness of a two unit cold standby repairable system by taking into account the failure possibility of a unit in standby mode. The system starts functioning with the operation of one unit while another unitstays in cold standby mode. Whenever an operating unit crosses a prefix span of time (called Maximum operation time), it is sent for preventive maintenance and further if repair of the unit is not completed within a prefix span of time (called Maximum repair time) then it is replaced by a new one with some replacement time. There is provision of a server to conduct all type of repairs, maintenances, replacements and inspections. All of the random variables included in the present study follow the arbitrary distribution. The expressions for various explicit measures of system effectiveness are derived by adopting semi- Markov approach and regenerative point technique. Finally, the practical importance of the model is illustrated through numerical example and depicted in tabular form.

Keywords: Repairable system, Standby failure, Maximum operation and repair time, Semi- Markov approach.

1 INTRODUCTION

With the elevation of modern science and technology, electronic and communication systems have become more and more complicated. So, for the purpose of making systems more reliable and profitable, the concept of cold standby redundancy technique has gained immense popularity during the design and development phase of sophisticated and complex systems.Immense number of researchers and reliability engineers have extensively discussed the models with cold standby redundant systems because such type of systems are frequently used in business and industries. Most of the researchers and academicians, including Singh and Agrafiotis (1995),Vanderperre (2004), Yuan Lin Zhang et al. (2007), Jia and Wu (2009), Mahmoud and Moshref (2010), Malik (2013), Bhardwaj and Singh (2014, 2015), Mahmoud et al. (2015), Malik et al. (2015), Bhardwaj and Singh (2017), Yu and Tang (2017) etc., presumed that the cold standby unit resumes working with full efficiency at the failure of operative unit. But this presumption may not be true always due to the fact that sometimes there are situations where the standby unit is unable to activate at the failure of operative unit. Here, the failure can happen due to some environmental factors or due to poor maintenance activity.So the analysis of possibility of standby failure is very important to meet the challenges of modern sophisticated and complex systems. For example, the Ram Air Turbine system together with two engines forms a cold standby system in an aircraft. If both the engines stop working then the standby Ram Air Turbine system is expected to come in operation immediately. Therefore, the readiness of the standby unit is mandatory to avoid serious consequences in terms of tremendous financial loss, menace to human life or serious damage to territory.

In reliability literature, Osaki and Nakagawa (1971)studied a two unit standby redundant system with the standby failure but they have derived only the mean time and Laplace – Stieltjes transform of the distribution of the first time to system failure. Since the work of the stated researchers was limited to few performance indices, there was a dire need to explore cold standby systems more extensively to have complete idea about the system performance.

Keeping all the facts in view, we have proposed to develop and analyze the models of cold standby redundant system with the chances of standby failure. In this paper, we extend the work of Bhardwaj and Kaur (2021) to evaluate the various reliability measures of a two unit cold standby repairable system by taking into account the failure of the unit in standby mode. In this research model, the expressions are derived for various explicit measures of system effectiveness such as mean time to system failure (MTSF), steady state availability, busy period of server, expected number of repairs, replacements, inspections and preventive maintenances by identifying the system at suitable regenerative epochs.

2 THE MODEL

2.1 Notations

$E/\overline{E}$	: The set of regenerative/ Non-regenerative states
N_0	: The unit is operative and in normal mode
C_s	: The unit is in cold-standby mode
a/b	: Probability that repair / replacement feasible
F_{ui}/F_{UI}	: Failed unit under inspection /under inspection continuously from previous state
F_{ur}/F_{UR}	: Failed unit under repair / under repair continuously from previous state
F_{urp}/F_{URP}	: Failed unit under replacement / under replacement continuously from previous state
F_{wi}/F_{WI}	: Failed unit waiting for inspection / waiting for inspection continuously from previous state
F_{wr}/F_{WR}	: Failed unit waiting for repair / waiting for repair continuously from previous state
P_m/P_M	: The unit is under preventive maintenance/ under preventive maintenance continuously from previous state
WP_m/WP_M	: The unit is waiting for preventive maintenance/ waiting for preventive maintenance continuously from previous state
λ	: Constant failure rate of operative unit

$s(t)/S(t)$	: pdf / cdf of failure time of cold-standby unit
$o(t)/O(t)$	: pdf/ cdf of maximum operation time
$r(t)/R(t)$	: pdf/ cdf of maximum repair time
$h(t)/H(t)$	: pdf / cdf of inspection time of unit
$g(t)/G(t)$	: pdf / cdf of repair time of unit
$f(t)/F(t)$	: pdf / cdf of replacement time of unit
$m_p(t)/M_p(t)$	: pdf/ cdf of preventive maintenance of unit
$q_{i,j}(t)/Q_{i,j}(t)$	: pdf/cdf of first passage time from regenerative state S_i to regenerative State S_j or failed state S_j without visiting any other regenerative state in $(0,t]$
$q_{i,j.k,r}(t)/Q_{i,j.k,r}(t)$	: pdf/cdf of first passage time from regenerative state S_i to regenerative state S_j or failed state S_j visiting state S_k, S_r once in $(0,t]$
$\mu_i(t)$	: Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$	: Probability that server busy in the state S_i up to time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states
$[s]/[c]$	: Symbol for Laplace-Stietjes convolution/Laplace convolution
~ /*	: Symbol for Laplace- stietjes Transform (LST)/Laplace transform (LT)
'$(desh)$	: Symbol used to represent alternative result

2.2 Description and Assumptions

The considered cold standby system has two identical units.At the beginning, both of the units are new: one of them is in operation and other is in cold standby mode.Whenthe cold standby unit crosses the pre-specified time (called Maximum Redundancy Time), it may fail so it goes under repair but whenever an operating unit crosses a prefix span of time (called Maximum operation time) it undergoes preventive maintenance and further if repair of the unit is not completed within prefix span of time (called Maximum repair time) then it is replaced by new one with some replacement time.The model is developed under the following assumptions:

Assumption 1. A single server is available to perform all type of repairs whenever needed.

Assumption 2. All the random variables are statistically independent and follows arbitrary distributions.

Assumption 3. The switching is perfect and there is no switchover delays.

Assumption 4. Each type of repair is perfect and repaired unit works as good as new.

Using the theory of renewals (Cox 1962), the model exhibits the following mutually exclusive states:

Regenerative states:

$S_0 = (N_o, C_S)$, $S_1 = (F_{ur}, N_o)$, $S_2 = (N_o, F_{ui})$, $S_7 = (N_o, F_{urp})$, $S_{10} = (P_m, N_o)$

Non regenerative states:

$S_3 = (F_{UR}, F_{wr})$, $S_4 = (F_{wr}, F_{UI})$, $S_5 = (F_{WR}, F_{ur})$, $S_6 = (F_{wr}, F_{URp})$, $S_8 = (F_{WR}, F_{urp})$, $S_9 = (WP_m, F_{URp})$, $S_{11} = (WP_m, F_{UI})$,
$S_{12} = (F_{UR}, WP_m)$, $S_{13} = (WP_M, F_{ur})$, $S_{14} = (WP_M, F_{urp})$, $S_{15} = (P_M, WP_m)$, $S_{16} = (P_M, F_{wr})$

2.3 Transition probabilities

Let $p_{i,j} = \Pr ob\{The \text{ system transits from state } S_i \text{ and } S_j\}$ and

$Q_{i,j}(t) = P\{X_{n+1} = j, T_n \le t \ge 0 \mid X_n = i\}$

be the conditional probability that the process will be in state S_j next, given that it is currently in state S_i and the waiting time in the current state S_i is no more than *t* i.e. $q_{i,j}(t)/Q_{i,j}(t)$ is the p.d.f./ c.d.f. of direct transition time from a regenerative state S_i to regenerative state S_j without visiting any other regenerative state.

Using probabilistic considerations we obtained the following expressions for the non- zero elements $p_{i,j} \ge 0$; i, j $\in \Omega$ and $\sum_j p_{i,j} = 1$, where

$$p_{i,j} = \lim_{t\to\infty} \Pr\{X_{n+1} = j, T_n \le t \ge 0 \mid X_n = i\} = \lim_{t\to\infty} Q_{i,j}(t) = \int_0^\infty q_{i,j}(t)dt \qquad (2.3.1)$$

$$p_{0,1} = \int_0^\infty \lambda e^{-\lambda t}\overline{O}(t)\overline{S}(t)dt,\ p_{0,2} = \int_0^\infty s(t)\overline{O}(t)e^{-\lambda t}dt,\ p_{0,10} = \int_0^\infty o(t)\overline{S}(t)e^{-\lambda t}dt,\ p_{1,0} = \int_0^\infty g(t)\overline{O}(t)\overline{R}(t)e^{-\lambda t}dt,$$

$$p_{1,3} = \int_0^\infty \lambda e^{-\lambda t}\overline{R}(t)\overline{G}(t)\overline{O}(t)dt,\ p_{1,7} = \int_0^\infty r(t)\overline{G}(t)\overline{O}(t)e^{-\lambda t}dt,\ p_{1,12} = \int_0^\infty o(t)\overline{R}(t)\overline{G}(t)e^{-\lambda t}dt,$$

$$p_{2,1} = \int_0^\infty ah(t)\overline{O}(t)e^{-\lambda t}dt,\ p_{2,4} = \int_0^\infty \lambda e^{-\lambda t}\overline{O}(t)\overline{H}(t)dt,\ p_{2,7} = \int_0^\infty bh(t)\overline{O}(t)e^{-\lambda t}dt,\ p_{2,11} = \int_0^\infty o(t)\overline{H}(t)e^{-\lambda t}dt,$$

$$p_{3,1} = \int_0^\infty g(t)\overline{R}(t)dt,\ p_{3,8} = \int_0^\infty r(t)\overline{G}(t)dt,\ p_{4,5} = \int_0^\infty ah(t)dt,\ p_{4,8} = \int_0^\infty bh(t)dt,\ p_{5,1} = \int_0^\infty g(t)\overline{R}(t)dt,\ p_{5,8} = \int_0^\infty r(t)\overline{G}(t)dt,$$

$$p_{6,1} = \int_0^\infty f(t)dt,\ p_{7,0} = \int_0^\infty f(t)\overline{O}(t)e^{-\lambda t}dt,\ p_{7,6} = \int_0^\infty \lambda e^{-\lambda t}\overline{O}(t)\overline{F}(t)dt,\ p_{7,9} = \int_0^\infty o(t)\overline{F}(t)e^{-\lambda t}dt,\ p_{8,1} = \int_0^\infty f(t)dt,$$

$$p_{9,10} = \int_0^\infty f(t)dt,\ p_{10,0} = \int_0^\infty m_p(t)\overline{O}(t)e^{-\lambda t}dt,\ p_{10,15} = \int_0^\infty o(t)\overline{M}_p(t)e^{-\lambda t}dt,\ p_{10,16} = \int_0^\infty \lambda e^{-\lambda t}\overline{O}(t)\overline{M}_p(t)dt,\ p_{11,13} = \int_0^\infty ah(t)dt,$$

$$p_{11,14} = \int_0^\infty bh(t)dt,\ p_{12,10} = \int_0^\infty g(t)\overline{R}(t)dt,\ p_{12,14} = \int_0^\infty r(t)\overline{G}(t)dt,\ p_{13,10} = \int_0^\infty g(t)\overline{R}(t)dt,\ p_{13,14} = \int_0^\infty r(t)\overline{G}(t)dt,$$

$$p_{14,10} = \int_0^\infty f(t)dt,\ p_{15,10} = \int_0^\infty m_p(t)dt,\ p_{16,1} = \int_0^\infty m_p(t)dt,\ p_{1,1.3} = p_{1,3}[c]p_{3,1},\ p_{1,1.3,8} = p_{1,3}[c]p_{3,8}[c]p_{8,1},\ p_{1,10.12} = p_{1,12}[c]p_{12,10},$$

$$p_{1,10.12,14} = p_{1,12}[c]p_{12,14}[c]p_{14,10},\ p_{2,1.4,5} = p_{2,4}[c]p_{4,5}[c]p_{5,1},\ p_{2,1.4,8} = p_{2,4}[c]p_{4,8}[c]p_{8,1},$$

$$p_{2,1.4,5,8} = p_{2,4}[c]p_{4,5}[c]p_{5,8}[c]p_{8,1},$$

$$p_{2,10.11,13} = p_{2,11}[c]p_{11,13}[c]p_{13,10},\ p_{2,10.11,14} = p_{2,11}[c]p_{11,14}[c]p_{14,10},\ p_{2,10.11,13,14} = p_{2,11}[c]p_{11,13}[c]p_{13,14}[c]p_{14,3},$$

$$p_{7,10.9} = p_{7,9}[c]p_{9,10},\ p_{7,1.6} = p_{7,6}[c]p_{6,1},\ p_{10,1.16} = p_{10,16}[c]p_{16,1},\ p_{10,10.15} = p_{10,15}[c]p_{15,10}$$

Here it can be observed that $\sum_{j} p_{i,j} = 1$

2.4 Mean Sojourn Times

The unconditional mean time taken by the system to transit to any regenerative state S_j when it is counted from the epoch of entrance into that state S_i is given by

$$m_{i,j} = \int_0^\infty t d\{Q_{i,j}(t)\} \quad (2.4.1)$$

Let T denotes the time to system failure then the mean sojourn time in the state S_i is given by

$$\mu_i = E(t) = \int_0^\infty P(T > t)dt \qquad (2.4.2)$$

Using these formulations, the following results are obtained

$$\mu_0 = \int_0^\infty e^{-\lambda t}\overline{O}(t)\overline{S}(t)dt,\ \mu_1 = \int_0^\infty e^{-\lambda t}\overline{O}(t)\overline{G}(t)\overline{R}(t)dt,\ \mu_2 = \int_0^\infty e^{-\lambda t}\overline{O}(t)\overline{H}(t)dt,\ \mu_7 = \int_0^\infty e^{-\lambda t}\overline{O}(t)\overline{F}(t)dt,$$

$$\mu_{10} = \int_0^\infty e^{-\lambda t}\overline{O}(t)\overline{M}_p(t)dt,\ \mu_1' = m_{1,0} + m_{1,7} + m_{1,1.3} + m_{1,1.3,8} + m_{1,10.12} + \mathrm{m}_{1,10.12,14},$$

$$\mu_2' = m_{2,1} + m_{2,7} + m_{2,1.4,5} + m_{2,1.4,8} + m_{2,1.4,5,8} + m_{2,10.11,13} + m_{2,10.11,14} + m_{2,10.11,13,14},$$

$$\mu_7' = m_{7,0} + m_{7,1.6} + m_{7,10.9},\ \mu_{10}' = m_{10,0} + m_{10,1.16} + m_{10,10.15}$$

3. SYSTEM PERFORMANCE MEASURES

3.1 Reliability and MTSF

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_0(t) = Q_{0,1}(t)[s]\phi_1(t) + Q_{0,2}(t)[s]\phi_2(t) + Q_{0,10}(t)[s]\phi_{10}(t)$$

$$\phi_1(t) = Q_{1,0}(t)[s]\phi_0(t) + Q_{1,3}(t) + Q_{1,7}(t)[s]\phi_7(t) + Q_{1,12}(t)$$

$$\phi_2(t) = Q_{2,1}(t)[s]\phi_1(t) + Q_{2,4}(t) + Q_{2,7}(t)[s]\phi_7(t) + Q_{2,11}(t)$$

$$\phi_7(t) = Q_{7,0}(t)[s]\phi_0(t) + Q_{7,6}(t) + Q_{7,9}(t)$$

$$\phi_{10}(t) = Q_{10,0}(t)[s]\phi_0(t) + Q_{10,15}(t) + Q_{10,16}(t) \quad (3.1.1)$$

Taking LST of above relations (3.1.1), we get the following matrix form.

$$\begin{bmatrix} \tilde{\phi}_0 \\ \tilde{\phi}_1 \\ \tilde{\phi}_2 \\ \tilde{\phi}_7 \\ \tilde{\phi}_{10} \end{bmatrix} = \begin{bmatrix} 1 & -\tilde{Q}_{0,1} & -\tilde{Q}_{0,2} & 0 & -\tilde{Q}_{0,10} \\ -\tilde{Q}_{1,0} & 1 & 0 & -\tilde{Q}_{1,7} & 0 \\ 0 & -\tilde{Q}_{2,1} & 1 & -\tilde{Q}_{2,7} & 0 \\ -\tilde{Q}_{7,0} & 0 & 0 & 1 & 0 \\ -\tilde{Q}_{10,0} & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \tilde{Q}_{1,3} + \tilde{Q}_{1,12} \\ \tilde{Q}_{2,4} + \tilde{Q}_{2,11} \\ \tilde{Q}_{7,6} + \tilde{Q}_{7,9} \\ \tilde{Q}_{10,15} + \tilde{Q}_{10,16} \end{bmatrix} \qquad (3.1.2)$$

Here it should be noted that the argument s is omitted for brevity. Solving equations (3.1.2) for $\tilde{\phi}_0(s)$, we get

$$\tilde{\phi}_0(s) = \frac{[\tilde{Q}_{1,3}+\tilde{Q}_{1,12}]\{\tilde{Q}_{0,1}+\tilde{Q}_{0,2}\tilde{Q}_{2,1}\}+\tilde{Q}_{0,2}[\tilde{Q}_{2,11}+\tilde{Q}_{2,4}]+[\tilde{Q}_{7,6}+\tilde{Q}_{7,9}]\{\tilde{Q}_{0,1}\tilde{Q}_{1,4}+\tilde{Q}_{0,2}[\tilde{Q}_{2,7}+\tilde{Q}_{1,7}\tilde{Q}_{2,1}]\} + \tilde{Q}_{0,10}[\tilde{Q}_{10,15}+\tilde{Q}_{10,16}]}{1-\tilde{Q}_{0,1}\tilde{Q}_{1,7}\tilde{Q}_{7,0}-\tilde{Q}_{0,2}\tilde{Q}_{7,0}\{\tilde{Q}_{2,7}+\tilde{Q}_{1,7}\tilde{Q}_{2,1}\}-\tilde{Q}_{0,10}\tilde{Q}_{10,0}-\tilde{Q}_{1,0}\{\tilde{Q}_{0,1}+\tilde{Q}_{0,2}\tilde{Q}_{2,1}\}}$$

The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s\to 0}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{\mu_0+[p_{0,1}+p_{0,2}p_{2,1}]\mu_1+p_{0,2}\mu_2+[p_{0,1}p_{1,7}+p_{0,2}\{p_{2,7}+p_{1,7}p_{2,1}\}]\mu_7+p_{0,10}\mu_{10}}{1-p_{0,1}[p_{1,0}+p_{1,7}p_{7,0}]-p_{0,2}[p_{1,0}p_{2,1}+p_{7,0}\{p_{2,7}+p_{1,7}p_{2,1}\}]-p_{0,10}p_{10,0}}$$

(3.1.3)

The reliability of system model can be obtained as follows:

$$R(t) = L^{-1}\left[\frac{1-\tilde{\phi}_0(s)}{s}\right] \qquad (3.1.4)$$

3.2 Steady State Availability

Let $M_i(t)$ be the probability that system is up initially in state $S_i \in E$ is up at time t without visiting any other regenerative state, we have

$M_0(t)=e^{-\lambda t}\bar{S}(t)\bar{O}(t)$, $M_1(t)=e^{-\lambda t}\bar{G}(t)\bar{R}(t)\bar{O}(t)$, $M_2(t)=e^{-\lambda t}\bar{H}(t)\bar{O}(t)$, $M_7(t)=e^{-\lambda t}\bar{F}(t)\bar{O}(t)$,

$M_{10}(t)=e^{-\lambda t}\bar{M}_p(t)\bar{O}(t)$

Let $A_i(t)$ be the probability that the system is in upstate at instant ‘t’ given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are as follows:

$$A_0(t)=M_0(t)+q_{0,1}(t)[c]A_1(t)+q_{0,2}(t)[c]A_2(t)+q_{0,10}(t)[c]A_{10}(t)$$

$$A_1(t)=M_1(t)+q_{1,0}(t)[c]A_0(t)+[q_{1,1.3}(t)+q_{1,1.3,8}(t)][c]A_1(t)+q_{1,7}(t)[c]A_7(t)+[q_{1,10.12}(t)+q_{1,10.12,14}(t)][c]A_{10}(t)$$

$$A_2(t)=M_2(t)+[q_{2,1}(t)+q_{2,1.4,5}(t)+q_{2,1.4,8}(t)+q_{2,1.4,5,8}(t)][c]A_1(t)+q_{2,7}(t)[c]A_7(t)+[q_{2,10.11,13}(t)+q_{2,10.11,14}(t) + q_{2,10.11,13,14}(t)][c]A_{10}(t)$$

$$A_7(t)=M_7(t)+q_{7,0}(t)[c]A_0(t)+q_{7,1.6}(t)[c]A_1(t)+q_{7,10.9}(t)[c]A_{10}(t)$$

$$A_{10}(t)=M_{10}(t)+q_{10,0}(t)[c]A_0(t)+q_{10,1.16}(t)[c]A_1(t)+q_{10,10.15}(t)[c]A_{10}(t) \qquad (3.2.1)$$

Taking LST of above relation (3.2.1) and putting them in matrix form we get

$$\begin{bmatrix} A_0^* \\ A_1^* \\ A_2^* \\ A_7^* \\ A_{10}^* \end{bmatrix} = \begin{bmatrix} 1 & -q_{0,1}^* & -q_{0,2}^* & 0 & -q_{0,10}^* \\ -q_{1,0}^* & 1-q_{1,1.3}^* * -q_{1,1.3,8}^* & 0 & -q_{1,7}^* & -[q_{1,10.12}^*+q_{1,10.12,14}^*] \\ 0 & \begin{matrix}-[q_{2,1}^*+q_{2,1.4,5}^*+q_{2,1.4,8}^* \\ +q_{2,1.4,5,8}^*]\end{matrix} & 1 & -q_{2,7}^* & \begin{matrix}-[q_{2,10.11,13}^*+q_{2,10.11,14}^* \\ +q_{2,10.11,13,14}^*]\end{matrix} \\ -q_{7,0}^* & -q_{7,1.6}^* & 0 & 1 & -q_{7,10.9}^* \\ -q_{10,0}^* & -q_{10,1.16}^* & 0 & 0 & 1-q_{10,10.15}^* \end{bmatrix} \begin{bmatrix} M_0^* \\ M_1^* \\ M_2^* \\ M_7^* \\ M_{10}^* \end{bmatrix} \qquad (3.2.2)$$

Solve equations (3.2.2) for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

Where

$$\begin{aligned}
N_2(s) = {} & [-M_0^* q_{1,7}^* + M_1^* q_{0,2}^* q_{2,7}^* - M_2^* q_{0,2}^* q_{1,7}^*]\{q_{10,1.16}^* q_{7,10.9}^* + q_{7,1.6}^*[1 - q_{10,10.15}^*]\} + [M_0^* + M_2^* q_{0,2}^* + M_7^* q_{0,2}^* q_{2,7}^*]\{[1- \\
& q_{10,10.15}^*][1 - q_{1,1.3}^* - q_{1,1.3,8}^*] - q_{10,1.16}^*\{q_{1,10.12}^* + q_{1,10.12,14}^*\}\} + [M_1^* q_{0,2}^* + M_7^* q_{0,2}^* q_{1,7}^*]\{[q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.4,5,8}^*] \\
& [1 - q_{10,10.15}^*] + q_{10,1.16}^*[q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*]\} + [M_1^* + M_7^* q_{1,7}^*]\{q_{0,1}^*[1 - q_{10,10.15}^*] + q_{0,10}^* q_{10,1.16}^*\} \\
& + M_{10}^* q_{0,2}^* q_{1,7}^*\{q_{7,10.9}^*[q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.4,5,8}^*] - q_{7,1.6}^*[q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*]\} + M_{10}^* q_{0,2}^* q_{2,7}^* \\
& \{q_{7,10.9}^*[1 - q_{1,1.3}^* - q_{1,1.3,8}^*] + q_{7,1.6}^*[q_{1,10.12}^* + q_{1,10.12,14}^*]\} + M_{10}^* q_{0,2}^*\{[1 - q_{1,1.3}^* - q_{1,1.3,8}^*][q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*] \\
& + [q_{1,10.12}^* + q_{1,10.12,14}^*][q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.4,5,8}^*]\} + M_{10}^* q_{1,7}^*\{q_{0,1}^* q_{7,10.9}^* - q_{0,10}^* q_{7,1.6}^*\} + M_{10}^*\{q_{0,1}^*[q_{1,10.12}^* \\
& + q_{1,10.12,14}^*] + q_{0,10}^*[1 - q_{1,1.3}^* - q_{1,1.3,8}^*]\}
\end{aligned}$$

$$\begin{aligned}
D_2(s) = {} & -\{q_{10,0}^* q_{7,10.9}^* + q_{7,0}^*[1 - q_{10,10.15}^*]\}[q_{0,2}^* q_{1,7}^*[q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.4,5,8}^*] + q_{0,2}^* q_{2,7}^*[1 - q_{1,1.3}^* - q_{1,1.3,8}^*] \\
& + q_{0,1}^* q_{1,7}^*] + \{q_{10,0}^* q_{7,1.6}^* - q_{7,0}^* q_{10,1.16}^*\}[q_{0,2}^* q_{1,7}^*[q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*] - q_{0,2}^* q_{2,7}^*[q_{1,10.12}^* + q_{1,10.12,14}^*] \\
& + q_{0,10}^* q_{1,7}^*] - \{q_{10,1.16}^* q_{7,10.9}^* + q_{7,1.6}^*[1 - q_{10,10.15}^*]\}[q_{0,2}^* q_{2,7}^* q_{1,0}^* + q_{1,7}^*] - q_{0,2}^* q_{1,0}^*\{[q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* \\
& + q_{2,1.4,5,8}^*][1 - q_{10,10.15}^*] + [q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*]q_{10,1.16}^*\} - q_{0,2}^* q_{10,0}^*\{[q_{2,10.11,13}^* + q_{2,10.11,14}^* \\
& + q_{2,10.11,13,14}^*][1 - q_{1,1.3}^* - q_{1,1.3,8}^*] + [q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.14,5,8}^*][q_{1,10.12}^* + q_{1,10.12,14}^*]\} + [1 - q_{1,1.3}^* - q_{1,1.3,8}^*] \\
& [1 - q_{10,10.15}^*] - q_{10,1.16}^*[q_{1,10.12}^* + q_{1,10.12,14}^*] - q_{0,1}^*\{q_{1,0}^*[1 - q_{10,10.15}^*] + q_{10,0}^*[q_{1,10.12}^* + q_{1,10.12,14}^*]\} - q_{0,10}^*\{q_{1,0}^* q_{10,1.16}^* \\
& + q_{10,0}^*[1 - q_{1,1.3}^* - q_{1,1.3,8}^*]\}
\end{aligned}$$

Using Tauberian theorem (Yakimiv 2005), the steady state availability is given by

$$A_0(\infty) = \lim_{s \to 0} sA_0^*(s) = \frac{\begin{aligned} & [-p_{1,7}\mu_0 + p_{0,2}p_{2,7}\mu_1 - p_{0,2}p_{1,7}\mu_2]\{p_{10,0}p_{7,1.6} - p_{10,1.16}p_{7,0}\} + [\mu_0 + p_{0,2}\mu_2 + p_{0,2}p_{2,7}\mu_7] \\ & \{[1 - p_{1,1.3} - p_{1,1.3,8}]p_{10,0} + p_{1,0}p_{10,1.16}\}[p_{0,2}\mu_1 + p_{0,2}p_{1,7}\mu_7]p_{10,0}[1 - p_{2,7}] + [p_{10,0}(-\mu_1 - p_{1,7}\mu_7) \\ & + (p_{1,7}p_{7,0} + p_{1,0})\mu_{10}]p_{0,2}\{p_{2,10.11,13} + p_{2,10.11,14} + p_{2,10.11,13,14}\} + p_{7,10.9}[p_{0,2}p_{1,0}p_{2,7} + p_{1,7}]\mu_{10} \\ & + [p_{1,10.12} + p_{1,10.12,14}][1 - p_{0,2}p_{2,7}p_{7,0}]\mu_{10} + p_{0,10}[p_{1,0} + p_{1,7}p_{7,0}]\mu_{10} + \{p_{0,1}p_{10,0} + p_{10,1.16}\} \\ & [\mu_1 + \mu_7 p_{1,7}] \end{aligned}}{D_2} \tag{3.2.3}$$

where

$$\begin{aligned}
D_2 = {} & [-p_{1,7}\mu_0 + p_{0,2}p_{2,7}\mu_1' - p_{0,2}p_{1,7}\mu_2']\{p_{10,0}p_{7,1.6} - p_{10,1.16}p_{7,0}\} + [\mu_0 + p_{0,2}\mu_2' + p_{0,2}p_{2,7}\mu_7']\{[1 - p_{1,1.3} - p_{1,1.3,8}] \\
& p_{10,0} + p_{1,0}p_{10,1.16}\}[p_{0,2}\mu_1' + p_{0,2}p_{1,7}\mu_7']p_{10,0}[1 - p_{2,7}] + [p_{10,0}(-\mu_1' - p_{1,7}\mu_7') + (p_{1,7}p_{7,0} + p_{1,0})\mu_{10}']p_{0,2}\{p_{2,10.11,13} \\
& + p_{2,10.11,14} + p_{2,10.11,13,14}\} + p_{7,10.9}[p_{1,7} + p_{0,2}p_{1,0}p_{2,7}]\mu_{10}' + [p_{1,10.12} + p_{1,10.12,14}][1 - p_{0,2}p_{2,7}p_{7,0}]\mu_{10}' + p_{0,10}[p_{1,0} \\
& + p_{1,7}p_{7,0}]\mu_{10}' + \{p_{0,1}p_{10,0} + p_{10,1.16}\}[\mu_1' + \mu_7' p_{1,7}]
\end{aligned}$$

3.3 Busy Period Due to Inspection

Let $W_i^I(t)$ be the probability that the server is busy in state S_i due to inspection of the unit upto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_2^I(t) = e^{-\lambda t}\overline{H}(t)\overline{O}(t) + (o(t)e^{-\lambda t}[c]1)\overline{H}(t) + (\lambda e^{-\lambda t}\overline{O}(t)[c]1)\overline{H}(t) \tag{3.3.1}$$

Let $B_i^I(t)$ be the probability that the server is busy in inspection of the unit due to cold-standby failure at an instant 't' given that the system entered S_i state at time t=0. The recursive relations for $B_i^I(t)$ are as follows

$$B_0^I(t) = q_{0,1}(t)[c]B_1^I(t) + q_{0,2}(t)[c]B_2^I(t) + q_{0,10}(t)[c]B_{10}^I(t)$$

$$B_1^I(t) = q_{1,0}(t)[c]B_0^I(t) + [q_{1,1.3}(t) + q_{1,1.3,8}(t)][c]B_1^I(t) + q_{1,7}(t)[c]B_7^I(t) + [q_{1,10.12}(t) + q_{1,10.12,14}(t)][c]B_{10}^I(t)$$

$$B_2^I(t) = W_2^I(t) + [q_{2,1}(t) + q_{2,1.4,5}(t) + q_{2,1.4,8}(t) + q_{2,1.4,5,8}(t)][c]B_1^I(t) + q_{2,7}(t)[c]B_7^I(t) + [q_{2,10.11,13}(t) + q_{2,10.11,14}(t) + q_{2,10.11,13,14}(t)][c]B_{10}^I(t)$$

$$B_7^I(t) = q_{7,0}(t)[c]B_0^I(t) + q_{7,1.6}(t)[c]B_1^I(t) + q_{7,10.9}(t)[c]B_{10}^I(t)$$

$$B_{10}^I(t) = q_{10,0}(t)[c]B_0^I(t) + q_{10,1.16}(t)[c]B_1^I(t) + q_{10,10.15}(t)[c]B_{10}^I(t) \qquad (3.3.2)$$

Taking LST of above relation (3.3.1), we get the relations in following matrix form.

$$\begin{bmatrix} B_0^{I^*} \\ B_1^{I^*} \\ B_2^{I^*} \\ B_7^{I^*} \\ B_{10}^{I^*} \end{bmatrix} = \begin{bmatrix} 1 & -q_{0,1}^* & -q_{0,2}^* & 0 & -q_{0,10}^* \\ -q_{1,0}^* & 1-q_{1,1.3}^* * -q_{1,1.3,8}^* & 0 & -q_{1,7}^* & -[q_{1,10.12}^* + q_{1,10.12,14}^*] \\ 0 & -[q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.4,5,8}^*] & 1 & -q_{2,7}^* & -[q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*] \\ -q_{7,0}^* & -q_{7,1.6}^* & 0 & 1 & -q_{7,10.9}^* \\ -q_{10,0}^* & -q_{10,1.16}^* & 0 & 0 & 1-q_{10,10.15}^* \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ W_2^{I^*} \\ 0 \\ 0 \end{bmatrix} \qquad (3.3.3)$$

Solve equations (3.3.3) for $B_0^{I^*}(s)$, we get

$$B_0^{I^*}(s) = \frac{W_2^{I^*} q_{0,2}^* \{-q_{1,7}^*[q_{10,1.16}^* q_{7,10.9}^* + q_{7,1.6}^*[1-q_{10,10.15}^*]] + [1-q_{1,1.3}^* - q_{1,1.3,8}^*][1-q_{10,10.15}^*] - [q_{1,10.12}^* + q_{1,10.12,14}^*] q_{10,1.16}^*\}}{D_2(s)}$$

Where $D_2(s)$ is already mentioned.

The time for which server is busy due to inspection is given by

$$B_0^I(\infty) = \lim_{s\to 0} sB_0^{I^*}(s) = \frac{W_2^{I^*}(0) p_{0,2}\{-p_{1,7}[p_{10,1.16} p_{7,10.9} + p_{7,1.6}[1-p_{10,10.15}]] + [1-p_{1,1.3} - p_{1,1.3.8}]p_{10,0} + [p_{1,0} + p_{1,7}]p_{10,1.16}\}}{D_2}$$

(3.3.4)

Where D_2 is already mentioned.

3.4 Busy Period Due to Repair

Let $W_i^R(t)$ be the probability that the server is busy in state S_i due to repair of the unit upto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_1^R(t) = e^{-\lambda t}\overline{G}(t)\overline{R}(t)\overline{O}(t) + (e^{-\lambda t}\overline{O}(t)\overline{G}(t)[c]1)\overline{R}(t) + (o(t)e^{-\lambda t}\overline{G}(t)[c]1)\overline{R}(t) \qquad (3.4.1)$$

Let $B_i^R(t)$ be the probability that the server is busy in repair of the unit due to failure at an instant 't' given that the system entered S_i state at time t=0. The recursive relations for $B_i^R(t)$ are as follows

$$B_0^R(t) = q_{0,1}(t)[c]B_1^R(t) + q_{0,2}(t)[c]B_2^R(t) + q_{0,10}(t)[c]B_{10}^R(t)$$

$$B_1^R(t) = W_1^R(t) + q_{1,0}(t)[c]B_0^R(t) + [q_{1,1.3}(t) + q_{1,1.3,8}(t)][c]B_1^R(t) + q_{1,7}(t)[c]B_7^R(t) + [q_{1,10.12}(t) + q_{1,10.12,14}(t)][c]B_{10}^R(t)$$

$$B_2^R(t) = [q_{2,1}(t) + q_{2,1.4,5}(t) + q_{2,1.4,8}(t) + q_{2,1.4,5,8}(t)][c]B_1^R(t) + q_{2,7}(t)[c]B_7^R(t) + [q_{2,10.11,13}(t) + q_{2,10.11,14}(t) + q_{2,10.11,13,14}(t)][c]B_{10}^R(t)$$

$$B_7^R(t) = q_{7,0}(t)[c]B_0^R(t) + q_{7,1.6}(t)[c]B_1^R(t) + q_{7,10.9}(t)[c]B_{10}^R(t)$$

$$B_{10}^R(t) = q_{10,0}(t)[c]B_0^R(t) + q_{10,1.16}(t)[c]B_1^R(t) + q_{10,10.15}(t)[c]B_{10}^R(t) \quad (3.4.2)$$

Taking LST of above relation (3.4.1), we get the relations in following matrix form.

$$\begin{bmatrix} B_0^{R^*} \\ B_1^{R^*} \\ B_2^{R^*} \\ B_7^{R^*} \\ B_{10}^{R^*} \end{bmatrix} = \begin{bmatrix} 1 & -q_{0,1}^* & -q_{0,2}^* & 0 & -q_{0,10}^* \\ -q_{1,0}^* & 1-q_{1,1.3}^* * -q_{1,1.3,8}^* & 0 & -q_{1,7}^* & -[q_{1,10.12}^* + q_{1,10.12,14}^*] \\ 0 & -[q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.4,5,8}^*] & 1 & -q_{2,7}^* & -[q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*] \\ -q_{7,0}^* & -q_{7,1.6}^* & 0 & 1 & -q_{7,10.9}^* \\ -q_{10,0}^* & -q_{10,1.16}^* & 0 & 0 & 1-q_{10,10.15}^* \end{bmatrix} \begin{bmatrix} 0 \\ W_1^{R^*} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.4.3)$$

Solve equations (3.4.3) for $B_0^{R^*}(s)$, we get

$$B_0^{R^*}(s) = \frac{\begin{array}{l} W_1^{R^*} q_{0,2}^* \{q_{2,7}^*[q_{10,1.16}^* q_{7,10.9}^* + q_{7,1.6}^*\{1-q_{10,10.15}^*\}] + [1-q_{10,10.15}^*][q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.4,5,8}^*] + q_{10,1.16}^* \\ [q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*]\} + W_1^{R^*}\{q_{0,1}^*[1-q_{10,10.15}^*] + q_{0,10}^* q_{10,1.16}^*\} \end{array}}{D_2(s)}$$

Where $D_2(s)$ is already mentioned.

The time for which server is busy due to repair is given by

$$B_0^R(\infty) = \lim_{s\to 0} sB_0^{R^*}(s) = \frac{\begin{array}{l} W_1^{R^*}(0) p_{0,2}\{p_{2,7}[p_{10,1.16} p_{7,10.9} + p_{7,1.6}\{1-p_{10,10.15}\}] + p_{10,0}[p_{2,1} + p_{2,1.4,5} + p_{2,1.4,8} + p_{2,1.4,5,8}] \\ + p_{10,1.16}[1-p_{2,7}]\} + W_1^{R^*}(0)\{p_{0,1}[1-p_{10,10.15}] + p_{0,10} p_{10,1.16}\} \end{array}}{D_2}$$

(3.4.4)

Where D_2 is already mentioned.

3.5 Busy Period Due to Replacement

Let $W_i^{Rp}(t)$ be the probability that the server is busy in state S_i due to replacement of the unit upto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_7^{Rp}(t) = e^{-\lambda t}\overline{F}(t)\overline{O}(t) + (o(t)e^{-\lambda t}[c]1)\overline{F}(t) + (\lambda e^{-\lambda t}\overline{O}(t)[c]1)\overline{F}(t) \qquad (3.5.1)$$

Let $B_i^{Rp}(t)$ be the probability that the server is busy in replacement of the unit due to failure at an instant 't' given that the system entered S_i state at time t=0. The recursive relations for $B_i^{Rp}(t)$ are as follows

$$B_0^{Rp}(t) = q_{0,1}(t)[c]B_1^{Rp}(t) + q_{0,2}(t)[c]B_2^{Rp}(t) + q_{0,10}(t)[c]B_{10}^{Rp}(t)$$

$$B_1^{Rp}(t) = q_{1,0}(t)[c]B_0^{Rp}(t) + [q_{1,1.3}(t) + q_{1,1.3,8}(t)][c]B_1^{Rp}(t) + q_{1,7}(t)[c]B_7^{Rp}(t) + [q_{1,10.12}(t) + q_{1,10.12,14}(t)][c]B_{10}^{Rp}(t)$$

$$B_2^{Rp}(t) = [q_{2,1}(t) + q_{2,1.4,5}(t) + q_{2,1.4,8}(t) + q_{2,1.14,5,8}(t)][c]B_1^{Rp}(t) + q_{2,7}(t)[c]B_7^{Rp}(t) + [q_{2,10.11,13}(t) + q_{2,10.11,14}(t) + q_{2,10.11,13,14}(t)][c]B_{10}^{Rp}(t)$$

$$B_7^{Rp}(t) = W_7^{Rp}(t) + q_{7,0}(t)[c]B_0^{Rp}(t) + q_{7,1.6}(t)[c]B_1^{Rp}(t) + q_{7,10.9}(t)[c]B_{10}^{Rp}(t)$$

$$B_{10}^{Rp}(t) = q_{10,0}(t)[c]B_0^{Rp}(t) + q_{10,1.16}(t)[c]B_1^{Rp}(t) + q_{10,10.15}(t)[c]B_{10}^{Rp}(t) \qquad (3.5.2)$$

Taking LST of above relation (3.5.1), we get the relations in following matrix form.

$$\begin{bmatrix} B_0^{Rp^*} \\ B_1^{Rp^*} \\ B_2^{Rp^*} \\ B_7^{Rp^*} \\ B_{10}^{Rp^*} \end{bmatrix} = \begin{bmatrix} 1 & -q^*_{0,1} & -q^*_{0,2} & 0 & -q^*_{0,10} \\ -q^*_{1,0} & 1-q^*_{1,1.3} * -q^*_{1,1.3,8} & 0 & -q^*_{1,7} & -[q^*_{1,10.12} + q^*_{1,10.12,14}] \\ 0 & \begin{matrix} -[q^*_{2,1} + q^*_{2,1.4,5} + q^*_{2,1.4,8} \\ + q^*_{2,1.4,5,8}] \end{matrix} & 1 & -q^*_{2,7} & \begin{matrix} -[q^*_{2,10.11,13} + q^*_{2,10.11,14} \\ + q^*_{2,10.11,13,14}] \end{matrix} \\ -q^*_{7,0} & -q^*_{7,1.6} & 0 & 1 & -q^*_{7,10.9} \\ -q^*_{10,0} & -q^*_{10,1.16} & 0 & 0 & 1-q^*_{10,10.15} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ W_7^{Rp^*} \\ 0 \end{bmatrix} \qquad (3.5.3)$$

Solve equations (3.5.3) for $B_0^{Rp^*}(s)$, we get

$$B_0^{Rp^*}(s) = \frac{\begin{matrix} W_7^{Rp^*} q^*_{0,2}\{q^*_{1,7}\{[1-q^*_{10,10.15}][q^*_{2,1} + q^*_{2,1.4,5} + q^*_{2,1.4,8} + q^*_{2,1.4,5,8}] + q^*_{10,1.16}[q^*_{2,10.11,13} + q^*_{2,10.11,14} \\ + q^*_{2,10.11,13,14}]\} + q^*_{2,7}\{[1-q^*_{10,10.15}][1-q^*_{1,1.3} - q^*_{1,1.3,8}] - q^*_{10,1.16}\{q^*_{1,10.12} + q^*_{1,10.12,14}]\}\} \\ + W_7^{Rp^*} q^*_{1,7}\{q^*_{0,1}[1-q^*_{10,10.15}] + q^*_{0,10} q^*_{10,1.16}\} \end{matrix}}{D_2(s)}$$

Where $D_2(s)$ is already mentioned.

The time for which server is busy due to replacement is given by

$$B_0^{Rp}(\infty) = \lim_{s\to 0} sB_0^{Rp^*}(s) = \frac{\begin{matrix} W_7^{Rp^*}(0) p_{0,2}\{p_{1,7}\{p_{10,0}[p_{2,1} + p_{2,1.4,5} + p_{2,1.4,8} + p_{2,1.4,5,8}] + p_{10,1.16}[1-p_{2,7}]\} \\ + p_{2,7}\{p_{10,0}[1-p_{1,1.3} - p_{1,1.3,8}] + p_{10,1.16}[p_{1,0} + p_{1,7}]\}\} + W_7^{Rp^*}(0) p_{1,7}\{p_{01} \\ [1-p_{10,10.15}] + p_{0,10} p_{10,1.16}\} \end{matrix}}{D_2}$$

(3.5.4)

Where D_2 is already mentioned.

3.6 Busy Period Due to Preventive Maintenance

Let $W_i^{PM}(t)$ be the probability that the server is busy in state S_i due to preventive maintenance of the unit upto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_{10}^{PM}(t) = e^{-\lambda t}\overline{M}_p(t)\overline{O}(t) + (o(t)e^{-\lambda t}[c]1)\overline{M}_p(t) + (\lambda e^{-\lambda t}\overline{O}(t)[c]1)\overline{M}_p(t) \tag{3.6.1}$$

Let $B_i^{PM}(t)$ be the probability that the server is busy in preventive maintenance of the unit due to failure at an instant 't' given that the system entered S_i state at time t=0. The recursive relations for $B_i^{PM}(t)$ are as follows

$$B_0^{PM}(t) = q_{0,1}(t)[c]B_1^{PM}(t) + q_{0,2}(t)[c]B_2^{PM}(t) + q_{0,10}(t)[c]B_{10}^{PM}(t)$$

$$B_1^{PM}(t) = q_{1,0}(t)[c]B_0^{PM}(t) + [q_{1,1.3}(t) + q_{1,1.,3,8}(t)][c]B_1^{PM}(t) + q_{1,7}(t)[c]B_7^{PM}(t) + [q_{1,10.12}(t) + q_{1,10.12,14}(t)][c]B_{10}^{PM}(t)$$

$$B_2^{PM}(t) = [q_{2,1}(t) + q_{2,1.4,5}(t) + q_{2,1.4,8}(t) + q_{2,1.4,5,8}(t)][c]B_1^{PM}(t) + q_{2,7}(t)[c]B_7^{PM}(t) + [q_{2,10.11,13}(t) + q_{2,10.11,14}(t)$$
$$+ q_{2,10.11,13,14}(t)][c]B_{10}^{PM}(t)$$

$$B_7^{PM}(t) = q_{7,0}(t)[c]B_0^{PM}(t) + q_{7,1.6}(t)[c]B_1^{PM}(t) + q_{7,10.9}(t)[c]B_{10}^{PM}(t)$$

$$B_{10}^{PM}(t) = W_{10}^{PM}(t) + q_{10,0}(t)[c]B_0^{PM}(t) + q_{10,1.16}(t)[c]B_1^{PM}(t) + q_{10,10.15}(t)[c]B_{10}^{PM}(t) \tag{3.6.2}$$

Taking LST of above relation (3.6.1), we get the relations in following matrix form.

$$\begin{bmatrix} B_0^{PM^*} \\ B_1^{PM^*} \\ B_2^{PM^*} \\ B_7^{PM^*} \\ B_{10}^{PM^*} \end{bmatrix} = \begin{bmatrix} 1 & -q_{0,1}^* & -q_{0,2}^* & 0 & -q_{0,10}^* \\ -q_{1,0}^* & 1-q_{1,1.3}^* * -q_{1,1.3,8}^* & 0 & -q_{1,7}^* & -[q_{1,10.12}^* + q_{1,10.12,14}^*] \\ 0 & \begin{matrix} -[q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* \\ + q_{2,1.4,5,8}^*] \end{matrix} & 1 & -q_{2,7}^* & \begin{matrix} -[q_{2,10.11,13}^* + q_{2,10.11,14}^* \\ + q_{2,10.11,13,14}^*] \end{matrix} \\ -q_{7,0}^* & -q_{7,1.6}^* & 0 & 1 & -q_{7,10.9}^* \\ -q_{10,0}^* & -q_{10,1.16}^* & 0 & 0 & 1-q_{10,10.15}^* \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ W_7^{PM^*} \end{bmatrix} \tag{3.6.3}$$

Solve equations (3.6.3) for $B_0^{PM^*}(s)$, we get

$$B_0^{PM^*}(s) = \frac{\begin{matrix} W_{10}^{PM^*} q_{0,2}^*[q_{1,4}^*\{q_{7,10.9}^*[q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.4,5,8}^*] - q_{7,1.6}^*[q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*]\} + q_{2,7}^* \\ \{q_{7,10.9}^*[1 - q_{1,1.3}^* - q_{1,1.3,8}^*] + q_{7,1.6}^*[q_{1,10.12}^* + q_{1,10.12,14}^*]\} + [1 - q_{1,1.3}^* - q_{1,1.3,8}^*][q_{2,10.11,13}^* + q_{2,10.11,14}^* + q_{2,10.11,13,14}^*] \\ + [q_{1,10.12}^* + q_{1,10.12,14}^*][q_{2,1}^* + q_{2,1.4,5}^* + q_{2,1.4,8}^* + q_{2,1.4,5,8}^*] + W_{10}^{PM^*}[q_{1,7}^*\{q_{0,1}^* q_{7,10.9}^* - q_{0,10}^* q_{7,1.6}^*\} + q_{0,1}^*[q_{1,10.12}^* \\ + q_{1,10.12,14}^*] + q_{0,10}^*[1 - q_{1,1.3}^* - q_{1,1.3,8}^*]] \end{matrix}}{D_2(s)}$$

Where $D_2(s)$ is already mentioned.

The time for which server is busy due to preventive maintenance is given by

$$B_0^{PM}(\infty)=\lim_{s\to 0} sB_0^{PM^*}(s)=\frac{\begin{aligned}&W_{10}^{PM^*}(0)p_{0,2}[p_{1,7}\{p_{7,10.9}[1-p_{2,7}]-[1-p_{7,0}][p_{2,10.11,13}+p_{2,10.11,14}+p_{2,10.11,13,14}]\}\\&+p_{2,7}\{p_{7,10.9}[p_{1,0}+p_{1,7}]+[1-p_{7,0}][p_{1,10.12}+p_{1,10.12,14}]\}+[p_{1,0}+p_{1,7}][p_{2,10.11,13}\\&+p_{2,10.11,14}+p_{2,10.11,13,14}]+[p_{1,10.12}+p_{1,10.12,14}][1-p_{2,7}]]+W_{10}^{PM^*}(0)[p_{1,7}\{p_{0,1}p_{7,10.9}\\&-p_{0,10}p_{7,1.6}\}+p_{0,1}[p_{1,10.12}+p_{1,10.12,14}]+p_{0,10}[1-p_{1,1.3}-p_{1,1.3,8}]]\end{aligned}}{D_2}$$

(3.6.4)

Where D_2 is already mentioned.

3.7 Expected Number of Inspections of the Unit

Let $I_i(t)$ be the expected number ofinspections of the unit by the server in (0,t] given that the system entered S_i state at time t=0. The recursive relations for $I_i(t)$ are as follows:

$$I_0(t)=Q_{0,1}(t)[s]I_1(t)+Q_{0,2}(t)[s][1+I_2(t)]+Q_{0,10}(t)[s]I_{10}(t)$$

$$I_1(t)=Q_{1,0}(t)[s]I_0(t)+[Q_{1,1.3}+Q_{1,1.3,8}(t)](t)[s]I_1(t)+Q_{1,7}(t)[s]I_7(t)+[Q_{1,10.12}(t)+Q_{1,10.12,14}(t)][s]I_{10}(t)$$

$$\begin{aligned}I_2(t)=&[Q_{2,1}(t)+Q_{2,1.4,5}(t)+Q_{2,1.4,8}(t)+Q_{2,1.4,5,8}(t)][s]I_1(t)+Q_{2,7}(t)[s]I_7(t)+[Q_{2,10.11,13}(t)+Q_{2,10.11,14}(t)\\&+Q_{2,10.11,13,14}(t)][s]I_{10}(t)\end{aligned}$$

$$I_7(t)=Q_{7,0}(t)[s]I_0(t)+Q_{7,1.6}(t)[s]I_1(t)+Q_{7,10.9}(t)[s]I_{10}(t)$$

$$I_{10}(t)=Q_{10,0}(t)[s]I_0(t)+Q_{10,1.16}(t)[s]I_1(t)+Q_{10,10.15}(t)[s]I_{10}(t) \quad (3.7.1)$$

Taking LST of above relation (3.7.1), we get the relations in following matrix form.

$$\begin{bmatrix}I_0^*\\I_1^*\\I_2^*\\I_7^*\\I_{10}^*\end{bmatrix}=\begin{bmatrix}1 & -\tilde{Q}_{0,1} & -\tilde{Q}_{0,2} & 0 & -\tilde{Q}_{0,10}\\ -\tilde{Q}_{1,0} & 1-\tilde{Q}_{1,1.3}-\tilde{Q}_{1,1.3,8} & 0 & -\tilde{Q}_{1,7} & -[\tilde{Q}_{1,10.12}+\tilde{Q}_{1,10.12,14}]\\ 0 & \begin{matrix}-[\tilde{Q}_{2,1}+\tilde{Q}_{2,1.4,5}+\tilde{Q}_{2,1.4,8}\\+\tilde{Q}_{2,1.4,5,8}]\end{matrix} & 1 & -\tilde{Q}_{2,7} & \begin{matrix}-[\tilde{Q}_{2,10.11,13}+\tilde{Q}_{2,10.11,14}\\+\tilde{Q}_{2,10.11,13,14}]\end{matrix}\\ -\tilde{Q}_{7,0} & -\tilde{Q}_{7,1.6} & 0 & 1 & -\tilde{Q}_{7,10.9}\\ -\tilde{Q}_{10,0} & -\tilde{Q}_{10,1.16} & 0 & 0 & 1-\tilde{Q}_{10,10.15}\end{bmatrix}\begin{bmatrix}\tilde{Q}_{0,2}\\0\\0\\0\\0\end{bmatrix} \quad (3.7.2)$$

Solve equations (3.7.2) for $I_0^*(s)$, we get

$$I_0^*(s)=\frac{\tilde{Q}_{0,2}\{-\tilde{Q}_{1,7}[\tilde{Q}_{10,1.16}\tilde{Q}_{7,10.9}+\tilde{Q}_{7,1.6}[1-\tilde{Q}_{10,10.15}]]+[1-\tilde{Q}_{1,1.3}-\tilde{Q}_{1,1.3,8}][1-\tilde{Q}_{10,10.15}]-\tilde{Q}_{10,1.16}[\tilde{Q}_{1,10.12}+\tilde{Q}_{1,10.12,14}]\}}{D_2(s)}$$

Where $D_2(s)$ is already mentioned.

The expected number of inspections of unit is given by

$$I_0(\infty)=\lim_{s\to 0} sI_0^*(s)=\frac{p_{0,2}\{-p_{1,7}[p_{10,1.16}p_{7,10.9}+p_{7,1.6}[1-p_{10,10.15}]]+[1-p_{1,1.3}-p_{1,1.3,8}]p_{10,0}+[p_{1,0}+p_{1,7}]p_{10,1.16}\}}{D_2}$$

(3.7.3)

Where D_2 is already mentioned.

3.8 Expected Number of Repairs of the Unit

Let $R_i(t)$ be the expected number ofrepairs of the unit by the server in (0,t] given that the system entered S_i state at time t=0. The recursive relations for $R_i(t)$ are as follows:

$$R_0(t) = Q_{0,1}(t)[s]R_1(t) + Q_{0,2}(t)[s]R_2(t) + Q_{0,10}(t)[s]R_{10}(t)$$

$$R_1(t) = Q_{1,0}(t)[s][1+R_0(t)] + Q_{1,1.3}(t)[s][1+R_1(t)] + Q_{1,1.3,8}(t) + [s]R_1(t) + Q_{1,7}(t)[s]R_7(t) + Q_{1,10.12}(t)[s][1+R_{10}(t)] + Q_{1,10.12,14}(t)[s]R_{10}(t)$$

$$R_2(t) = [Q_{2,1}(t) + Q_{2,1.4,8}(t) + Q_{2,1.4,5,8}(t)][s]R_1(t) + Q_{2,1.4,5}(t)[s][1+R_1(t)] + Q_{2,7}(t)[s]R_7(t) + Q_{2,10.11,13}(t)[s][1 + R_{10}(t)] + [Q_{2,10.11,14}(t) + Q_{2,10.11,13,14}(t)][s]R_{10}(t)$$

$$R_7(t) = Q_{7,0}(t)[s]R_0(t) + Q_{7,1.6}(t)[s]R_1(t) + Q_{7,10.9}(t)[s]R_{10}(t)$$

$$R_{10}(t) = Q_{10,0}(t)[s]R_0(t) + Q_{10,1.16}(t)[s]R_1(t) + Q_{10,10.15}(t)[s]R_{10}(t) \quad (3.8.1)$$

Taking LST of above relation (3.8.1), we get the relations in following matrix form.

$$\begin{bmatrix} R_0^* \\ R_1^* \\ R_2^* \\ R_7^* \\ R_{10}^* \end{bmatrix} = \begin{bmatrix} 1 & -\tilde{Q}_{0,1} & -\tilde{Q}_{0,2} & 0 & -\tilde{Q}_{0,10} \\ -\tilde{Q}_{1,0} & 1-\tilde{Q}_{1,1.3}-\tilde{Q}_{1,1.3,8} & 0 & -\tilde{Q}_{1,7} & -[\tilde{Q}_{1,10.12}+\tilde{Q}_{1,10.12,14}] \\ 0 & \begin{matrix} -[\tilde{Q}_{2,1}+\tilde{Q}_{2,1.4,5}+\tilde{Q}_{2,1.4,8} \\ +\tilde{Q}_{2,1.4,5,8}] \end{matrix} & 1 & -\tilde{Q}_{2,7} & \begin{matrix} -[\tilde{Q}_{2,10.11,13}+\tilde{Q}_{2,10.11,14} \\ +\tilde{Q}_{2,10.11,13,14}] \end{matrix} \\ -\tilde{Q}_{7,0} & -\tilde{Q}_{7,1.6} & 0 & 1 & -\tilde{Q}_{7,10.9} \\ -\tilde{Q}_{10,0} & -\tilde{Q}_{10,1.16} & 0 & 0 & 1-\tilde{Q}_{10,10.15} \end{bmatrix} \begin{bmatrix} 0 \\ \tilde{Q}_{1,0}+\tilde{Q}_{1,1.3}+\tilde{Q}_{1,10.12} \\ \tilde{Q}_{2,1.4,5}+\tilde{Q}_{2,10.11,13} \\ 0 \\ 0 \end{bmatrix}$$

(3.8.2)

Solve equations (3.7.2) for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{\begin{matrix} \tilde{Q}_{0,2}[\tilde{Q}_{1,0}+\tilde{Q}_{1,1.3}+\tilde{Q}_{1,10.12}][\tilde{Q}_{2,7}\{\tilde{Q}_{10,1.16}\tilde{Q}_{7,10.9}+\tilde{Q}_{7,1.6}[1-\tilde{Q}_{10,10.15}]\}+[\tilde{Q}_{2,1}+\tilde{Q}_{2,1.4,5}+\tilde{Q}_{2,1.4,8}+\tilde{Q}_{2,1.4,5,8}] \\ [1-\tilde{Q}_{10,10.15}]+\tilde{Q}_{10,1.16}[\tilde{Q}_{2,10.11,13}+\tilde{Q}_{2,10.11,14}+\tilde{Q}_{2,10.11,13,14}]]+[\tilde{Q}_{1,0}+\tilde{Q}_{1,1.3}+\tilde{Q}_{1,10.12}]\{\tilde{Q}_{0,1}[1-\tilde{Q}_{10,10.15}] \\ +\tilde{Q}_{0,10}\tilde{Q}_{10,1.16}\}+\tilde{Q}_{0,2}[\tilde{Q}_{2,1.4,5}+\tilde{Q}_{2,10.11,13}][-\tilde{Q}_{1,7}\{\tilde{Q}_{10,1.16}\tilde{Q}_{7,10.9}+\tilde{Q}_{7,1.6}[1-\tilde{Q}_{10,10.15}]\}+[1-\tilde{Q}_{1,1.3}-\tilde{Q}_{1,1.3,8}] \\ [1-\tilde{Q}_{10,10.15}]-\tilde{Q}_{10,1.16}[\tilde{Q}_{1,10.12}+\tilde{Q}_{1,10.12,14}]] \end{matrix}}{D_2(s)}$$

Where $D_2(s)$ is already mentioned.

The expected number of repairs of unit is given by

$$I_0(\infty) = \lim_{s\to 0} sI_0^*(s) = \frac{\begin{matrix} p_{0,2}[p_{1,0}+p_{1,1.3}+p_{1,10.12}][p_{2,7}\{p_{10,1.16}p_{7,10.9}+p_{7,1.6}[1-p_{10,10.15}]\}+[1-p_{2,7}]p_{10,1.16}+[p_{2,1} \\ +p_{2,1.4,5}+p_{2,1.4,8}+p_{2,1.4,5,8}]p_{10,0}]+[p_{1,0}+p_{1,1.3}+p_{1,10.12}]\{p_{0,1}[1-p_{10,10.12}]+p_{0,10}p_{10,1.16}\} \\ +p_{0,2}[p_{2,1.4,5}+p_{2,10.11,13}][-p_{1,7}\{p_{10,1.16}p_{7,10.9}+p_{7,1.6}[1-p_{10,10.15}]\}+[1-p_{1,1.3}-p_{1,1.3,8}]p_{10,0} \\ +[p_{1,0}+p_{1,7}]p_{10,1.16}] \end{matrix}}{D_2}$$

(3.8.3)

Where D_2 is already mentioned.

3.9 Expected Number of Replacements of the Unit

Let $R_i^C(t)$ be the expected number of replacements of the unit failed in cold-standby by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $R_i^C(t)$ are as follows:

$$R_0^C(t) = Q_{0,1}(t)[s]R_1^C(t) + Q_{0,2}(t)[s]R_2^C(t) + Q_{0,10}(t)[s]R_{10}^C(t)$$

$$R_1^C(t) = Q_{1,0}(t)[s]R_0^C(t) + Q_{1,1.3}(t)[s]R_1^C(t) + Q_{1,1.3,8}(t)[s][1+R_1^C(t)] + Q_{1,10.12}(t)[s]R_{10}^C(t) + Q_{1,10.12,14}(t)[s][1+R_{10}^C(t)] + Q_{1,7}(t)[s]R_7^C(t)$$

$$R_2^C(t) = [Q_{2,1}(t) + Q_{2,1.4,5}(t)][s]R_1^C(t) + [Q_{2,1.4,8}(t) + Q_{2,1.4,5,8}(t)][s][1+R_1^C(t)] + Q_{2,7}(t)[s]R_7^C(t) + Q_{2,10.11,13}(t)[s]R_{10}^C(t) + [Q_{2,10.11,14}(t) + Q_{2,10.11,13,14}(t)][s][1+R_{10}^C(t)]$$

$$R_7^C(t) = Q_{7,0}(t)[s][1+R_0^C(t)] + Q_{7,1.6}(t)[s][1+R_1^C(t)] + Q_{7,10.9}(t)[s][1+R_{10}^C(t)]$$

$$R_{10}^C(t) = Q_{10,0}(t)[s]R_0^C(t) + Q_{10,1.16}(t)[s]R_1^C(t) + Q_{10,10.15}(t)[s]R_{10}^C(t) \qquad (3.9.1)$$

Taking LST of above relation (3.9.1), we get the relations in following matrix form.

$$\begin{bmatrix} \tilde{R}_0^C \\ \tilde{R}_1^C \\ \tilde{R}_2^C \\ \tilde{R}_7^C \\ \tilde{R}_{10}^C \end{bmatrix} = \begin{bmatrix} 1 & -\tilde{Q}_{0,1} & -\tilde{Q}_{0,2} & 0 & -\tilde{Q}_{0,10} \\ -\tilde{Q}_{1,0} & 1-\tilde{Q}_{1,1.3}-\tilde{Q}_{1,1.3,8} & 0 & -\tilde{Q}_{1,7} & -[\tilde{Q}_{1,10.12}+\tilde{Q}_{1,10.12,14}] \\ 0 & \begin{matrix} -[\tilde{Q}_{2,1}+\tilde{Q}_{2,1.4,5}+\tilde{Q}_{2,1.4,8} \\ +\tilde{Q}_{2,1.4,5,8}] \end{matrix} & 1 & -\tilde{Q}_{2,7} & \begin{matrix} -[\tilde{Q}_{2,10.11,13}+\tilde{Q}_{2,10.11,14} \\ +\tilde{Q}_{2,10.11,13,14}] \end{matrix} \\ -\tilde{Q}_{7,0} & -\tilde{Q}_{7,1.6} & 0 & 1 & -\tilde{Q}_{7,10.9} \\ -\tilde{Q}_{10,0} & -\tilde{Q}_{10,1.16} & 0 & 0 & 1-\tilde{Q}_{10,10.15} \end{bmatrix} \begin{bmatrix} 0 \\ \tilde{Q}_{1,1.3,8}+\tilde{Q}_{1,10.12,14} \\ \begin{matrix} [\tilde{Q}_{2,1.4,8}+\tilde{Q}_{2,1.4,5,8}+\tilde{Q}_{2,10.11,14} \\ +\tilde{Q}_{2,10.11,13,14}] \end{matrix} \\ \tilde{Q}_{7,0}+\tilde{Q}_{7,1.6}+\tilde{Q}_{7,10.9} \\ 0 \end{bmatrix}$$

(3.9.2)

Solve equations (3.9.2) for $\tilde{R}_0^C(s)$, we get

$$\tilde{R}_0^C(s) = \frac{\begin{matrix} \tilde{Q}_{0,2}\{\tilde{Q}_{10,1.16}\tilde{Q}_{7,10.9} + \tilde{Q}_{7,1.6}[1-\tilde{Q}_{10,10.15}]\}\{-\tilde{Q}_{1,7}[\tilde{Q}_{2,1.4,8} + \tilde{Q}_{2,1.4,5,8} + \tilde{Q}_{2,10.11,14} + \tilde{Q}_{2,10.11,13,14}] + \tilde{Q}_{2,7}[\tilde{Q}_{1,1.3,8} \\ + \tilde{Q}_{1,10.12,14}]\} + \tilde{Q}_{0,2}\{[\tilde{Q}_{2,1} + \tilde{Q}_{2,1.4,8} + \tilde{Q}_{2,1.4,5} + \tilde{Q}_{2,1.4,5,8}][1-\tilde{Q}_{10,10.15}] + \tilde{Q}_{10,1.16}[\tilde{Q}_{2,10.11,13} + \tilde{Q}_{2,10.11,14} + \tilde{Q}_{2,10.11,13,14}]\} \\ \{\tilde{Q}_{1,7}[\tilde{Q}_{7,0} + \tilde{Q}_{7,1.6} + \tilde{Q}_{7,10.9}] + [\tilde{Q}_{1,1.3,8} + \tilde{Q}_{1,10.12,14}] + \tilde{Q}_{0,2}\{[1-\tilde{Q}_{1,1.3} - \tilde{Q}_{1,1.3,8}][1-\tilde{Q}_{10,10.15}] - \tilde{Q}_{10,1.16}[\tilde{Q}_{1,10.12} \\ + \tilde{Q}_{1,10.12,14}]\}\{\tilde{Q}_{2,7}[\tilde{Q}_{7,0} + \tilde{Q}_{7,1.6} + \tilde{Q}_{7,10.9}] + [\tilde{Q}_{2,1.4,8} + \tilde{Q}_{2,1.4,5,8} + \tilde{Q}_{2,10.11,14} + \tilde{Q}_{2,10.11,13,14}]\} + \{\tilde{Q}_{0,1} + [1-\tilde{Q}_{10,10.15}] \\ + \tilde{Q}_{0,10}\tilde{Q}_{10,1.16}\}\{\tilde{Q}_{1,7}[\tilde{Q}_{7,0} + \tilde{Q}_{7,1.6} + \tilde{Q}_{7,10.9}] + [\tilde{Q}_{1,1.3,8} + \tilde{Q}_{1,10.12,14}]\} \end{matrix}}{D_2(s)} \text{ W}$$

here $D_2(s)$ is already mentioned.

The expected number of replacements of unit is given by

$$R_0^C(\infty) = \lim_{s\to 0} s\tilde{R}_0^C(s) = \frac{\begin{matrix} p_{0,2}\{p_{10,1.16}[1-p_{7,0}] + p_{10,0}p_{7,1.6}\}\{-p_{1,7}[p_{2,1.4,8} + p_{2,1.4,5,8} + p_{2,10.11,14} + p_{2,10.11,13,14}] + p_{2,7}[p_{1,1.3,8} \\ + p_{1,10.12,14}]\} + \{p_{1,7} + p_{1,1.3,8} + p_{1,10.12,14}\}\{p_{0,2}p_{10,0}[p_{2,1} + p_{2,1.4,8} + p_{2,1.4,5} + p_{2,1.4,5,8}] + p_{0,1}p_{10,0} \\ - p_{0,2}p_{2,7}p_{10,1.16} + p_{10,1.16}\} + p_{0,2}\{[1-p_{1,1.3} - p_{1,1.3,8}]p_{10,0} + (p_{1,0} + p_{1,7})p_{10,1.16}\}\{1-p_{2,1} - p_{2,1.4,5} \\ - p_{2,10.11,13}\} \end{matrix}}{D_2}$$

(3.9.3)

Where D_2 is already mentioned.

3.10 Expected Number of Preventive Maintenance

Let $P_i^M(t)$ be the expected number of preventive maintenance of the units by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $P_i^M(t)$ are as follows:

$$P_0^M(t) = Q_{0,1}(t)[s]P_1^M(t) + Q_{0,2}(t)[s]P_2^M(t) + Q_{0,10}(t)[s]P_{10}^M(t)$$

$$P_1^M(t) = Q_{1,0}(t)[s]P_0^M(t) + [Q_{1,1.3}(t) + Q_{1,1.3,8}(t)][s]P_1^M(t) + [Q_{1,10.12}(t) + Q_{1,7}(t)[s]P_7^M(t) + Q_{1,10.12,14}(t)][s]P_{10}^M(t)$$

$$P_2^M(t) = [Q_{2,1}(t) + Q_{2,1.4,5}(t) + Q_{2,1.4,8}(t) + Q_{2,1.4,5,8}(t)][s]P_1^M(t) + Q_{2,7}(t)[s]P_7^M(t) + [Q_{2,10.11,13}(t) + Q_{2,10.11,14}(t) + Q_{2,10.11,13,14}(t)][s]P_{10}^M(t)$$

$$P_7^M(t) = Q_{7,0}(t)[s]P_0^M(t) + Q_{7,1.6}(t)[s]P_1^M(t) + Q_{7,10.9}(t)[s]P_{10}^M(t)$$

$$P_{10}^M(t) = Q_{10,0}(t)[s][1 + P_0^M(t)] + Q_{10,1.16}(t)[s][1 + P_1^M(t)] + Q_{10,10.15}(t)[s][1 + P_{10}^M(t)] \quad (3.10.1)$$

Taking LST of above relation (3.10.1), we get the relations in following matrix form.

$$\begin{bmatrix} \tilde{P}_0^M \\ \tilde{P}_1^M \\ \tilde{P}_2^M \\ \tilde{P}_7^M \\ \tilde{P}_{10}^M \end{bmatrix} = \begin{bmatrix} 1 & -\tilde{Q}_{0,1} & -\tilde{Q}_{0,2} & 0 & -\tilde{Q}_{0,10} \\ -\tilde{Q}_{1,0} & 1-\tilde{Q}_{1,1.3} - \tilde{Q}_{1,1.3,8} & 0 & -\tilde{Q}_{1,7} & -[\tilde{Q}_{1,10.12} + \tilde{Q}_{1,10.12,14}] \\ 0 & \begin{matrix} -[\tilde{Q}_{2,1} + \tilde{Q}_{2,1.4,5} + \tilde{Q}_{2,1.4,8} \\ +\tilde{Q}_{2,1.4,5,8}] \end{matrix} & 1 & -\tilde{Q}_{2,7} & \begin{matrix} -[\tilde{Q}_{2,10.11,13} + \tilde{Q}_{2,10.11,14} \\ +\tilde{Q}_{2,10.11,13,14}] \end{matrix} \\ -\tilde{Q}_{7,0} & -\tilde{Q}_{7,1.6} & 0 & 1 & -\tilde{Q}_{7,10.9} \\ -\tilde{Q}_{10,0} & -\tilde{Q}_{10,1.16} & 0 & 0 & 1-\tilde{Q}_{10,10.15} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tilde{Q}_{10,0} + \tilde{Q}_{10,1.16} + \tilde{Q}_{10,10.15} \end{bmatrix}$$

(3.10.2)

Solve equations (3.10.2) for $\tilde{P}_0^M(s)$, we get

$$\tilde{P}_0^M(s) = \frac{\begin{matrix} \tilde{Q}_{0,2}[\tilde{Q}_{10,0} + \tilde{Q}_{10,1.16} + \tilde{Q}_{10,10.15}][[1-\tilde{Q}_{1,1.3} - \tilde{Q}_{1,1.3,8}][\tilde{Q}_{2,10.11,13} + \tilde{Q}_{2,10.11,14} + \tilde{Q}_{2,10.11,13,14} + \tilde{Q}_{2,7}\tilde{Q}_{7,10.9}] + [\tilde{Q}_{1,10.12} \\ + \tilde{Q}_{1,10.12,14} + \tilde{Q}_{1,7}\tilde{Q}_{7,10.9}][\tilde{Q}_{2,1} + \tilde{Q}_{2,1.4,5} + \tilde{Q}_{2,1.4,8} + \tilde{Q}_{2,1.4,5,8}] + \tilde{Q}_{7,1.6}\{\tilde{Q}_{2,7}[\tilde{Q}_{1,10.12} + \tilde{Q}_{1,10.12,14}] - \tilde{Q}_{1,7}[\tilde{Q}_{2,10.11,13} \\ + \tilde{Q}_{2,10.11,14} + \tilde{Q}_{2,10.11,13,14}]\}] + \tilde{Q}_{1,7}\{\tilde{Q}_{0,1}\tilde{Q}_{7,10.9} - \tilde{Q}_{0,10}\tilde{Q}_{7,1.6}\}[\tilde{Q}_{10,0} + \tilde{Q}_{10,1.16} + \tilde{Q}_{10,10.15}] + [\tilde{Q}_{10,0} + \tilde{Q}_{10,1.16} \\ + \tilde{Q}_{10,10.15}]\{\tilde{Q}_{0,1}[\tilde{Q}_{1,10.12} + \tilde{Q}_{1,10.12,14}] + \tilde{Q}_{0,10}[1-\tilde{Q}_{1,1.3} - \tilde{Q}_{1,1.3,8}]\} \end{matrix}}{D_2(s)}$$

Where $D_2(s)$ is already mentioned.

The expected number of preventive maintenanceof unit is given by

$$P_0^M(\infty) = \lim_{s\to 0} s\tilde{P}_0^M(s) = \frac{\begin{matrix} [p_{1,10.12} + p_{1,10.12,14}]\{p_{0,1} + p_{0,2}[1-p_{2,7}] + p_{0,2}p_{2,7}p_{7,1.6}\} + [1-p_{1,1.3} - p_{1,1.3,8}]\{p_{0,2}p_{2,7}p_{7,10.9} \\ + p_{0,10}\} + p_{0,2}[p_{2,10.11,13} + p_{2,10.11,14} + p_{2,10.11,13,14}]\{p_{1,0} + p_{1,7}[1-p_{7,1.6}]\} + p_{0,2}[p_{2,1} + p_{2,1.4,8} \\ + p_{2,1.4,5} + p_{2,1.4,5,8}]p_{1,7}p_{7,10.9} + p_{1,7}\{p_{0,1}p_{7,10.9} - p_{0,10}p_{7,1.6}\} \end{matrix}}{D_2}$$

(3.10.3)

Where D_2 is already mentioned.

4 COST ANALYSIS

The Profit incurred to the system model in (0,t] is given as

$$P(t) = K_0 A_0(t) - \sum_{i=1}^{8} K_i X_i(t) \tag{4.1}$$

As t→∞, we obtain the profit attained asymptotically i.e.

$$P_0 = \lim_{t\to\infty} P(t)/t = K_0 A_0 - \lim_{t\to\infty} \sum_{i=1}^{8} K_i X_i(t) \tag{4.2}$$

Where $X_i(\infty) = \begin{cases} B_0^I; & \text{if } i=1 \\ B_0^R; & \text{if } i=2 \\ B_0^{Rp}; & \text{if } i=3 \\ I_0; & \text{if } i=4 \\ R_0; & \text{if } i=5 \\ R_0^C; & \text{if } i=6 \\ B_0^{PM}; & \text{if } i=7 \\ P_0^M; & \text{if } i=8 \end{cases}$ (4.3)

Where

K_0 = Revenue per unit up-time of the system, K_1 =Cost per unit time for which server is busy in inspection of the failure of cold-standby unit, K_2 = Cost per unit time for which server is busy due to repair, K_3 = Cost per unit time for which server is busy due to replacement, K_4 = Cost per unit inspection, K_5 = Cost per unit repair, K_6 = Cost per unit replacement, K_7 = Cost per unit time for which server is busy due to preventive maintenance, K_8 = Cost per unit preventive maintenance.

5 NUMERICAL ILLUSTRATION

In this section, for practical illustration, the steady state behavior of system performance measures viz. mean time to system failure, availability and profit, is studied with failure rate (λ), the rate (μ)with which the standby goes under failure, inspection rate of standby unit (α), repair rate of failed unit (β), the replacement rate of unit (γ), the rate with which the operative unit goes under preventive maintenance(ξ), the preventive maintenance rate of unit (ν) and the rate with which the unit goes under replacement(ρ).

Here, the failure time of cold standby unit, maximum operation time of the unit, maximum repair time of the unit, preventive maintenance time, replacement time and inspection time follows the exponential distribution with different parameters but repair time of the unit follows gamma distribution and their pdf's are given as :

$s(t) = \mu \exp(-\mu t)$, $o(t) = \xi \exp(-\xi t)$, $r(t) = \rho \exp(-\rho t)$, $m_p(t) = \upsilon \exp(-\upsilon t)$, $f(t) = \gamma \exp(-\gamma t)$,

$h(t) = \alpha \exp(-\alpha t)$, $g(t) = \dfrac{\exp(-\beta t) t^{\eta-1} \beta^{\eta}}{\Gamma\eta}$

where $t \ge 0$ and $\eta, \mu, \xi, \rho, \upsilon, \gamma, \alpha, \beta > 0$ respectively.

Initially we fix the values of parameters as:

a=0.3, b=0.7, α=0.14, β=0.12, γ=0.2, δ=0.07, μ=0.1, ν=0.25, ξ=0.041

Similarly, setting different costs as:
$K_0 = 30000,\ K_1 = 100,\ K_2 = 500,\ K_3 = 150, K_4 = 800, K_5 = 1000, K_6 = 700, K_7 = 300, K_8 = 300$
We obtained the results shown in Table1.

When shape parameter η=2 then the repair time distribution reduces to: $g(t) = \exp(-\beta\, t) t \beta^2$

Table 1: Effect of α, β, γ, ν, ξ, μ and ρ on system performance w.r.t.λ (**η=2.0**, a=0.3, b=0.7, *α=0.14, β=0.12*, γ=0.2,υ=0.25, ξ=0.041, *μ=0.1, ρ=0.77*)

Performance Index	λ		*α=0.3*	*β=0.2*	γ=0.4	υ=0.3	ξ=0.06	*μ=0.5*	*ρ=0.86*
MTSF	0.01	38.83	43.14	38.90	41.96	39.20	28.98	24.10	38.88
	0.02	32.67	35.87	32.73	35.36	32.97	25.45	20.36	32.72
	0.03	28.21	30.65	28.27	30.57	28.45	22.68	17.66	28.26
	0.04	24.82	26.74	24.89	26.94	25.03	20.46	15.63	24.87
	0.05	22.16	23.69	22.23	24.07	22.34	18.62	14.04	22.21
Availability	0.01	0.8162	0.8695	0.8168	0.8510	0.8186	0.7725	0.7313	0.8167
	0.02	0.7900	0.8450	0.7907	0.8308	0.7927	0.7498	0.6986	0.7906
	0.03	0.7657	0.8211	0.7665	0.8123	0.7686	0.7284	0.6696	0.7664
	0.04	0.7428	0.7978	0.7438	0.7952	0.7460	0.7082	0.6436	0.7438
	0.05	0.7213	0.7751	0.7225	0.7793	0.7247	0.6888	0.6201	0.7224
Profit	0.01	24226.89	25786.11	24242.44	25309.36	24300.23	22914.28	21593.87	24241.68
	0.02	23417.69	25028.46	23437.64	24683.52	23500.54	22211.74	20602.83	23437.10
	0.03	22665.03	24288.87	22689.68	24109.74	22755.94	21548.89	19722.36	22689.35
	0.04	21959.84	23568.80	21989.46	23578.94	22057.51	20920.46	18933.05	21989.30
	0.05	21295.28	22869.37	21330.03	23083.98	21398.49	20322.40	18219.81	21330.00

INTERPRETATION

From Table 1, we notice that MTSF, Availability and Profit decreases with increasing values of failure rate λ, μ and ξ. The higher value of ξ states the smaller value for maximum operation time threshold. It indicates that the shorter limit on the operation time of the operating unit declines the system reliability, availability and profit.System reliability, availability and profit improves with higher values of inspection rate α, repair rate β, replacement rate γ and preventive maintenance rate ν of the unit. All the three measures shows uprising trend with higher value of ρ. The increase in the value of ρ indicates decrease in the value of upper time limit for repair, accordingly cutting the system down time and ensuring the system functioning by replacing the failed unit. Thus, a reasonable higher value of ρ accounts for higher reliability, availability and profit. Hence, the results clearly declare that the system becomes less effective with lower operation time threshold. However, there is significant hike in the system performance with efficient restoration remedies.

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