# Math Tasks of Fuzzy Numbers and Intuitionistic Fuzzy Number

**Priya Rathore and Arihant Jain**
Department of Mathematics,
Dr. A. P. J. Abdul Kalam University, Indore (M.P.) - 452016, India
Corresponding Author Email : priyarathore0608@gmail.com

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***Abstract***
The aim of this paper is to apply the math tasks of fuzzy numbers and intuitionistic fuzzy numbers just as to execute the current strategies with number-crunching activities of fuzzy numbers/intuitionistic fuzzy numbers in a programming language.

**Keywords:** Fuzzy numbers, intuitionistic fuzzy, number-crunching.

## 1. INTRODUCTION

The concept of fuzzy set was initially introduced by Zadeh [26]. The knowledge of a fuzzy set manages the cost of an appropriate situation of departure for the erection of a hypothetical design. A class of things with a scope of scores of participation is named as a fuzzy set. Previously mentioned set is exemplified by an affiliation (trademark) work which transfers to each thing a rating of participation range flanked by nothing and one. The idea of incorporation, association, crossing point, supplement, connection, convexity, and so forth, is extended to those sets, and various plots of this idea in the structure of fuzzy sets are perceived. Mental outlines through natural language conditions like low temperature, short young lady, colossal mass, and so on and alongside things like creature, stepping stool, etc are the part of things referenced in Zadeh's delineation. Customary rationale is unreasonably firm to report for such gathering where it arises that participation is a continuous understanding to a limited degree than an all/or nothing matter. Kosko in his composition entitles this as a difference issue: the world is dim despite the fact that science is highly contrasting. Exhaustively, the fuzzy hypothesis is that "the entire thing is subject of degree". In this way the relationship in a fuzzy set isn't a subject of affirmation or nullification, yet generally a subject of degree. Accordingly, the center rationale is the fuzzy rationale. Let β be a fuzzy set in a non-void set X and it is portrayed by an enrollment work $\mu\beta(x)$ for all $x \in X$, which relates with each point in Y is a genuine number in the stretch [0,1], with the worth of $\mu\beta(x)$ at x addressing the "Evaluation of Membership" of x in β. It is hard to apply the math tasks of fuzzy numbers and intuitionistic fuzzy numbers just as to execute the current strategies with number-crunching activities of fuzzy numbers/intuitionistic fuzzy numbers in a programming language. It is much simple to apply the math tasks of genuine numbers when contrasted with fuzzy numbers and intuitionistic fuzzy numbers just as to execute a strategy with number-crunching activities of genuine numbers in a programming language as contrasted a technique and number-crunching tasks of fuzzy numbers and intuitionistic fuzzy numbers.

Keeping, something very similar as a top priority, the point of the theory is to show that utilizing the positioning capacity, the fuzzy transportation issue and intuitionistic fuzzy transportation issues can be changed into comparable fresh transportation issues just as the arrangement of the fuzzy transportation issue and intuitionistic fuzzy transportation issues can be gotten utilizing the arrangement of the changed fresh transportation issue.

## 2. PRELIMINARY NOTES

### 2.1 Tabular Representation

A balanced transportation problem (total availability equal to total demand) having *m* sources ( $O_i$, i = 1, 2,…, m) and *n* destinations ( $D_j$, i = 1, 2,…, n) can be represented as shown in Table..

**Table 1: Tabular representation of a transportation problem**

| Destinations Origins | $D_1$ | $D_2$ | … | $D_n$ | Availability ($a_i$) |
|---|---|---|---|---|---|
| $O_1$ | $c_{11}$ | $c_{12}$ | … | $c_{1n}$ | $a_1$ |
| $O_2$ | $c_{21}$ | $c_{22}$ | … | $c_{2n}$ | $a_2$ |
| ⋮ | ⋮ | ⋮ | ⋱ | ⋮ | |
| $O_m$ | $c_{m1}$ | $c_{m2}$ | … | $c_{mn}$ | $a_m$ |
| Demand $b_j$ | $b_1$ | $b_2$ | … | $b_n$ | $\sum_i a_i = \sum_j b_j$ |

Where,

$a_i$ : quantity of sources of material available at origin $O_i, i = 1, 2, ..., m$ .

$b_j$ : quantity of sources of material required at destinations at $D_j$, $j = 1, 2, ..., n$ .

$c_{ij}$ : unit cost of transportation from source $O_i$ to destination $D_j$ .

### 2.2 Linear Programming Formulation

A balanced transportation issue, addressed by Table 1, can likewise be defined into the accompanying linear programming issue:

$$Minimize\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$Subject\ to$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2,\dots,m$$

$$\sum_{j=1}^{n} x_{ij} = b_j, \qquad j = 1,2,\dots,n$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j.$$

where, $x_{ij}$: quantity taken from the ith source to the jth destination.

### 2.3 Fuzzy Transportation Problem

**Basic Definitions**

In this segment, some essential meanings of fuzzy set, fuzzy number, triangular fuzzy number and trapezoidal fuzzy number are evaluated.

**Definition 1 . 1** Let $X$ be a non-empty set. A fuzzy set which is A of X is written as

$$\tilde{A} = \left\{\left\langle x, \mu_{\tilde{A}}(x)\right\rangle / x \in X\right\} \text{ where } \mu_{\tilde{A}}(x)$$

is called membership function which maps each value of $X$ to a quantity between 0 and1.

**Definition 1.2** A fuzzy no. $\tilde{F}$ is a converging normalized set on the real line R such that:

- $\tilde{F}$ is normal. It implies that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$
- $\tilde{F}$ is convex. It implies that for each $x_1, x_2 \in R$

$\mu_{\tilde{A}}(x)(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \lambda \in [0,1].$

- $\mu_{\tilde{A}}(x)$ is upper semi-continuous.

sup($F$) is bounded in $\tilde{R}$.

**Definition 1.3** A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be triangular fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2\\ 1, & x=a_2\\ \dfrac{x-a_3}{a_2-a_3}, & a_2 \le x \le a_3\\ 0, & \text{otherwise}\end{cases}$$

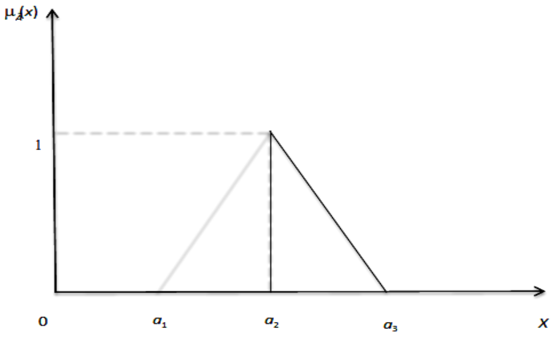


**Figure 1: Triangular fuzzy number**

**Definition 1.4:** A fuzzy number $\tilde{A}$ = (a1, a2, a3, a4) is said to be trapezoidal fuzzy number if its membership function is given by □ (*x*)

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, a_1 \le x < a_2\\ 1, \quad a_2 \le x \le a_3\\ \dfrac{x-a_4}{a_3-a_4}, a_3 < x \le a_4\\ 0, \quad \text{otherwise}\end{cases}$$

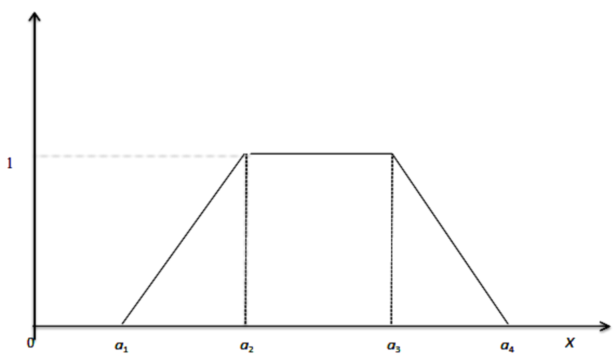


**Figure 2: Trapezoidal fuzzy number**

## 3. ARITHMETIC OPERATIONS

In this section arithmetic operations of triangular and trapezoidal fuzzy numbersare presented.

### Arithmetic Operations of Triangular Fuzzy Numbers

In this segment mathematical operations of triangular fuzzy numbers are mentioned.

Let A = ( $a_1$, $a_2$, $a_3$) and B = ( $b_1$, $b_2$, $b_3$) be two triangular fuzzy numbers. Then,

i. $\tilde{A}+\tilde{B}=(a_1+b_1, a_2+b_2, a_3+b_3)$.

ii. $\tilde{A}-\tilde{B}=(a_1-b_3, a_2-b_2, a_3-b_1)$.

iii. $\tilde{A}\times\tilde{B}=(\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$.

iv. $\lambda\tilde{A}=\begin{cases}(\lambda a_1, \lambda a_2, \lambda a_3) & \lambda\geq 0\\ (\lambda b_3, \lambda b_2, \lambda b_1) & \lambda<0\end{cases}$

### Arithmetic Operations of Trapezoidal Fuzzy Numbers

In this segment mathematical operations of trapezoidal fuzzy numbers are mentioned. Let two trapezoidal fuzzy numbers is if A = ($a_1$, $a_2$, $a_3$, $a_4$) and B = ($b_1$, $b_2$, $b_3$, $b_4$) . Then,

i. $\tilde{A}+\tilde{B}=(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$.

ii. $\tilde{A}-\tilde{B}=(a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$.

iii. $\tilde{A}\times\tilde{B}=(a,b,c,d)$,

Where

$a$ □ $\min(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$, $b$ □ $\min(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$,

$c$ □ $\max(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$, $d$ □ $\max(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$

$$\lambda\tilde{A}=\begin{cases}(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4) & \lambda\geq 0\\ (\lambda b_4, \lambda b_3, \lambda b_2, \lambda b_1) & \lambda<0\end{cases}$$

### Comparison of Fuzzy Numbers

Examination of fuzzy numbers, assume a significant part in dynamic issues. Fuzzy numbers should be analyzed before a move is made by chief. Jain [14] proposed the idea of positioning capacity for looking at ordinary fuzzy numbers. Chen [16] brought up that much of the time it's anything but to conceivable limiting the enrollment capacity to the typical and the proposed idea of summed up fuzzy numbers. From that point forward, enormous endeavors are spent and critical advances are made on the improvement of various systems. A proficient methodology for contrasting the fuzzy numbers is by the utilization of a positioning capacity

$$\Re : F(\mathbb{R}) \rightarrow \mathbb{R}, \text{ where } F(\mathbb{R})$$

is a bunch of fuzzy numbers characterized on set of genuine numbers, which maps each fuzzy number into the genuine line, where a characteristic request exists.

Let two trapezoidal fuzzy numbers A = ($a_1$, $a_2$, $a_3$, $a_4$) and B = ($b_1$, $b_2$, $b_3$, $b_4$) be. Then,

(i) $\tilde{A} >_{\Re} \tilde{B}$ *iff* $\Re(\tilde{A}) > \Re(\tilde{B})$

(ii) $\tilde{A} <_{\Re} \tilde{B}$ *iff* $\Re(\tilde{A}) < \Re(\tilde{B})$

(iii) $\tilde{A} =_{\Re} \tilde{B}$ *iff* $\Re(\tilde{A}) = \Re(\tilde{B})$

where,

$\Re(\tilde{A}) = (a_1 + b_1 + c_1 + d_1)/4$, and $\Re(\tilde{B}) = (a_2 + b_2 + c_2 + d_2)/4$.

**Tabular Representation**

A balanced fuzzy transportation problem (total fuzzy availability equal to total fuzzy demand) having $m$ sources ( $O_i$ , $i$ □ 1, 2,..., $m$ ) and $n$ destinations ( $D_j$, J = 1,2,..,n) can be represented as shown in Table 3.2 .

**Table 2: Tabular portrayal of a FTP**

| | | | | | |
|---|---|---|---|---|---|
| ⋮ | ⋮ | ⋮ | ⋱ | ⋮ | |
| $O_m$ | $c_{m1}$ | $c_{m2}$ | … | $c_{mn}$ | $a_m$ ~ |
| Demand $b_j$ | $b_1$ | ~ $b_2$ | ~… | $b_n$ | ~ □ $a_i$ □□ $b_j$ $i$ $j$ ~ |
| $O_2$ | $c_{21}$ | $c_{22}$ | … | $c_{2n}$ | $a_2$ |

| Destinations | $D_1$ | $D_2$ | … | $D_n$ | Fuzzy Availability |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| Origins | | | | | $(a_i)$ |
| $O_1$ | $c_{11}$ | $c_{12}$ | $\dots$ | $c_{1n}$ | $a_1$ |

**Linear Programming Formulation**

A balanced fuzzy transportation issue, addressed by Table 1.2, can likewise be detailed into the accompanying fuzzy linear programming issue:

$$Minimize\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$$

Subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i \qquad i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j \qquad j = 1,2,\dots,m$$

$$\tilde{x}_{ij} \geq 0 \ \forall\ i,j.$$

$$\sum_{j=1}^{n} \tilde{a}_i = \sum_{i=1}^{m} \tilde{b}_j$$

## 4. INTUITIONISTIC FUZZY TRANSPORTATION PROBLEM

**Basic Definitions**

In this section, some basic definitions of intuitionistic fuzzy set, intuitionistic fuzzy number, triangular intuitionistic fuzzy number and trapezoidal intuitionistic fuzzy number are reviewed.

**Definition 1.5** Let $X$ be non-empty set. An intuitionistic fuzzy set $A^I$ of $X$ is defined as

$$\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)\rangle / x \in X\} \text{ where } \mu_{\tilde{A}^I}(x) \text{ and } \nu_{\tilde{A}^I}(x)$$

Are membership and non-membership functions such that

$\mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) : X \to [0,1]$ and $0 \leq \mu_{\tilde{A}^I}(x) \leq \nu_{\tilde{A}^I}(x) \leq 1$ for all $x \in X$.

**Definition 1.6** An subset of intuitionistic fuzzy

$$\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)\rangle / x \in X\}$$

of real line X is called an intuitionistic fuzzy number if the following conditions hold:

(i) There exists a real number $m$ such that $\mu_{\tilde{A}^I}(m)=1$ and $\nu_{\tilde{A}^I}(m)=0$

(ii) $\mu_{\tilde{A}^I}:\mathbb{R}\to[0,1]$ is a continuous function such that $0\le\mu_{\tilde{A}^I}(x)+\nu_{\tilde{A}^I}(x)\le 1$ for all $x\in X$.

(iii) The membership and non-membership functions $\tilde{A}^I$ are the following form:

$$\mu_{\tilde{A}^I}(x)=\begin{cases}0, & -\infty<x\le a_1\\ f(x), & a_1\le x\le a_2\\ 1, & x=a_2\\ g(x), & a_2\le x\le a_3\\ 0, & a_2\le x<\infty\end{cases}\qquad \nu_{\tilde{A}^I}(x)=\begin{cases}1, & -\infty<x\le a_1^{'}\\ f^{'}(x), & a_1^{'}\le x\le a_2\\ 0, & x=a_2\\ g^{'}(x), & a_2\le x\le a_3^{'}\\ 1, & a_3\le x<\infty\end{cases}$$

Where $f,f^{'},g,g^{'}$ are functions from $\mathbb{R}\to[0,1]$, $f$ and $g^{'}$ are strictly increasing functions and $g$ and $f^{'}$ are strictly decreasing functions with the conditions $0\le f(x)+f^{'}(x)\le 1$ and $0\le g(x)+g^{'}(x)\le 1$.

**Definition 1.7** A triangular intuitionistic fuzzy number $\tilde{A}^I$ is denoted by $\tilde{A}^I=(a_1,a_2,a_3)(a_1^{'},a_2,a_3^{'})$ where $a_1^{'}\le a_1\le a_2\le a_3\le a_3^{'}$ with the following membership $\mu_{\tilde{A}^I}(x)$ and non-membership function $\nu_{\tilde{A}^I}(x)$.

$$\mu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & \text{for } a_1\le x\le a_2\\ 1, & \text{for } x=a_2\\ \dfrac{x-a_3}{a_2-a_3}, & \text{for } a_2\le x\le a_3\\ 0, & \text{otherwise}\end{cases}$$

$$\nu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{x-a_2}{a_1'-a_2}, & \text{for } a_1'\le x\le a_2\\ 0, & \text{for } x=a_2\\ \dfrac{x-a_2}{a_3'-a_2}, & \text{for } a_2\le x\le a_3'\\ 1, & \text{otherwise}\end{cases}$$

**Definition 1.8** A trapezoidal intuitionistic fuzzy number is denoted by

$$\tilde{A}^{I} = (a_1, a_2, a_3, a_4)(a_1^{'}, a_2, a_3, a_4^{'}) \text{ where } \quad a_1^{'} \le a_1 \le a_2 \le a_3 \le a_4 \le a_4^{'}$$

with the following membership □ $I\ (x)$ and non-membership function□ $I\ (x)$.

$$\mu_{\tilde{A}^{I}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & \text{for } a_1 \le x \le a_2 \\ 1, & \text{for } a_2 \le x \le a_3 \\ \dfrac{x-a_4}{a_3-a_4}, & \text{for } a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}^{I}}(x) = \begin{cases} \dfrac{x-a_2}{a_1'-a_2}, & \text{for } a_1' \le x \le a_2 \\ 0, & \text{for } a_2 \le x \le a_3 \\ \dfrac{x-a_3}{a_4'-a_3}, & \text{for } a_3 \le x \le a_4' \\ 1, & \text{otherwise} \end{cases}$$

**Arithmetic Operations**

In this section, arithmetic operations of triangular and trapezoidal intuitionistic fuzzy numbers are presented.

**Arithmetic Logic Operator in Intuitionistic Fuzzy Numbers for Triangular Set**

In this segment mathematical operations of triangular intuitionistic fuzzy numbers are shown

Let $\tilde{A}^{I} = (a_1, a_2, a_3)(a_1', a_2, a_3')$ and $\tilde{B}^{I} = (b_1, b_2, b_3)(b_1', b_2, b_3')$ be any two triangular intuitionistic fuzzy numbers. Then,

I. $\tilde{A}^{I} + \tilde{B}^{I} = (a_1+b_1, a_2+b_2, a_3+b_3)(a_1'+b_1', a_2+b_2, a_3'+b_3')$

II. $\tilde{A}^{I} - \tilde{B}^{I} = (a_1-b_3, a_2-b_2, a_3-b_1)(a_1'-b_3', a_2-b_2, a_3'-b_1')$

III. $\tilde{A}^{I} \times \tilde{B}^{I} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)(a_1' \times b_1', a_2 \times b_2, a_3' \times b_3')$ where $\tilde{A}^{I}, \tilde{B}^{I}$ are non-negative triangular intuitionistic fuzzy numbers.

**Arithmetic Logic Operator in Intuitionistic Fuzzy Numbers for Trapezoidal Set**

In the given section trapezoidal intuitionistic fuzzy numbers are presented and arithmaic opertation applied on them.

Let $\tilde{A}^I = (a_1, a_2, a_3, a_4)(a_1', a_2, a_3, a_4')$ and $\tilde{B}^I = (b_1, b_2, b_3, b_4)(b_1', b_2, b_3, b_4')$ be any two trapezoidal intuitionistic fuzzy numbers. Then,

I. $\tilde{A}^I - \tilde{B}^I = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)(a_1' - b_4', a_2 - b_3, a_3 - b_2, a_4' - b_1')$

II. $\tilde{A}^I + \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)(a_1' + b_1', a_2 + b_2, a_3 + b_3, a_4' + b_4')$

III. $\tilde{A}^I \times \tilde{B}^I = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4; a_1' b_1', a_2 b_2, a_3 b_3, a_4' b_4')$ Where $\tilde{A}^I, \tilde{B}^I$ are non-negative trapezoidal intuitionistic fuzzy numbers.

**Comparison of Intuitionistic Fuzzy Numbers:**

Let $\tilde{A}^I = (a_1, a_2, a_3,)(a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3)(b_1', b_2, b_3')$ be two triangular intuitionistic fuzzy numbers. Then,

(i) $\Re(\tilde{A}^I) > \Re(\tilde{B}^I)$ *iff* $\tilde{A}^I > \tilde{B}^I$

(ii) $\Re(\tilde{A}^I) < \Re(\tilde{B}^I)$ *iff* $\tilde{A}^I < \tilde{B}^I$

(iii) $\Re(\tilde{A}^I) = \Re(\tilde{B}^I)$ *iff* $\tilde{A}^I = \tilde{B}^I$

Where,

$$\Re(\tilde{A}^I) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

$$\Re(\tilde{B}^I) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

**Tabular Representation**

A balanced intuitionistic fuzzy transportation problem (total intuitionistic fuzzyavailability equal to total intuitionistic fuzzy demand) having $m$ sources ($O_i$, $i$ □ $1, 2, ..., m$) and $n$ destinations ($D_j$, $j$ □ $1, 2, ..., n$) can be represented as shown in Table 3.3.

**Table 3: Tabular representation of an intuitionisticfuzzy transportation problem**

| Destinations / Origins | $D_1$ | $D_2$ | $\cdots$ | $D_n$ | Intuitionistic fuzzy Availability $(a_i)$ |
|---|---|---|---|---|---|
| $O_1$ | $\tilde{c}_{11}{}^{I}$ | $\tilde{c}_{12}{}^{I}$ | $\cdots$ | $\tilde{c}_{1n}{}^{I}$ | $\tilde{a}_1{}^{I}$ |
| $O_2$ | $\tilde{c}_{21}{}^{I}$ | $\tilde{c}_{22}{}^{I}$ | $\cdots$ | $\tilde{c}_{2n}{}^{I}$ | $\tilde{a}_2{}^{I}$~ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | |
| $O_m$ | $\tilde{c}_{m1}{}^{I}$ | $\tilde{c}_{m2}{}^{I}$ | $\cdots$ | $\tilde{c}_{mn}{}^{I}$ | $\tilde{a}_m{}^{I}$ |
| Demand $\tilde{b}_j{}^{I}$ | $\tilde{b}_1{}^{I}$ | $\tilde{b}_2{}^{I}$ | $\cdots$ | $\tilde{b}_n{}^{I}$ | $\sum_i \tilde{a}_i{}^{I} = \sum_j \tilde{b}_j{}^{I}$ |

**Linear Programming Formulation**

A balanced intuitionistic FTP, given by Table 3.3,can also be converted into the following fuzzy linear programming problem:

$$\text{Minimize } \tilde{Z}^{I} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}{}^{I} \tilde{x}_{ij}{}^{I}$$

Subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij}{}^{I} = \tilde{a}_{ij}{}^{I}, \quad i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij}{}^{I} = \tilde{b}_{ij}{}^{I}, \quad j = 1,2,\ldots,n$$

*and*

$$\tilde{x}_{ij}{}^{I} \geq 0, \quad \forall i,j$$

**Intuitionistic Fuzzy Initial Basic Feasible Solution (IFIBFS)**

Thamaraiselvi and Santhi proposed the intuitionistic fuzzy Vogel's approximation method (IFVAM) to obtain the intuitionistic fuzzy basic feasible solution. The method proceeds as follows.

**Step 1:** Calculate the magnitude of difference between the minimum and next to minimum transportation cost in each row and column and write it as "Diff." along the side of the table against the corresponding row/column.

**Step 2:** In the row /column corresponding to maximum "Diff.", make the maximum allotment at the box having minimum transportation cost in that row/ column.

**Step 3:** If the maximum "Diff." corresponding to two or more rows or columns are equal,select the top most row and the extreme left column. Repeat the above procedure until all the "IF" supplies are fully used and "IF" demands are fully received.

**Intuitionistic Fuzzy Optimal Solution (IFOS)**

Thamaraiselvi and Santhi proposed the following method to obtain the intuitionistic fuzzy optimal solution of the intuitionistic fuzzy transportation problem.

**Step 1:** Write the intuitionistic fuzzy transportation problem in tabular form.

**Step 2:** Subtract each row entries of the table from the row minimum or row reduced. **Step 3:** In the table, obtained in Step 1, subtract each column entries from the columnminimum. We get at least one zero in each row and each column in the resultant table.

**Step 4:** In the resultant table, obtained in Step 3, for every zero cost cell, count the total number of zeros in the corresponding row and column. let $(i, j)^{th}$ zero cost cell is selected, calculate the all the 0 in the $i^{th}$ and $j^{th}$ column.

**Step 5:** Now select a cell of zero cost for which the number of zeros counted in Step 4 is minimum and allocate the maximum possible hexagonal intuitionistic fuzzy quantity to that cell. If tie occurs for some zeros in Step 4 then find the sum of all the given elements in the corresponding column. and row Now select the zero with maximum sum and allocate the maximum possible quantity to that cell.

**Step 6:** After every allotment, remove the row or column for which the demand f ulfilled and the supply is depleted.

**Step 7:** Repeat Step 4 to Step 6 until all the demands are satisfied and all the supplies are exhausted.

## 5. NUMERICAL EXAMPLE

Thamaraiselvi and Santhi solved the intuitionistic fuzzy transportation problem, represented by

Table 3.4, to illustrate their proposed method.

**Table 4 : Intuitionistic fuzzy transportation problem**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply |
|---|---|---|---|---|---|
| $O_1$ | 6 | 7 | 13 | 10 | (7,9,11,13,16, 20) (5,7,11,13,19, 20) |
| $O_2$ | 4 | 3 | 9 | 5 | (6,8,11,14,19, 21) (4,7,11,14,21, 27) |
| $O_3$ | 8 | 12 | 21 | 10 | (9,11,13,15, 18, 20) (8,10,13,15, 19, 22) |
| IF Demand | (3,4,5, 6,8,10) (2,4,5, 6,10,12) | (3,5,7, 9,12,16) (2,4,7, 9,13,17) | (6,7,9, 11,13,24) (5,6,9, 11,16,18) | (10,12,14, 16,20, 24) (8,10,14,16,20, 25) | |

**Intuitionistic Fuzzy Initial Basic Feasible Solution (IFIBFS)**

From the above method we got the table now applying the steps in given problem:-

**Step 1:** The magnitude of difference in the min. and next to min. transportation cost in each row and column is shown in Table 5.

**Table 5: Intuitionistic fuzzy transportation problem with “Diff”**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply | Diff |
|---|---|---|---|---|---|---|
| $O_1$ | 6 | 7 | 13 | 10 | (7,9,11,13, 16,20) (5,7,11,13, | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | 19,20) | |
| $O_2$ | 4 | 3 | 9 | 5 | (6,8,11,14, 19,21) (4,7,11,14, 21,27) | 1 |
| $O_3$ | 8 | 12 | 21 | 10 | (9,11,13,15 ,18,20) (8,10,13,15 ,19,22) | 2 |
| IF Demand | (3,4,5, 6,8,10) (2,4,5, 6,10,12) | (3,5,7, 9,12,16) (2,4,7, 9,13,17) | (6,7,9, 11,13,24) (5,6,9, 11,16,18) | (10,12,14, 16,20,24) (8,10,14, 16,20,25) | | |
| Diff | 2 | 4 | 4 | 5 □ | | |

**Step 2:** Utilizing the Step 2 distinguish the line/segment relating to the most elevated worth of "Diff". For this situation it happens at section 4. In this section least expense cell is (2, 4) and the comparing interest and supply are (10, 12, 14, 16, 20, 24) (8, 10, 14, 16, 20, 25) furthermore, (6, 8, 11, 14, 19, 25) (4, 7, 11, 14, 21, 27) separately. Presently distribute the (base of the above interest and supply) most extreme potential units (6, 8, 11, 14, 19, 25) (4, 7, 11, 14, 21, 27) to the base expense position (2, 4) and compose the excess in segment 4. In the wake of eliminating the subsequent column rehashes the Step 1, we acquired new Table.6

**Table 6: First Allocation**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply | Diff |
|---|---|---|---|---|---|---|
| $O_1$ | 6 | 7 | 13 | 10 | (7,9,11, 13,16,20) (5,7,11, 13,19,20) | 1 |
| $O_2$ | - | - | - | (6,8,11, 14,19,25) | - | - |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | (4,7,11, 14,21,27) | | |
| $O_3$ | 8 | 12 | 21 | 10 | (9,11,13, 15,18,220) (8,10,13, 15,19,22) | 2 |
| IF Demand | (3,4,5, 6,8,10) (2,4,5, 6,10,12) | (3,5,7, 9,12,16) (2,4,7, 9,13,17) | (6,7,9, 11,13,24) (5,6,9, 11,16,18) | $(-15,-7,0,$ $5,12,18)$ $(-19,-1,0,$ $5,13,21)$ | | |
| Diff | 2 | 5 | 8 ↑ | 0 | | |

**Step 3**: In Table 2.3 the most extreme worth of "Diff" happens at the third section. So allot the greatest potential units (6, 7, 9, 11, 13, 16) (5, 6, 9, 11, 16, 18) to the base expense position (1, 3) and compose the excess in first column. Subsequent to eliminating the third section rehash the means 1 to 3. Presently most noteworthy worth of "Diff" happens at second segment. Presently apportion the most extreme potential units (-9, -4, 0, 4, 9, 14) (-13, - 9, 0, 4, 13, 18) to the base expense position (1, 2). Subsequent to composing the leftover in section 2, eliminating the main line and rehashing the Step1, Table 7 is the acquired table.

**Table 7: The next allocations**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply | Diff |
|---|---|---|---|---|---|---|
| $O_1$ | - | $(-9,-4,0,$ $4,9,1)$ $(13,-9,0,$ $4,13,18)$ | (6,7,9, 11,13,16) (5,6,9, 11,16,18) | - | - | - |
| $O_2$ | - | - | - | (6,8,11, 14,19,25) (4,7,11, 14,21,27) | - | - |
| $O_3$ | 8 | 12 | - | 10 | (9,11,1 3, | 2 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | 15,18, 20) (8,10,13, 15,19, 22) | |
| IF Demand | (3,4,5, 6,8,10) (2,4,5, 6,10,12) | $(-11,-4,3, 9,16,24)$ $(-19,-9,3, 9,22,30)$ | - | $(-15,-7,0, 5,12,18)$ $(-19,-11,0, 5,13,21)$ | | |
| Diff | 2 | - | - | - | | |

**Step 4**: Repeating the same procedure, the optimal solution shown in Table 8, is obtained.

**Table 8: The results**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply | Diff |
|---|---|---|---|---|---|---|
| $O_1$ | - | $(-9,-4,0, 4,9,1)$ $(-13,-9,0, 4,13,18)$ | (6,7,9, 11,13,16) (5,6,9, 11,16,18) | - | - | - |
| $O_2$ | - | - | - | (6,8,11, 14,19,25) (4,7,11, 14,21,27) | - | - |
| $O_3$ | (3,4,5, 6,8,10) (2,4,5, 6,10,12) | $(-11,-4,3, 9,16,24)$ $(-19,-9,3, 9,22,30)$ | - | $(-15,-7,0, 5,12,18)$ $(-19,-11,0, 5,13,21)$ | - | - |
| IF Demand | - | - | - | - | | |
| Diff | - | - | - | - | | |

Solution for the given problem is given by above table :-

$x_{12} = (-9,-4,0,4,9,14)(-13,-9,0,4,13,18),$

$x_{13} = (6,7,9,11,13,16)(5,6,9,11,16,18)$

$x_{24} = (6,8,11,14,19,25)(4,7,11,14,21,27)$

$x_{31} = (3,4,5,6,8,10)(2,4,5,6,10,12)$

$x_{32} = (-11,-4,3,9,16,24)(-19,-11,0,5,13,21)$

$x_{33} = (-15,-7,0,5,12,18)(-19,-11,0,5,13,21)$

and the minimum total intuitionistic fuzzy transportation cost is

$$Z^I = 7(-9,-4,0,4,9,14)(-13,-9,0,4,13,18) + 13(6,7,9,11,13,16)(5,6,9,11,16,18)$$
$$+5(6,8,11,14,19,25)(4,7,11,14,21,27) + 8(3,4,5,6,8,10)(2,4,5,6,10,12)$$
$$+12(-11,-4,3,9,16,24)(-19,-11,0,5,13,21) + 10(-15,-7,0,5,12,18)(-19,-11,0,5,13,21)$$
$$= (-213,17,248,447,703,979)(-408,-160,212,399,770,1053)$$

**Intuitionistic Fuzzy Optimal Solution (IFOS)**

Using the method, proposed by Thamaraiselvi and Santhi [17], an intuitionistic fuzzy optimal solution of the IFTP, represented by Table 3.4, can be obtained as follows:

**Step 1:** Using Step 2 and Step 3 of the method, proposed by Thamaraiselvi and Santhi, Table 9 is obtained.

**Table 9: represent step 1 and 2**

| Destinations<br>Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply |
|---|---|---|---|---|---|
| $O_1$ | 0 | 1 | 1 | 2 | (7,9,11,13,16,20)<br>(5,7,11,13,19,20) |
| $O_2$ | 1 | 0 | 0 | 0 | (6,8,11,14,19,21)<br>(4,7,11,14,21,27) |
| $O_3$ | 0 | 4 | 7 | 0 | (9,11,13,15,18,20)<br>(8,10,13,15,19,22) |
| IF Demand | (3,4,5,<br>6,8,10) | (3,5,7,<br>9,12,16) | (6,7,9,<br>11,13,24) | (10,12,14,<br>16,20,24) | |

| | | | | | |
|---|---|---|---|---|---|
| | (2,4,5, 6,10,12) | (2,4,7, 9,13,1 7) | (5,6,9, 11,16,18) | (8,10,14, 16,20,25) | |

**Step 2**: follow Step 5 and Step 4 for the next Table 3.9. For each 0, tally the all out number of 0's in the relating line and segment. Since, the zero in the cell (1, 1) has least number (1) of zeros with most extreme total (5) of components in first line and first section. Presently distribute the greatest potential units (3, 4, 5, 6, 8, 10) (2, 4, 5, 6, 10, 12) to the position (1,1) and compose the excess in line 1. In the wake of eliminating the first section we get the accompanying table.

**Table 10: First allocation**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply |
|---|---|---|---|---|---|
| $O_1$ | (3,4,5,6, 8,10) (2,4,5,6, 10,12) | 1 | 1 | 2 | (−3,1,5,8,12,17) (−7,-3,5,8,15,18) |
| $O_2$ | - | 0 | 0 | 0 | (6,8,11,14,19,21) (4,7,11,14,21,27) |
| $O_3$ | - | 4 | 7 | 0 | (9,11,13,15,18,20) (8,10,13,15,19,22) |
| IF Demand | - | (3,5,7,9, 121,16) (2,4,7,9, 13,17) | (6,7,9, 11,13,24) (5,6,9, 11,16,18) | (10,12,14,1 6,20,24) (8,10,14,16 ,20,25) | |

**Step 3**: Applying the Step 4 and Step 5 and apportion the greatest potential units (9, 11, 13, 15, 18, 20)(8, 10, 13, 15, 19, 22) to the position (3, 4) and compose the leftover in fourth line.

**Table 11: Second allocation**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply |
|---|---|---|---|---|---|
| $O_1$ | (3,4,5,6,8,10) (2,4,5,6,10,12) | 1 | 1 | 2 | (−3,1,5,8,12,17) (-7,-3,5,8,15,18) |
| $O_2$ | - | 0 | 0 | 0 | (6,8,11,14,19,21) (4,7,11,14,21,27) |
| $O_3$ | - | - | - | (9,11,13,15,18,20) (8,10,13,15,19,22) | - |
| IF Demand | - | (3,5,7,9,12,16) (2,4,7,9,13,17) | (6,7,9,11,13,24) (5,6,9,11,16,18) | (-10,-6,-1,3,9,16) (-14,-9,-1,3,10,17) | |

**Step 4**: Applying the Step 4 and Step 5 and apportion the greatest potential units (3, 5, 7, 9, 12, 16)(2, 4, 7, 9, 13, 17) to the position (2, 2) and compose the leftover in second row.

**Table 12: Third allocation**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply |
|---|---|---|---|---|---|
| $O_1$ | (3,4,5,6,8,10) (2,4,5,6,10,12) | - | 1 | 2 | (−3,1,5,8,12,17) (-7,-3,5,8,15,18) |
| $O_2$ | - | (3,5,7,9,12,16) (2,4,7,9,13,17) | 0 | 0 | (-10,-4,2,7,14,18) (-13,-6,2,7,17,25) |
| $O_3$ | - | - | - | (9,11,13,15,18,20) (8,10,13,15,19,22) | - |

| | | | | | |
|---|---|---|---|---|---|
| IF Demand | - | - | (6,7,9, 11,13,24) (5,6,9,11,16,18) | (-10,-6,-1,3,9,16) (-14,-9,-1,3,10,17) | |

**Step 5**: Applying the Step 4 and Step 5 and apportion the greatest potential units (-10, -4, 2, 7, 14, 18)(-13, -6, 2, 7, 17, 25) to the position (2, 3) and compose the leftover in second row.

**Table 13: fourth allocation**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ | IF Supply |
|---|---|---|---|---|---|
| $O_1$ | (3,4,5,6, 8,10) (2,4,5,6, 10,12) | - | 1 | 2 | (−3,1,5,8,12,17) (-7,-3,5,8,15,18) |
| $O_2$ | - | (3,5,7,9,12, 16) (2,4,7,9,13,17) | (-10,-4,2,7,14,18) (-13,-6,2,7,17, 25) | - | - |
| $O_3$ | - | - | - | (9,11,13,15,18,20) (8,10,13,15,19,22) | - |
| IF Demand | - | - | (-12,-7,2,9,17,34) (-20,-11,2,9,22,31) | (-10,-6,-1,3,9,16) (-14,-9,-1,3,10, 17) | |

**Step 5**: Applying the Step 4 and Step 5 and apportion the greatest potential units (-10, -4, 2, 7, 14, 18)(-13, -6, 2, 7, 17, 25) to the position (2, 3) and compose the leftover in second row.

**Table 14: Optimal solution**

| Destinations Origins | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $O_1$ | (3,4,5,6,8,10) (2,4,5,6,10,12) | - | 1 | 2 |
| $O_2$ | - | (3,5,7,9,12, 16) (2,4,7,9,1 3,17) | (-10,-4,2,7,14,18) (-13,-6,2,7,17,25) | - |
| $O_3$ | - | - | - | (9,11,13,15,18,20) (8,10,13,15,19,22) |

Thus, the intuitionistic fuzzy optimal solution in terms of hexagonal intuitionistic fuzzy numbers (HIFN) is,

$\tilde{x}_{11} = (3, 4, 5, 6, 8, 10)(2, 4, 5, 6, 10, 12)$, $\tilde{x}_{22} = (3, 5, 7, 9, 12, 16)(2, 4, 7, 9, 13, 17)$,

$\tilde{x}_{23} = (-10, -4, 2, 7, 14, 18)(-13, -6, 2, 7, 17, 25)$, $x_{34} = (9, 11, 13, 15, 18, 20)(8, 10, 13, 15, 19, 22)$,

And the total minimize intuitionistic fuzzy transportation cost is

Z = 6(3, 4, 5, 6, 8, 10)(2, 4, 5, 6, 10, 12) + 3(3, 5, 7, 9, 12, 16)(2, 4, 7, 9, 13, 17) + 9(-10, -4, 2, 7, 14, 18)(-13, -6, 2, 7, 17, 25) +10(9, 11, 13, 15, 18, 20)( 8, 10, 13, 15, 19, 22)

= (27, 113, 199, 276, 390, 470) (-19, 82, 199, 276, 442, 568)

## 5. PARADOX IN TRANSPORTATION PROBLEMS UNDER FUZZY ENVIRONMENTS

Sometimes of the transportation issue, an increment in the provisions and requests or all in all, expansion in the stream results a lessening in the ideal transportation cost. This kind of conduct which implies dumbfounding is called transportation oddity. Acharya et al. proposed an adequate condition for the presence of paradox in a transportation issue under fuzzy climate.

**Existing Algorithm**

Acharya et al. proposed the following algorithm to find all the paradoxical pairs of the problem (P).

**Step 1**: $i = 0$.

**Step 2**: Find the cost flow pair $(\tilde{Z}_0, \tilde{F}_0)$ for the optimum solution $\tilde{x}_0$.

**Step 3**: Find all cells $(r,s) \notin B$ such that $(u_r + v_s) \tilde{<} \tilde{0}$ if it exists, otherwise go to Step 8.

**Step 4**: Find min flow for $l = (1,\tilde{0},0,0), l = (0,\tilde{1},0,0), l = (0,\tilde{0},1,0),\ l \approx (0,\tilde{0},0,1)$ or $l = (1,\tilde{1},1,1)$, which enters into existing basis whose corresponding cost is minimum. Let $(Z_i, F_i)$ be the new cost flow pair corresponding to the optimum solution $X_i$.

**Step 5**: $i = i + 1$.

**Step 6**: Write $(\tilde{Z}_i, \tilde{F}_i)$.

**Step 7**: Find all cells $(r,s) \notin B$ such that $(u_r + v_s) \tilde{<} \tilde{0}$ if it exists go to Step 4, otherwise goto Step 9.

**Step 8**: Write paradox does not exist and go to Step 10.

**Step 9**: Write paradox exists and the best paradox pair $(Z_*, F_*) = (Z_i, F_i)$ for the optimum solution $x_* = x_i$.

**Step 10**: End.

**Numerical Example**

Acharya et al. considered a mathematical model which comprises of three beginnings and four objections, the dubious number of supply, request and cost per unit are arranged in Table 15.

**Table 15: Fuzzy transportation problem**

| Destination → Origin ↓ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $\tilde{a}_i$ |
|---|---|---|---|---|---|
| $O_1$ | (1,2,4,5) | (6,7,9,10) | (2,3,5,6) | (7,8,10,11) | (39,40,42, 43) |
| $O_2$ | (5,6,8,9) | (1,2,2,3) | (8,9,11,12) | (3,4,6,7) | (50,51,53,54) |
| $O_3$ | (2,3,5,6) | (7,8,10,11) | (1,2,4,5) | (9,10,12,13) | (39,40,42,43) |
| | (19,20,22 ,23) | (24,25,27,28) | (49,50,52,53) | (34,35,37,38) | |

Utilizing fuzzy Vogel's estimate strategies, takes the principal push and pick its littlest section and deduct this from the following littlest passage, and write before the line on the right. This is the fuzzy punishment for first column. Ascertain the punishments for different lines. Likewise figure fuzzy punishments for every one of the segments and keep in touch with them in the lower part of the Fuzzy transportation table beneath comparing segments. Select the most noteworthy fuzzy punishment and notice the line or segment for which this compares. Decide the littlest fuzzy expense in the chose line or segment. Let it be $c_{ij}$. Allocate $x_{ij} = \min(a_i, b_j)$ in the $(i, j)^{th}$ cell of the given fuzzy transportation table. So we can find the first allocation on $(x_{22}) = (24, 25, 27, 28)$ after allocating the value, the fuzzy availability is changed to $\tilde{a}_1 = (22, \tilde{24}, \tilde{28}, \tilde{30})$. The resulting matrix is shown in table 16

**Table 16: first allocation**

| Destination→ Origin ↓ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $\tilde{a}_i$ |
|---|---|---|---|---|---|
| $O_1$ | (1,2,4,5) | (6,7,9,10) | (2,3,5,6) | (7,8,10,11) | (39,40,42, 43) |
| $O_2$ | (5,6,8,9) | (1,2,2,3)<br>(24,25,27,28) | (8,9,11,12) | (3,4,6,7) | (50,51,53,54)<br>(22,24,28,30) |

| | | | | | |
|---|---|---|---|---|---|
| $O_3$ | (2,3,5,6) | (7,8,10,11) | (1,2,4,5) | (9,10,12,13) | (39,40,42,43) |
| | (19,20,22,23) | (24,25,27,28) | (49,50,52,53) | (34,35,37,38) | |

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