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A New Form of Soft Functions in Soft Topological Spaces

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Abstract

In this paper, we introduce a new category of soft function called Š α^* - continuous function. Also we study in detail the properties of Š α^* - continuous function, Š α^* irresolute function and its relation with other Š function. All these findings will provide a base to researchers who want to work in the field of soft topology and will help to establish a general frame work for applications in practical fields.

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1. Introduction

Molodtsov introduced the concept of soft sets from which the difficulties of fuzzy sets, intutuionistic fuzzy sets, vague sets, interval mathematics and rough sets have been rectified. Application of soft sets in decision making problems has been found by Maji et al. whereas Chen gave a parametrization reduction of soft sets and a comparison of it with attribute reduction in rough set theory. Further soft sets are a class of special information.

Shabir and Naz introduced soft topological spaces in 2011 and studied some basic properties of them. Meanwhile generalized closed sets in topological spaces were introduced by Levine in 1970 and recent survey of them is in which is extended to soft topological spaces in the year 2012. Further Kannan and Rajalakshmi have introduced soft g – locally closed sets and soft semi star generalized closed sets. Soft strongly g – closed sets have been studied by Kannan, Rajalakshmi and Srikanth. Chandrasekhara Rao and Palaiappan introduced generalized star star closed sets in topological spaces and it is extended to the bitopological context by Chandrasekhara Rao and Kannan.

Recently papers about soft sets and their applications in various fields have increased largely. Modern topology depends strongly on the ideas of set theory. Any Research work should result in addition to the existing knowledge of a particular concept. Such an effort not only widens the scope of the concept but also encourages others to explore new and newer ideas. Therefore in this work we introduce a new soft generalized set called Š α^* open set and its related properties. This may be another starting point for the new soft set mathematical concepts and structures that are based on soft set theoretic operations.

2. Preliminaries

In this section, this project X be an initial universe and $\widehat{E}$ be a set of parameters. Let P (X) denote the power set of X and A be a non-empty subset of $\widehat{E}$. A pair $\left(F^{\breve{S}}, A\right)$ denoted by $F_a^{\breve{s}}$ is called a soft set over X, where $F^{\breve{S}}$ is a mapping given by $F^{\breve{s}} : A \to P(X)$.

Definition 2.1.1. [8] : For two soft sets $\left(F^{\breve{S}}, A\right)$ and (G,B) over a common universe X, we say that $\left(F^{\breve{S}}, A\right)$ is a soft subset of (G, B) denoted by $\left(F^{\breve{S}}, A\right) \subseteq_S (G, B)$, if

i. $\left(A \subseteq_S B\right)$ and

ii. $F^{\breve{S}}(\hat{e}) \subseteq_S G(\hat{e})$ for all $\hat{e} \in \hat{E}$

Definition 2.1.2. [8] : The complement of a soft set $\left(F^{\breve{S}}, A\right)$ denoted by $\left(F^{\breve{S}}, A\right)^c$, is defined by $\left((F^{\breve{S}}, A)^c\right) = \left(F^{\breve{S}c}, A\right)$, where $F^{\hat{S}c} : A \to P(X)$ is a mapping given by $F^{\hat{S}c}(\hat{e}) = X - F^{\hat{S}}(\hat{e})$, for all $\hat{e} \in E$

Definition 2.1.3. [7] : Let a $\breve{S}$ set $(F^{\hat{S}}, A)$ over X.

a. Null $\breve{S}$ set denoted by φ if for all $e \in A, F^{\breve{S}}(\hat{e}) = \varphi$.

b. Absolute $\breve{S}$ set denoted by X if for all $e \in A, F^{\breve{S}}(\hat{e}) = X$.

Clearly, $X^c = \varphi$ and $\varphi^c = X$.

Definition 2.1.4. [4]: The Union of two $\breve{S}$ sets of $(F^{\breve{S}}, A)$ and (G, B), over the common universe X is the $\breve{S}$ set (H, C), where $C = A \cup_S B$, and for all $\hat{e} \in C$

$H(e) = F^{\breve{S}}(\hat{e})$ if $\hat{e} \in A - B$, $H(\hat{e}) = G(\hat{e})$, if $\hat{e} \in B - A$ and $H(\hat{e}) = F^{\breve{S}}(\hat{e}) \cup_S G(\hat{e})$, if $\hat{e} \in A \cap_S B$. and is denoted as $(F^{\breve{S}}, A) \cup_S G(\hat{e}) = (H, C)$.

Definition 2.1.5. [4] : The Intersection (H, C) of two $\breve{S}$ sets of $(F^{\breve{S}}, A)$ and (G, B), over the common universe X, denoted by the $\breve{S}$ set (H, C). where $C = A \cap_S B$, and $H(\hat{e}) = F^{\breve{S}}(\hat{e}) \cap_S G(\hat{e})$, for all $\hat{e} \in C$.

Definition 2.1.6. [10]: Let τ be a collection of $\breve{S}$ sets over X with the fixed set of parameters. Then τ is called a $\breve{S}$ of Topology on X, if

i. Φ and X belongs to τ_s.

ii. The union of any number of Šoft sets in τ_s belongs to τ_s.

iii. The intersection of any two Šoft sets in τ_s belongs to τ_s.

The triplet $(X, \tau_s, \hat{E})$ is called Šoft Topological Spaces over X.

The members of τ_s are called Šoft open sets in X and complements of them are called Šoft Closed sets in X.

Definition 2.1.7. [10]: Let $(X, \tau_s, \hat{E})$ be a Šoft Topological Spaces over X and let $(F^{\breve{S}}, \hat{E})$ be a Šoft set over X.

1) The Šoft Interior of $(F^{\breve{S}}, \hat{E})$ is the Šoft set Šoft Int $(F^{\breve{S}}, \hat{E}) \cup_S (O, \hat{E}) : (O, \hat{E})$ which is Šoft open and $(O, \hat{E}) \subseteq_S (F^{\breve{S}}, \hat{E})\}$
2) The Šoft closure of $(F^{\breve{S}}, \hat{E})$ is the Šoft set Šoft cl $(F^{\breve{S}}, \hat{E}) \cap_S \{(A, \hat{E}) : (A, \hat{E})$ which is Šoft closed and $(F^{\breve{S}}, \hat{E}) \subseteq_S (A, \hat{E})\}$. Clearly Šoft cl $(F^{\breve{S}}, \hat{E})$ is the largest Šoft closed set over X which contains $(F^{\breve{S}}, \hat{E})$.

Definition 2.1.8. : A Sub set of a Šoft topological space $(X, \tau_s, \hat{E})$ is said to be

1. a Šoft Semi-Open set [3] if $(F^{\breve{S}}, \hat{E}) \subseteq_S \breve{S}Cl(\breve{S}int(F^{\breve{S}}, \hat{E})$ and a Šoft semi-Closed set if $\breve{S}$ int $Cl(F^{\breve{S}}, \hat{E}) \subseteq_S (F^{\breve{S}}, \hat{E})$.

2. a $\breve{S}$ Pre-open set [1] if $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \breve{S}\mathrm{Int}\left(\breve{S}\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right)\right)$ and a Šoft Pre-Closed set if $\breve{S}$ $\mathrm{Cl}\left(\breve{S}\mathrm{int}\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(F^{\breve{S}},\hat{E}\right)\right)$a $\breve{S}\alpha$ Open set [1] if $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \breve{S}\mathrm{Int}\left(\breve{S}\mathrm{Cl}(\mathrm{int}\left(F^{\breve{S}},\hat{E}\right)\right)$ and a $\breve{S}\alpha$ of -Closed set if $\breve{S}$ $\mathrm{Cl}\left(\breve{S}\mathrm{int}\left(\breve{S}\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(F^{\breve{S}},\hat{E}\right)\right)\right)$.

3. a $\breve{S}$ β -Open set [1] if $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \breve{S}\mathrm{Cl}\left(\breve{S}\mathrm{int}\,(\breve{S}\mathrm{cl}\,(F^{\breve{S}},\hat{E})\right)$and a $\breve{S}\beta$ Closed set if $\breve{S}$ int $\left(\breve{S}\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(F^{\breve{S}},\hat{E}\right)\right)$.

4. a $\breve{S}$ generalized Closed set (briefly $\breve{S}$ gs-Closed) if $\breve{S}$ $\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$whenever $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$and $\left(G,\hat{E}\right)$is $\breve{S}$ Open in $\left(X,\tau_s,\hat{E}\right)$. The complement of a $\breve{S}$ gs-Closed set is called a $\breve{S}$ gs-Open set.

5. a $\breve{S}$ Semi-generalized Closed set (briefly $\breve{S}$ Sg-Closed) if $\breve{S}$ $\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$whenever $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$ *and* $\left(G,\hat{E}\right)$ *is Šoft semi Open in* $\left(X,\tau_s,\hat{E}\right)$. The Complement of a šoft Semi g-Closed set is called a $\breve{S}$ Sg-Open set.

6. a generalized Šoft Semi-Closed set (briefly gs-Closed) if $\breve{S}$ $\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right) O_S\left(G,\hat{E}\right)$whenever $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$*and*$\left(G,\hat{E}\right)$*is Šoft Open in* $\left(X,\tau_s,\hat{E}\right)$. The complement of a Šoft gs-Closed set is called a $\breve{S}$ gs-Open set.

7. a Šoft - Closed [9] if $\breve{S}$ $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$ whenever $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)and\left(G,\hat{E}\right)$ is Šoft semi Openin $\left(X,\tau_s,\hat{E}\right)$

8. a Šoft ω-Closed [9] if $\breve{S}$ $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$whenever $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)and\left(G,\hat{E}\right)$ is Šoft semi Openin.

9. a Šoft alpha-generalized Closed set (briefly $\breve{S}$ α g-Closed) if $\alpha\breve{S}$ $\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$ whenever $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$ *and* $\left(G,\hat{E}\right)$ *is šoft semi Open in* $\left(X,\tau_s,\hat{E}\right)$. The Complement of a Šoft αg -Closed set is called a $\breve{S}$ α g-Open set.

10. a Šoft generalized alpha Closed set (briefly $\breve{S}$ g α-Closed) if α $\breve{S}$ $\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$ whenever $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$ and $\left(G,\hat{E}\right)$is šoft Openin $\left(X,\tau_s,\hat{E}\right)$. The Complement of a šoft gα -Closed set is called a $\breve{S}$ gα-Open set.

11. a Šoft generalized preClosed set (briefly $\breve{S}$ gp-Closed)[1] if p $\breve{S}$ $\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$ whenever a Šoft gp-Openset.

12. a Šoft generalized preregular Closed set (briefly $\breve{S}$ gpr-Closed)[5] if p $\breve{S}$ $\mathrm{Cl}\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$ whenever $\left(F^{\breve{S}},\hat{E}\right)\subseteq_S \left(G,\hat{E}\right)$ and $\left(G,\hat{E}\right)$ is Šoft regular Open in $\left(X,\tau_s,\hat{E}\right)$. The complement of a Šoft gpr-Closed set is called a $\breve{S}$ gpr-Open set.

3.1: $\breve{S}$ α* - continuous function

Definition 3.1.1:

A $\breve{S}$ of t function $f:(X,\tau_s.\grave{E})\to(\acute{Y},\sigma_s,\grave{E})$ is called a $\breve{S}$ α*- continuous, if the inverse image of every Šoft -open set in $(\acute{Y},\sigma_s,\grave{E})$ is $\breve{S}$ α*-Open set in $(X,\tau_s,\grave{E})$ (i.e) $f^{-1}(f^s,\grave{E})$ is a $\breve{S}$ α*-open set in $(X,\tau_s,\grave{E})$, for every $\breve{S}$ open set in $(\acute{Y},\sigma_s,\grave{E})$.

Example 3.1.2:

Let $X=\acute{Y}=\{x_1,x_2\}, \tau_s=\{f_3^s,f_{15}^s,f_{16}^s\}$ and $\breve{S}\ \alpha *-\hat{O}(X)=\{f_1^s,f_2^s,f_3^s,f_4^s,f_5^s,f_6^s$ $f_7^s,f_8^s,f_9^s,f_{10}^s,f_{11}^s,f_{12}^s,f_{13}^s,f_{14}^s,f_{15}^s,f_{16}^s\}$ and $\sigma_s=\{f_3^s,f_{11}^s,f_{12}^s,f_{15}^s,f_{16}^s\}$.

Let $f:(X,\tau_s,\grave{E})\to(\acute{Y},\sigma_s,\grave{E})$ be defined by $f(f_i^s)=f_i^s$ for all i=1 to 16, $f^{-1}(f_3^s)=f_3^s, f^{-1}(f_{11}^s)=f_{11}^s, f^{-1}(f_{12}^s)=f_{12}^s, f^{-1}(f_{15}^s)=f_{15}^s, f^{-1}(f_{16}^s)=f_{16}^s \Rightarrow \{f_3^s\},\{f_{11}^s\},\{f_{12}^s\},\{f_{15}^s\},\{f_{16}^s\}$ are in $\breve{S}$ α*- open, clearly f is $\breve{S}$ α* - continuous.

Theorem 3.1.3: Every Šoft - continuous function is $\breve{S}$ α*- continuous, but not conversely.

Proof:

Let $f:(X,\tau_s,\grave{E})\to(\acute{Y},\sigma_s,\grave{E})$ be a šoft – continuous function. Let $(f^s,\grave{E})$ be a $\breve{S}$ - Open in $(\acute{Y},\sigma_s,\grave{E})$. Since f is a šoft – continuous function, then, $f:(f^s,\hat{E})$ isŠoft –Open in $(X,\tau_s,\grave{E})\Rightarrow f^{-1}(f^s,\hat{E})$ is a $\breve{S}$ α*-Open in $(X,\tau_s,\grave{E})\Rightarrow f$ is a $\breve{S}$ α*- continuous function.

Example 3.1.4:

Let $X=\acute{Y}=\{x_1,x_2\}, \tau_s=\{f_2^s,f_4^s,f_9^s,f_{15}^s,f_{16}^s\}$ and $\sigma_s=\{f_6^s,f_{13}^s,f_{14}^s,f_{15}^s,f_{16}^s\}$, $\breve{S}\ \alpha *-\hat{O}(X)=\{f_1^s,f_2^s,f_3^s,f_4^s,f_5^s,f_6^s,f_7^s,f_8^s,f_9^s,f_{10}^s,f_{11}^s,f_{12}^s,f_{13}^s,f_{14}^s,f_{15}^s,f_{16}^s\}$, and $\breve{S}\ \alpha *-\hat{O}(\acute{Y})=\{f_4^s,f_5^s,f_6^s,f_7^s,f_8^s,f_9^s,f_{10}^s,f_{11}^s,f_{12}^s,f_{13}^s,f_{14}^s,f_{15}^s,f_{16}^s\}$.

Let $f:(X,\tau_s,\grave{E})\to(\acute{Y},\sigma_s,\grave{E})$ be defined by $f(f_1^s)=f_6^s, f(f_2^s)=f_1^s, f(f_3^s)=f_4^s$,

$f(f_4^s)=f_3^s, f(f_5^s)=f_7^s, f(f_6^s)=f_1^s, f(f_7^s)=f_5^s, f(f_8^s)=f_9^s, f(f_9^s)=f_8^s, f(f_{10}^s)=f_{11}^s$,

$f(f_{11}^s)=f_{10}^s, f(f_{12}^s)=f_{12}^s, f(f_{13}^s)=f_{13}^s, f(f_{14}^s)=f_{14}^s, f(f_{15}^s)=f_{15}^s, f(f_{16}^s)=f_{16}^s$.

Clearly f is $\breve{S}$ α* - continuous but not šoft -continuous function, because

$f^{-1}(f_1^s) = f_6^s\ f^{-1}(f_{13}^s) = f_{13}^s\ f^{-1}(f_{14}^s) = f_{14}^s$ are not $\breve{S}$oft -Open in $(X, \tau_s, \grave{E})$.

Theorem 3.1.5:

Every $\breve{S}$ α-continuous function is $\breve{S}$ α*-continuous, but not conversely.

Proof :

Let $(f^s, \grave{E})$ be a $\breve{S}$oft -Open in $(\acute{Y}, \sigma_s, \grave{E})$. Since f is a $\breve{S}$ α-continuous function, then, $f^{-1}(f^s, \grave{E})$ is a $\breve{S}$ α-Open in $(X, \tau_s, \grave{E}) \Rightarrow f^{-1}(f^s, \grave{E})$ is a $\breve{S}$ α*-Open in $(X, \tau_s, \grave{E}) \Rightarrow f$ is a $\breve{S}$ α*-continuous function.

Example 3.1.6 :

Let $X = Y = \{x_1, x_2\}, \tau_s = \{f_3^s, f_{11}^s, f_{12}^s, f_{15}^s, f_{16}^s\}, \breve{S}\alpha - \breve{O}(X) = \{f_3^s, f_{11}^s, f_{12}^s, f_{15}^s, f_{16}^s\}$,

$\breve{S}\ \alpha* - \hat{O}(X) = \{f_1^s, f_2^s, f_3^s, f_7^s, f_8^s, f_9^s, f_{10}^s, f_{11}^s, f_{12}^s, f_{13}^s, f_{14}^s, f_{15}^s, f_{16}^s\}$, $\sigma_s = \{f_4^s, f_7^s, f_{15}^s, f_{16}^s\}$.

Let $f : (X, \tau_s, \grave{E}) \to (\acute{Y}, \sigma_s, \grave{E})$ be defined by $f(f_1^s) = f_1^s, f(f_2^s) = f_5^s, f(f_3^s) = f_3^s$,
$f(f_4^s) = f_8^s, f(f_5^s) = f_2^s, f(f_6^s) = f_6^s, f(f_7^s) = f_7^s, f(f_8^s) = f_4^s, f(f_9^s) = f_{10}^s, f(f_{10}^s) = f_9^s$,
$f(f_{11}^s) = f_{13}^s, f(f_{12}^s) = f_{14}^s, f(f_{13}^s) = f_{11}^s, f(f_{14}^s) = f_{12}^s, f(f_{15}^s) = f_{15}^s, f(f_{16}^s) = f_{16}^s$.

Clearly f is $\breve{S}$ α* - continuous but not $\breve{S}$ α continuous function, because
$f^{-1}(f_4^s) = f_8^s\ f^{-1}(f_7^s) = f_7^s$ are not $\breve{S}$ α Open in $(X, \tau_s, \grave{E})$.

Theorem 3.1.7:

Every $\breve{S}$ g-continuous function is $\breve{S}$ α*-continuous, but not conversely.

Proof :

Let $(f^s, \grave{E})$ be a $\breve{S}$ -Open in $(\acute{Y}, \sigma_s, \grave{E})$. Since f is a $\breve{S}$ g-continuous function, then, $f^{-1}(f^s, \grave{E})$ is a $\breve{S}g - \hat{O}(X)$ in $(X, \tau_s, \grave{E}) \Rightarrow f^{-1}(f^s, \grave{E})$ is a $\breve{S}$ α*-Open in $(X, \tau_s, E) \Rightarrow f$ is a $\breve{S}$ α*-continuous function.

Example 3.1.8 :

Let $X = Y = \{x_1, x_2\}, \tau_s = \{f_1^s, f_2^s, f_3^s, f_{15}^s, f_{16}^s\}$,

$\breve{S}g - \breve{O}(X) = \{f_1^s, f_2^s, f_3^s, f_4^s, f_5^s, f_7^s, f_8^s, f_9^s, f_{10}^s, f_{11}^s, f_{12}^s, f_{15}^s, f_{16}^s\}$,
$\breve{S}\ \alpha* - \hat{O}(X) = \{f_1^s, f_2^s, f_3^s, f_4^s, f_5^s, f_6^s, f_7^s, f_8^s, f_9^s, f_{10}^s, f_{11}^s, f_{12}^s, f_{13}^s, f_{14}^s, f_{15}^s, f_{16}^s\}$, and $\sigma_s = \{f_4^s, f_7^s, f_{15}^s, f_{16}^s\}$.

Let $f:(X,\tau_s,È)\to(Ý,\sigma_s,È)$ be defined by $f(f_1^s)=f_1^s, f(f_2^s)=f_2^s, f(f_3^s)=f_3^s, f(f_4^s)=f_6^s,$ $f(f_5^s)=f_5^s, f(f_6^s)=f_4^s, f(f_7^s)=f_{13}^s, f(f_8^s)=f_8^s, f(f_9^s)=f_{10}^s, f(f_{10}^s)=f_9^s, f(f_{11}^s)=f_{11}^s, f(f_{12}^s)=f_{14}^s,$ $f(f_{13}^s)=f_7^s, f(f_{14}^s)=f_{12}^s, f(f_{15}^s)=f_{15}^s, f(f_{16}^s)=f_{16}^s$. Clearly f is $\breve{S}\,\alpha^*$ - continuous but not $\breve{S}\,g$ continuous function, because $f^{-1}(f_4^s)=f_6^s\ f^{-1}(f_7^s)=f_{13}^s$ are not $\breve{S}\,g$ Open in $(X,\tau_s,È)$.

3.2. $\breve{S}$ α*- irresolute mappings:

Definition: 3.2.1. A šoft function $f:(X,\tau_s,È)\to(Ý,\sigma_s,È)$ is $\breve{S}\,\alpha^*$ - irresolute mappings, if the $f^{-1}(\breve{S}\alpha^*\text{–Open})\text{in}(Ý,\sigma_s,È)\text{is}\breve{S}\alpha^*\text{–Open in}(X,\tau_s,È)$.

Example: 3.2.2

Let $X=Ý=\{x_1,x_2\}, \tau_s=\{f_3^s,f_{15}^s,f_{16}^s\}$, and $\breve{S}\,\alpha^*-\hat{O}(X)=\{f_1^s,f_2^s,f_3^s,f_4^s,f_5^s,f_6^s,f_7^s,f_8^s,f_9^s,$ $f_{10}^s,f_{11}^s,f_{12}^s,f_{13}^s,f_{14}^s,f_{15}^s,f_{16}^s\}$, and $\sigma_s=\{f_1^s,f_2^s,f_3^s, f_{15}^s,f_{16}^s\}$
$\breve{S}\,\alpha^*-\hat{O}(X)=\{f_1^s,f_2^s,f_3^s,f_4^s,f_5^s,f_6^s,f_7^s,f_8^s,f_9^s,f_{10}^s,f_{11}^s,f_{12}^s,f_{13}^s,f_{14}^s,f_{15}^s,f_{16}^s\}$.

Let $f:(X,\tau_s,È)\to(Ý,\sigma_s,È)$ be defined by $f(f_i^s)$ for all I = 1 to 16, Clearly f is $\breve{S}\,\alpha^*$ irresolute functions.

Theorem 3.2.3.

A $\breve{S}$ of t function $f:(X,\tau_s,È)\to(Ý,\sigma_s,È)$ is said to be $\breve{S}\,\alpha^*$ - irresolute if f the $f^{-1}(\breve{S}\,\alpha^*\text{–}Closed)in(Ý,\sigma_s,È)is\,\breve{S}\,\alpha^*\text{–}Closed\,in(X,\tau_s,È)$.

Assume that f is $\breve{S}\,\alpha^*$-irresolute. Let $(f^s,È)$ be any $\breve{S}\,\alpha^*$-Closed in $(Ý,\sigma_s,È)$. Then $(f^s,È)^c$ is $\breve{S}\,\alpha^*$-Open in $(Ý,\sigma_s,È)$. Since, f is $\breve{S}\,\alpha^*$ - irresolute, $f^{-1}((f^s,E)^c)$ is $\breve{S}\,\alpha^*$ - Open in $(X,\tau_s,È)$(i.e) $f^{-1}(f^s,È)$ is $\breve{S}\alpha^*$–Open in $(X,\tau_s,È)\Rightarrow f^{-1}(f^s,È)$ is $\breve{S}\alpha^*$– Closed in $(X,\tau_s,È)$. Hence the inverse image of every $\breve{S}\,\alpha^*$-Closed in $(Y,\sigma_s,È)$ is $\breve{S}\,\alpha^*$-Closed in $(X,\tau_s,È)$. Conversely, assume that the inverse image of every $\breve{S}\,\alpha^*$-Closed in $(Ý,\sigma_s,È)$ is $\breve{S}\,\alpha^*$-Closed in $(X,\tau_s,È)$. Let $(f^s,È)$ be any $\breve{S}\,\alpha^*$- Open in $(Ý,\sigma_s,È)$. Then $((f^s,È)^c)$ is $\breve{S}\,\alpha^*$-Closed in $(Ý,\sigma_s,È)$. By assumption, $f^{-1}((f^s,È)^c)$ is is $\breve{S}\,\alpha^*$ - Closed in

$(X,\tau_s,\grave{E})$(i.e) $f^{-1}(f^s,\grave{E})$ is $\breve{S}\,\alpha^*$–Closedin $(X,\tau_s,\grave{E}) \Rightarrow f^{-1}(f^s,\grave{E})$ is $\breve{S}\alpha^*–\hat{O}(X)$ in $(X,\tau_s,\grave{E})$. Thus f is $\breve{S}\,\alpha^*$ - irresolute.

Theorem 3.2.4.

Every $\breve{S}\,\alpha^*$-irresolute map is $\breve{S}\,\alpha^*$-continuous, but not conversely.

Proof :

Let $f:(X,\tau_s,\grave{E})\to(\acute{Y},\sigma_s,\grave{E})$ be a $\breve{S}\,\alpha^*$- irresolute map. Let $(f^s,\grave{E})$ be a Šoft – Open in $(\acute{Y},\sigma_s,\grave{E})$. Then $(f^s,\grave{E})$ is $\breve{S}\,\alpha^*$ Open in $(\acute{Y},\sigma_s,\grave{E})$. Since f is a $\breve{S}\,\alpha^*$ -irresolute map $f^{-1}(f^s,\grave{E})$ is a $\breve{S}\,\alpha^*$ -Open in $(X,\tau_s,\grave{E})$. Therefore, f is a $\breve{S}\,\alpha^*$-continuous function.

Example 3.2.5

Let $X=Y=\{x_1,x_2\}$, $\tau_s=\{f_3^s,f_{11}^s,f_{12}^s,f_{15}^s,f_{16}^s\}$, $\sigma_s=\{f_2^s,f_{10}^s,f_{11}^s,f_{15}^s,f_{16}^s\}$,
$\breve{S}\,\alpha^*-\hat{O}(X)=\{f_1^s,f_2^s,f_3^s,f_7^s,f_8^s,f_9^s,f_{10}^s,f_{11}^s,f_{12}^s,f_{13}^s,f_{14}^s,f_{15}^s,f_{16}^s\}$,
$\breve{S}\alpha^*\hat{O}(Y)=\{f_2^s,f_3^s,f_4^s,f_9^s,f_{10}^s,f_{11}^s,f_{12}^s,f_{14}^s,f_{15}^s,f_{16}^s\}$

Let $f:(X,\tau_s,\grave{E})\to(\acute{Y},\sigma_s,\grave{E})$ be defined by $f(f_1^s)=f_1^s, f(f_2^s)=f_2^s, f(f_3^s)=f_3^s$, $f(f_4^s)=f_4^s, f(f_5^s)=f_5^s, f(f_6^s)=f_7^s, f(f_7^s)=f_6^s, f(f_8^s)=f_8^s, f(f_9^s)=f_9^s, f(f_{10}^s)=f_{11}^s, f(f_{11}^s)=f_{13}^s$, $f(f_{12}^s)=f_{14}^s, f(f_{13}^s)=f_{13}^s, f(f_{14}^s)=f_{12}^s, f(f_{15}^s)=f_{15}^s, f(f_{16}^s)=f_{16}^s$. Clearly f is $\breve{S}\,\alpha^*$ - continuous but not $\breve{S}\,\alpha^*$-irresolute map, because $f^{-1}(f_4^s)=f_4^s$ are not in $\breve{S}\,\alpha^*$ - Open in $(X,\tau_s,\grave{E})$.

Theorem 3.2.6 :

Let $f:(X,\tau_s,\grave{E})\to(\acute{Y},\sigma_s,\grave{E})$ be a Šoft - continuous and Šoft – Open. Therefore, f is $\breve{S}\,\alpha^*$ - irresolute.

Proof:

Let $(f^s,\grave{E})$ be any Šoft– Open in $(\acute{Y},\sigma_s,\grave{E})$. Then $(f^s,\grave{E})\subseteq_s \breve{S}int^*(\breve{S}Cl(\breve{S}int^*(f^s,\grave{E})))$ Since f is Šoft - continuous and Šoft – Open, then it follows that,

$$f^{-1}(f^s,\grave{E})\subseteq_s f^{-1}(\breve{S}int^*(\breve{S}Cl(\breve{S}int^*(f^s,\grave{E}))))\subseteq_s f^{-1}(\breve{S}int^*(\breve{S}Cl(\breve{S}int^*(f^s,\grave{E}))))$$

$\Rightarrow f^{-1}(f^{s},\grave{E}) \subseteq_s f^{-1}(\breve{S}\mathrm{int}(f^{-1}(\breve{S}Cl(\breve{S}\mathrm{int}*(f^{s},\grave{E})))) \subseteq_s (\breve{S}\mathrm{int}*(\breve{S}Cl(\breve{S}\mathrm{int}*f^{-1}(f^{s},\grave{E}))))$

$\Rightarrow f^{-1}(f^{s},\grave{E})\, is\, a\, \breve{s}oft-Open in (X,\tau_s,\grave{E})$

$\Rightarrow f$ is $\breve{S}\alpha*-$irresolute

Theorem 3.2.7

Let $f:(X,\tau_s,\grave{E}) \rightarrow (\acute{Y},\sigma_s,\grave{E})$ be Šoft α*- irresolute iff for every šoft set $(f^{s},\grave{E})$ of X, $f(\breve{S}\alpha*-\mathrm{Open}) \subseteq_s \alpha*-\mathrm{Closed} f(f^{s},\grave{E})$.

Proof :

Suppose that f is $\breve{S}$ α* - irresolute, Now, $\breve{S}$ α* - Open is $\breve{S}$ α* - Open
$\Rightarrow$ f ($\breve{S}$ α*-Open) is $\breve{S}$ α*-Open set in Ý.
$\Rightarrow \breve{S}$ α*-Open is $\breve{S}$ α* - Open set in Y. Since f is $\breve{S}$ α*- irresolute, then $f^{-1}(\breve{S}\alpha*-\mathrm{Open}(f((f(f^{s},\grave{E})))) \breve{S}\alpha*-\hat{O}(X)$ and $((f^{s},\grave{E}) \subseteq_s f^{-1}(f(f^{s},\grave{E}))) \subseteq_s f^{-1}(\breve{S}\alpha*-\hat{O}(f(f^{s},\grave{E})))$
Hence by the definition of $\breve{S}$ α*-Open, $\breve{S}$ α*-Open $((f^{s},\grave{E})) \subseteq_s f^{-1}\breve{S}\alpha*-\hat{O}((f^{s},\grave{E}))$

$\Rightarrow f(\breve{S}\alpha*-\hat{O}((f^{s},\grave{E}))) \subseteq_s \breve{S}\alpha*-\hat{O}((f^{s},\grave{E}))$. Conversely, suppose that $(f^{s},\grave{E})$ is $\breve{S}\alpha*-\hat{O}$ set in Y. Now, by hypothesis,

$f(\breve{S}\alpha*-\hat{O}(f^{-1}(f^{s},\grave{E}))) \subseteq_s \breve{S}\alpha*-\hat{O} f(f^{-1}(f^{s},\grave{E})) \subseteq_s \breve{S}\alpha*-\hat{O}((f^{s},\grave{E})) \subseteq_s ((f^{s},\grave{E}))$

$\Rightarrow \breve{S}\alpha*-\hat{O}(f^{-1}(f^{s},\grave{E})) \subseteq_s f^{-1}(f(f^{s},\grave{E}))$ …………………………………… 1.

Also, $(f^{s},\grave{E}) \subseteq_s \breve{S}\alpha*-\hat{O}((f^{s},\grave{E}))$

$\Rightarrow f^{-1}(f^{s},\grave{E}) \subseteq f^{-1}(\breve{S}\alpha*-\hat{O}(f^{s},\grave{E}) \breve{S}\alpha*-\hat{O}(f^{-1}(f^{s},\grave{E}))$ ……………………………… 2.

From 1 and 2, we get $\breve{S}\alpha*-\hat{O}(f^{-1}(f^{s},E)) = f^{-1}(f^{s},E)$

$\Rightarrow f^{-1}(f^{s},\grave{E})$ is a $\breve{S}$ α*-Open.. Hence f is $\breve{S}$ α*- irresolute.

Theorem 3.2.8:

Let $f:(X,\tau_s,\grave{E})\rightarrow(\acute{Y},\sigma_s,\grave{E})$ be Šoft α*- irresolute iff for all Šoft set $(f^s,\grave{E})$ of Y, $f\left(\breve{S}\alpha*-\hat{O}(f^s,\grave{E})\right)\subseteq_s f^{-1}\left(\breve{S}\alpha*-\hat{O}(f(f^s,\grave{E}))\right)$.

Proof :

Suppose that f is $\breve{S}$ α*- irresolute, Now, $\breve{S}\alpha*-\hat{O}\left((f^s,\grave{E})\right)$ is $\breve{S}$ α*- O(Y) so that $f^{-1}\left(\breve{S}\alpha*-\bar{O}\left((f^s,\grave{E})\right)\right)$ is $\breve{S}$ α*- Open set in X. Since $\left((f^s,\grave{E})\subseteq_s \breve{S}\alpha*-\hat{O}(f^s,\grave{E})\right)$ $\Rightarrow f^{-1}(f^s,\grave{E})\subseteq_s f^{-1}\left(\breve{S}\alpha*-\hat{O}\left((f^s,\grave{E})\right)\right)$. By definition of $\breve{S}$ α*- closure, $\breve{S}\alpha*-\hat{O}\left(f^{-1}(f^s,\grave{E})\right)\subseteq_s f^{-1}\left(\breve{S}\alpha*-\hat{O}\left((f^s,\grave{E})\right)\right)$. Conversely, suppose that $(f^s,\grave{E})$ is $\breve{S}$ α*- O set it Y. Now, by hypothesis,

$\left(\breve{S}\alpha*-\hat{O}\left(f^{-1}(f^s,\grave{E})\right)\right)\subseteq_s \left(S\alpha*-\hat{O}\left((f^s,\grave{E})\right)\subseteq_s (f^s,\grave{E})\right)$ ………………………..1

Also, $f^{-1}(f^s,\grave{E})\subseteq_s \breve{S}\alpha*-\hat{O}\left(f^{-1}(f^s,\grave{E})\right)$ ……………………………… 2.

From 1 and 2, we get $\breve{S}\alpha*-\hat{O}\left(f^{-1}(f^s,\grave{E})\right)=f^{-1}(f^s,\grave{E})$

$\Rightarrow f^{-1}(f^s,\grave{E})$ is a $\breve{S}$ α*-Open in $(X,\tau_s,\grave{E})$

Hence f is $\breve{S}$ α*- irresolute.

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