

Brinkman-Forchheimer Slip Flow of Hybrid Nanofluid through a Flat Plate with Thermal Radiation: A Cattaneo-Christov Heat Flux Model

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Abstract: The purpose of this work is to examine the influence of the velocity slip parameter and viscous dissipation on the features of the flow of a radiative hybrid nanofluid (Ethylene Glycol + Graphene + Copper) past a flat plate. Additionally, Cattaneo-Christov model is merged in the energy equation. The equations required to represent the problem have been turned into a system, and this system has been solved using the bvp4c solver. The heat transmission rate and friction factor against the pertinent parameters are explained using bar graphs. It is observed that, at $0 \leq Mn \leq 3$, (Magnetic field parameter), friction factor declines at a proportion of 0.07462 and the rate of increment in the friction factor is 0.385017 when volume fraction of graphene nanoparticles (ϕ_g) is in the range $0 \leq \phi_g \leq 0.25$. Nusselt number is found to decrease by 0.85144 when Eckert number (Eck) is adjusted to $0.1 \leq Eck \leq 0.6$, while the same increases by 0.350461 when $0 \leq \Lambda \leq 0.25$ (thermal relaxation parameter) is used. Temperature has been seen to rise with increases in the thermal radiation parameter. In addition, it has been shown that a diminution in the velocity profile takes place whenever the porosity parameter is increased.

Keywords: Thermal radiation, Hybrid nanofluid, Flat plate, Cattaneo-Christov heat flux, Viscous dissipation, Velocity slip.

Article History

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Nomenclature

κ ($= \kappa_0 x$)	Permeability of porous medium	η	Similarity variable
σ	Electrical conductivity	σ^*	Stefan-Boltzmann constant
Pr	Prandtl number	Λ	Thermal relaxation parameter
k^*	Mean absorption coefficient	Mn	Magnetic field Parameter
ν	Kinematic viscosity	u, v	Components of velocity in two (x, y) directions
k	Thermal conductivity	$C_1 \left(= \frac{C_0}{\sqrt{x}} \right)$	Coefficient of porous inertia (second order)
ϕ_g	Volume fraction of Graphene		
ϕ_c	Volume fraction of Copper		

g	Acceleration due to gravity	L	Slip length
Eck	Eckert number	$\lambda_1 (= \lambda_{0,x})$	Thermal relaxation time of heat flux
S	Velocity slip parameter		
δ	Porosity parameter	Rd	Thermal radiation parameter
Γ	Porous media (second order) resistance parameter		
λ	Mixed convection parameter		
C_p	Specific heat capacity		

1 Background Information

Nanofluids are a novel and intriguing class of heat transmission fluids that can be used in place of more conventional options. As a result of their diminutive size, they may greatly reduce erosion and corrosion. Fuel cells, heat exchangers, and pharmaceutical process are just a few of the many uses for them. When more than one nanoparticle is present in a nanofluid, it is called a hybrid nanofluid, which differs from mono nanofluid. Thus, when compared to mono fluids, the heat transmission properties of hybrid fluids are superior. These are finding uses in a wide variety of fields, from solar energy to air conditioning. Huang et al. [1] conducted an experiment to investigate the characteristics of heat transmission by analyzing an HNF flow (water, alumina, and MWCNTs) that took place inside of a heat exchanger. According to the findings of Rahman et al. [2], the friction factor is up whenever there is an increase in the number of hybrid nanoparticles. According to research conducted by Rahman et al. [2], the friction coefficient rises with each additional hybrid nanoparticle. Theoretical work by Iqbal et al. [3] examined the passage of HNF through a rotating erect channel, using radiation and Hall current. An increase in temperature can be explained by an increase in the rate of thermal radiation. Amala and Mahanthesh [4] discussed an HNF flow that included a heat sink by taking into consideration a revolving plate. It has been discovered that increasing the nanoparticle volume percentage makes the temperature field stronger. According to Alharbi et al. [5], the hybrid nanofluid generates heat at a faster pace than the nanofluid itself. To examine the radiative flow of a HNF caused by a spinning disc, Acharya et al. [6] enacted a combination of shooting and RK4 procedures. Nadeem et al. [7] examined a HNF (Water + Cu + Al₂O₃) flow by a curved extending surface with suction/injection. Different behaviors were seen depending on the injection/suction when volume % of Al₂O₃ is applied on temperature profile. Waini et al. [8] scrutinized the stagnation point flow of the same HNF by the use of a cylinder and offered two different solutions. It has been observed that as the Reynolds' number surges, the temperature of the fluid drops. After that, a number of scholars [9-14] investigated different hybrid nanofluid flows by taking into consideration a variety of geometries.

The CCHF model (Cattaneo - Christov heat flux) does a good job of representing heat transport processes because it is based on a modified version of the model of heat conduction introduced by Fourier and subsequently enhanced by Cattaneo. Oldroyd's derivative and the time required for thermal relaxation are taken into account in this model. UCM fluid flow was studied by Shah et al. [15], who analyzed the impression of Ohmic heating using an expanding sheet and the CCHF model. The conclusion drawn is that a higher Deborah number causes a fluid's temperature to rise. With the same concept and geometry in mind, Khan and Khan [16] investigated the 3D flow of Burgers fluid. Temperatures were more evenly distributed according to Fourier's law than the CCHF model predicted. In their study of nanofluid flow, Xu and Chen [17] observed that the heat transmission rate is bigger for the minor shape feature of the nanoparticle. With the help of HAM, Vasu and Ray [18] discussed the Carreau fluid flow over an erect plate using the CCHF model. It has been demonstrated that shear thickening fluids have a lower surface shear stress than shear thinning fluids. Ali et al. [19] inspected a viscous fluid flow by a revolving disk with heat sink/source in the slip regime. Using the CCHF model, Abu-Hamdeh et al. [20] carried on a computational study of nanofluid flow through a porous extended surface. Research shows that when the Reynolds number increases, so does the entropy. In later studies [21–24], a variety of authors used the CCHF model to investigate diverse fluid flows.

In view of the preceding research, nobody has up until now tried to explain the significance of non-Fourier heat flux and viscous dissipation effects, for the radiative hybrid nanofluid (Ethylene glycol + Graphene + Copper) flow. Engineering parameters of concern including friction factor are explained using bar graphs. The concept of Cattaneo-Christov (C-C) Heat Flux (H - F) has been added into the energy equation with the purpose of discussing the characteristics of heat transmission.

2 Research Methodology

In this study, a laminar, steady and dissipative hybrid nanofluid (Ethylene Glycol + Graphene + Copper) flow through a flat plate with thermal radiation and velocity slip parameter is considered. Thermophysical characteristics of Ethylene Glycol (base fluid), Graphene and Copper (nanoparticles) are shown in Table 1. Following are the assumptions for the current study:

- (i) The x - axis is supposed to be parallel to the plate and the y - axis is perpendicular to it.
- (ii) Over the surface of the plate, the fluid is moving at a constant speed U_∞ (see Fig. 1).
- (iii) There is a magnetic field applied perpendicular to the direction of flow, with a strength of $B = \frac{B_0}{\sqrt{x}}$.
- (iv) Moreover, the momentum equation incorporates the Darcy-Brinkmann model and the energy equation includes the Cattaneo-Christov model of heat flux.

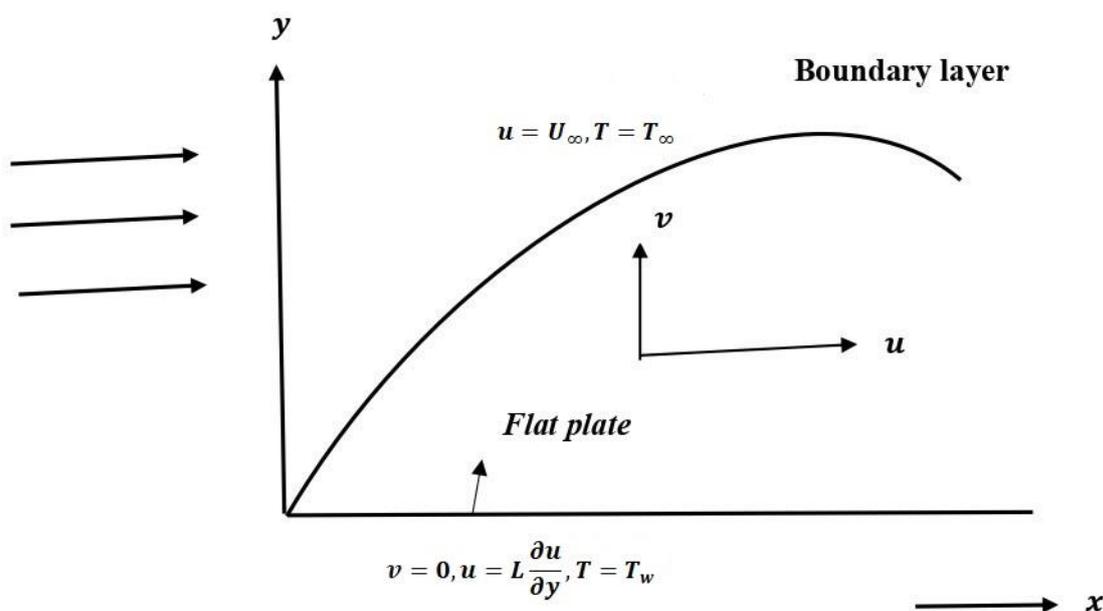


Fig. 1 Schematic diagram

These assumptions lead to the following conditions and equations: (Rao et al. [21], Makinde [25] and Shehzad et al. [26])

$$\frac{\partial}{\partial x}(u) + \frac{\partial}{\partial y}(v) = 0 \tag{1}$$

$$u \frac{\partial}{\partial x}(u) + v \frac{\partial}{\partial y}(u) = \nu_{hmf} \frac{\partial^2}{\partial y^2}(u) - \frac{\nu_{hmf}}{\kappa} u + g(T - T_\infty) \beta_T - \frac{C_1}{\rho_{hmf}} \frac{1}{\sqrt{\kappa}} u^2 - B^2 \frac{\sigma_{hmf}}{\rho_{hmf}} u \tag{2}$$

$$u \frac{\partial}{\partial x}(T) + v \frac{\partial}{\partial y}(T) = \left(\frac{1}{\rho C_p} \right)_{hmf} k_{hmf} \frac{\partial^2}{\partial y^2}(T) + \left(\frac{1}{\rho C_p} \right)_{hmf} \left(\frac{\partial}{\partial y}(u) \right)^2 \mu_{hmf} - \lambda_1 \left[\begin{aligned} & u^2 \frac{\partial^2}{\partial x^2}(T) + 2uv \frac{\partial}{\partial x} \left(\frac{\partial}{\partial y}(T) \right) + v^2 \frac{\partial^2}{\partial y^2}(T) + \frac{\partial}{\partial y}(T) \left(u \frac{\partial}{\partial x}(v) + v \frac{\partial}{\partial y}(v) \right) \\ & + \frac{\partial T}{\partial x} \left(u \frac{\partial}{\partial x}(u) + v \frac{\partial}{\partial y}(u) \right) \end{aligned} \right] \tag{3}$$

$$\left. \begin{aligned} & u(x, y)|_{y=0} = L \frac{\partial u}{\partial y}(x, y)|_{y=0}, v(x, y)|_{y=0} = 0, T(x, y)|_{y=0} = T_w, \\ & u(x, y)|_{y=\infty} = U_\infty, T(x, y)|_{y=\infty} = T_\infty. \end{aligned} \right\} \tag{4}$$

2.1 Hybrid nanofluid (HNF) thermophysical characteristics

$$\left. \begin{aligned}
 k_{hnf} &= k_{nf} \times \frac{k_c + 2k_{nf} - 2k_{nf}\phi_c + 2k_c\phi_c}{k_c + 2k_{nf} + k_{nf}\phi_c - k_c\phi_c}, \\
 \rho_{hnf} &= \left[(1 - \phi_g)\rho_f + \phi_g\rho_g \right] (1 - \phi_c) + \rho_c\phi_c, \\
 (\rho C_p)_{hnf} &= \left[(1 - \phi_g)(\rho C_p)_f + \phi_g(\rho C_p)_g \right] (1 - \phi_c) + (\rho C_p)_c\phi_c, \\
 \sigma_{nf} &= \sigma_f \times \frac{\sigma_g + 2\sigma_f - 2\sigma_f\phi_g + 2\sigma_{s_1}\phi_g}{\sigma_g + 2\sigma_f + \sigma_f\phi_g - \sigma_g\phi_g}, \\
 k_{nf} &= k_f \times \frac{k_g + 2k_f - 2k_f\phi_g + 2k_g\phi_g}{k_g + 2k_f + k_f\phi_g - k_g\phi_g}, \mu_{hnf} = \frac{\mu_f}{(1 - \phi_g)^{2.5} (1 - \phi_c)^{2.5}}, \\
 \sigma_{hnf} &= \sigma_{nf} \times \frac{\sigma_c + 2\sigma_{nf} - 2\sigma_{nf}\phi_c + 2\sigma_c\phi_c}{\sigma_c + 2\sigma_{nf} + \sigma_{nf}\phi_c - \sigma_c\phi_c}.
 \end{aligned} \right\}$$

Table 1 Estimations of distinct properties of ethylene glycol, graphene and copper (Alhowaity et al. [27])

S. No.		Ethylene Glycol (<i>f</i>)	Graphene (<i>s</i> ₁)	Copper (<i>s</i> ₂)
1	$\rho(kgm^{-3})$	1114	2250	8933
2	$k(W(m K)^{-1})$	0.252	2500	401
3	$\sigma(Sm^{-1})$	5.5×10^{-6}	1.0×10^7	5.96×10^7
4	$C_p(J(kg K)^{-1})$	2415	2100	385

Makinde [25] recommended the succeeding similarity variables to alter controlling equations:

$$\left. \begin{aligned}
 u &= U_\infty f'(\eta) = U_\infty \frac{d}{d\eta}(f), v = \sqrt{\frac{\nu U_\infty}{4x}} [\eta f'(\eta) - f(\eta)], \\
 \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \eta = y \sqrt{\frac{U_\infty}{\nu x}}.
 \end{aligned} \right\}$$

(5)

Then (5) makes (1) to fulfil and redraft (2 –4) as:

$$\frac{2}{Q_1 Q_2} f''' + 2\lambda\theta + ff'' - \frac{2Q_3}{Q_1} Mnf' - \frac{2}{Q_1 Q_2} \delta f' - \frac{2}{Q_1} \Gamma f'^2 = 0$$

(6)

$$\left(\frac{Q_4}{Q_5} \frac{2}{Pr} + \frac{Rd}{Q_5} \frac{2}{Pr} - \frac{\Lambda}{2} f^2 \right) \theta'' - \frac{3\Lambda}{2} ff'\theta' + f\theta'' + \frac{2}{Q_5 Q_2} Eckf'^2 = 0 \tag{7}$$

$$\left. \begin{aligned} f(\eta)|_{\eta=0} = 0, f'(\eta) = \frac{d}{d\eta}(f) \Big|_{\eta=0} = Sf''(\eta) = S \frac{d^2}{d\eta^2}(f) \Big|_{\eta=0}, \\ \theta(\eta)|_{\eta=0} = 1, f'(\eta) = \frac{d}{d\eta}(f) \Big|_{\eta=\infty} = 1, \theta(\eta)|_{\eta=\infty} = 0. \end{aligned} \right\} \tag{8}$$

where

$$\left. \begin{aligned} Rd = \frac{4}{3} \frac{4\sigma^* T_\infty^3}{k^* k}, \lambda = \frac{Gr_x}{Re_x^2}, Gr_x = \frac{g\beta_T(T_w - T_\infty)x^3}{\nu^2}, Re_x = \frac{U_\infty x}{\nu}, \delta = \frac{\nu}{\kappa_0 U_\infty}, \\ Mn = \frac{\sigma B_0^2}{\rho U_\infty}, \Gamma = \frac{C_0}{\rho \sqrt{\kappa_0}}, \Lambda = \lambda_0 U_\infty, Pr = \frac{\mu C_p}{k}, Eck = \frac{U_\infty^2}{C_p(T_w - T_\infty)}, S = L_0 \sqrt{\frac{U_\infty}{\nu}}. \end{aligned} \right\}$$

and

$$\left. \begin{aligned} Q_2 = (1 - \phi_g)^{2.5} (1 - \phi_c)^{2.5}, Q_5 = (1 - \phi_c) \left[(1 - \phi_g) + \phi_g \frac{(\rho C_p)_g}{(\rho C_p)_f} \right] + \phi_c \frac{(\rho C_p)_c}{(\rho C_p)_f}, \\ Q_{31} = \frac{\sigma_g + 2\sigma_f - 2\phi_g(\sigma_f - \sigma_g)}{\sigma_g + 2\sigma_f + \phi_g(\sigma_f - \sigma_g)}, Q_1 = (1 - \phi_c) \left[(1 - \phi_g) + \phi_g \frac{\rho_g}{\rho_f} \right] + \phi_c \frac{\rho_c}{\rho_f}, \\ Q_3 = \frac{\sigma_c + 2Q_{31}\sigma_f - 2\phi_c(Q_{31}\sigma_f - \sigma_c)}{\sigma_c + 2Q_{31}\sigma_f + \phi_c(Q_{31}\sigma_f - \sigma_c)}, Q_4 = \frac{k_c + 2Q_{41}k_f - 2\phi_c(Q_{41}k_f - k_c)}{k_c + 2Q_{41}k_f + \phi_c(Q_{41}k_f - k_c)} Q_{41}, \\ Q_{41} = \frac{k_g + 2k_f - 2\phi_g(k_f - k_g)}{k_g + 2k_f + \phi_g(k_f - k_g)}. \end{aligned} \right\}$$

Coefficient of skin friction (Cf_x) is defined as:

$$Cf_x = \frac{\tau_w}{\rho U_\infty^2} \Big|_{y=0}, \tag{9}$$

where $\tau_w = \mu_{mf} \left(\frac{\partial u}{\partial y} \right)$.

Rewriting the expression in (9) with (5) yields

$$\sqrt{\text{Re}_x} Cf_x = \frac{1}{Q_2} f''(0).$$

Nusselt number can be determined using the following formula:

$$Nu_x = - \left. \frac{xq_w}{k_f (T_w - T_\infty)} \right|_{y=0}. \quad (10)$$

The formula in (10) is reworked with the help of (5) as:

$$\sqrt{\frac{1}{\text{Re}_x}} Nu_x = -Q_4 \theta'(0).$$

3 Results and Discussion

In order to solve equations (6 - 7) with the conditions (8), the built-in function `bvp4c` of MATLAB is employed.

3.1 Profiles of Engineering Parameters of Concern

According to Figs. 2 - 3, it has been noted that Mn diminishes Cf_x , while ϕ_g increases it. It is observed that, at $0 \leq Mn \leq 3$, the skin friction coefficient declines at a rate of 0.07462 and the rate of increment in the friction factor is 0.385017 when ϕ_g is in the range $0 \leq \phi_g \leq 0.25$. It is noticed from Figs. 4 - 5 that Eck and Λ have different behaviours on heat transmission rate. When Eck is equal to $0.1 \leq Eck \leq 0.6$, the heat transfer rate (Nusselt number) is found to decrease by 0.85144 and at $0 \leq \Lambda \leq 0.25$, the heat transmission rate upsurges at a rate of 0.350461.

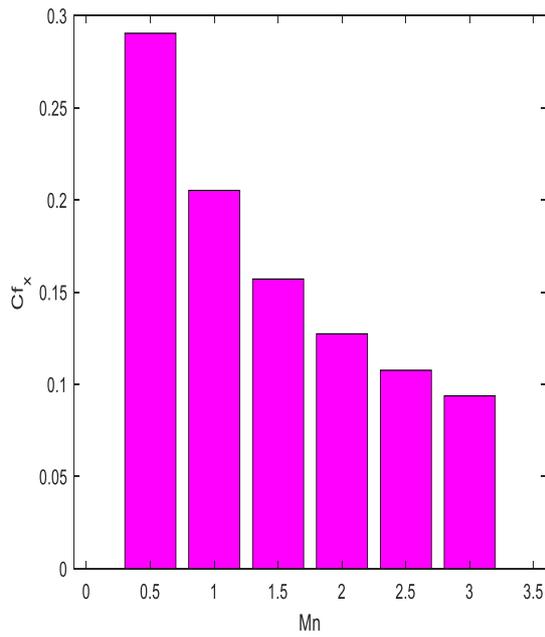


Fig. 2 Profile of skin friction coefficient with the impact of Mn

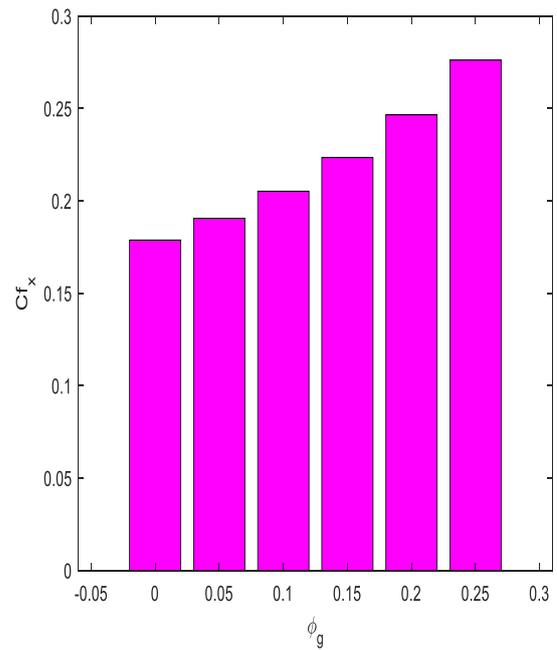


Fig. 3 Profile of skin friction coefficient with the impact of ϕ_g

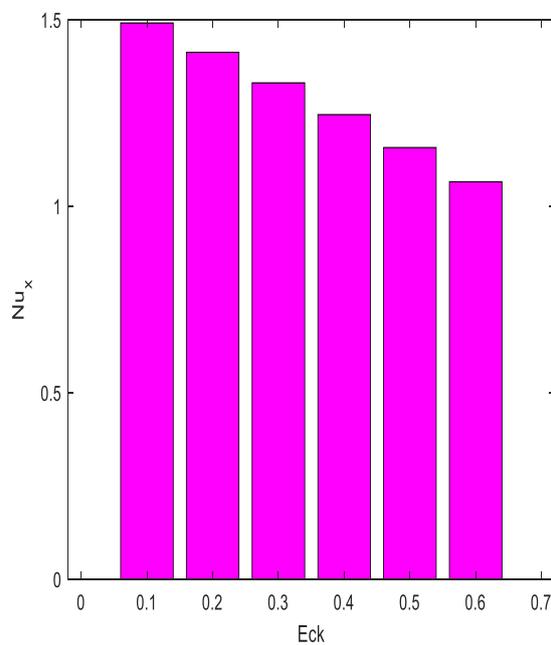


Fig. 4 Profile of Nusselt number with the impact of Eck

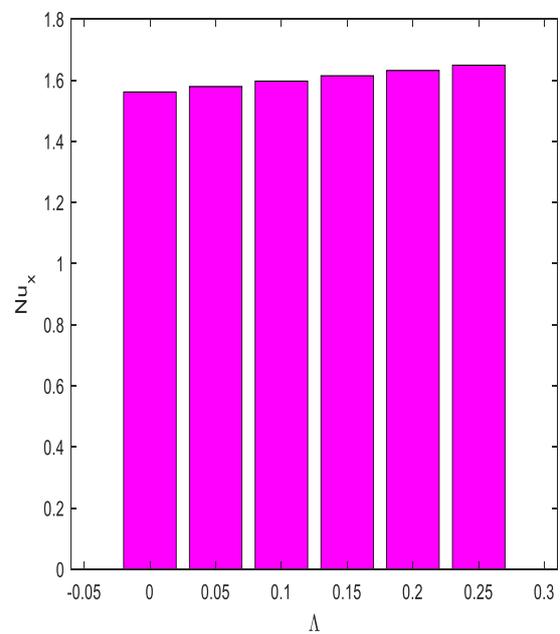


Fig. 5 Profile of Nusselt number with the impact of Λ

3.2 Temperature and Velocity Profiles

As a result of the thermal radiation parameter, heat is transferred to the fluid. There will be a greater transfer of heat energy from the source to the fluid if this value is made larger. Therefore, as seen in Fig. 6, the temperature rises as the fluid particles absorb heat energy. In When Eck grows higher, more of the kinetic energy is transformed into thermal energy by viscous forces. The fluid's temperature therefore increases (see Fig. 7). Temperature reduction in the fluid is seen when the thermal relaxation parameter increases (see Fig. 8). It has been found, as shown in Fig. 9, that when the magnetic field parameter upsurges, the velocity declines. Magnetic flow lines travelled normally through the fluid when an external magnetic field was applied around it. Particles' movement from one layer to another is impeded as a result of the Lorentz force that is created. Due to this, the fluid's molecules travel very slowly. The fluid's velocity slows as a result. In Fig. 10, we can see that when the porosity parameter rises, the fluid's velocity declines. According to the Fig. 11, it has been established that an increase in Γ results in a decrease in velocity.

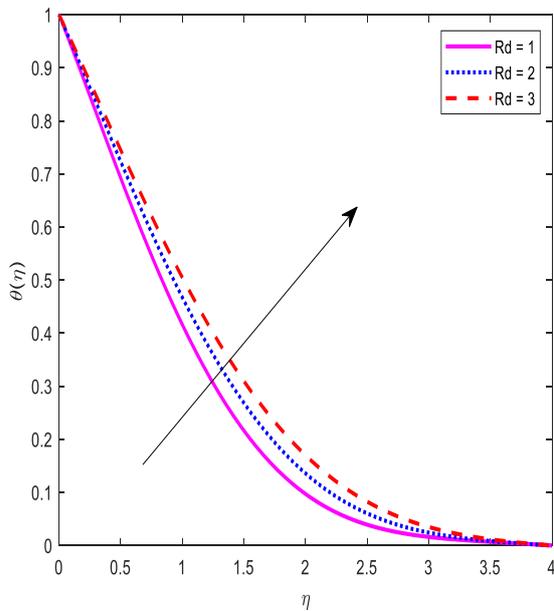


Fig. 6 Profile of $\theta(\eta)$ with the impact of Rd

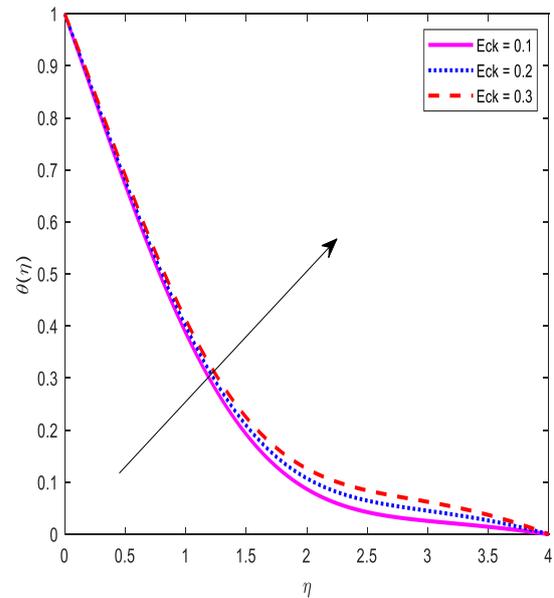


Fig. 7 Profile of $\theta(\eta)$ with the impact of Eck

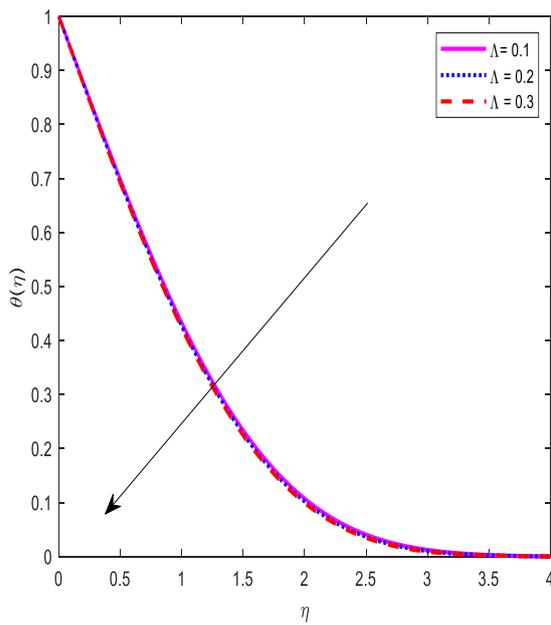


Fig. 8 Profile of $\theta(\eta)$ with the impact of Λ

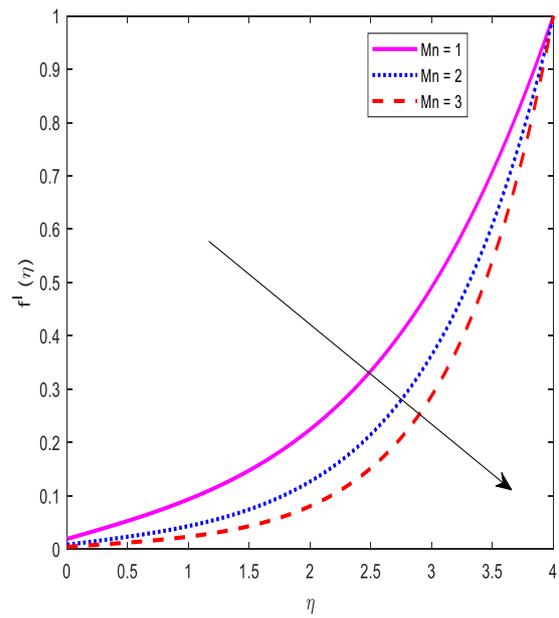


Fig. 9 Profile of $f'(\eta)$ with the impact of Mn

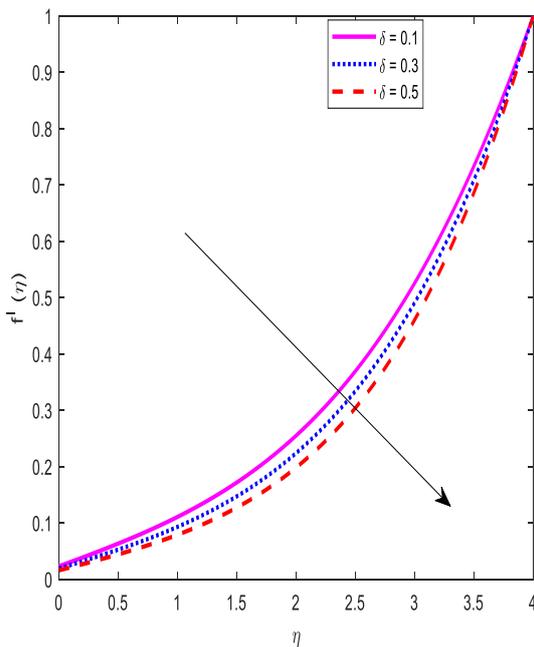


Fig. 10 Profile of $f'(\eta)$ with the impact of δ

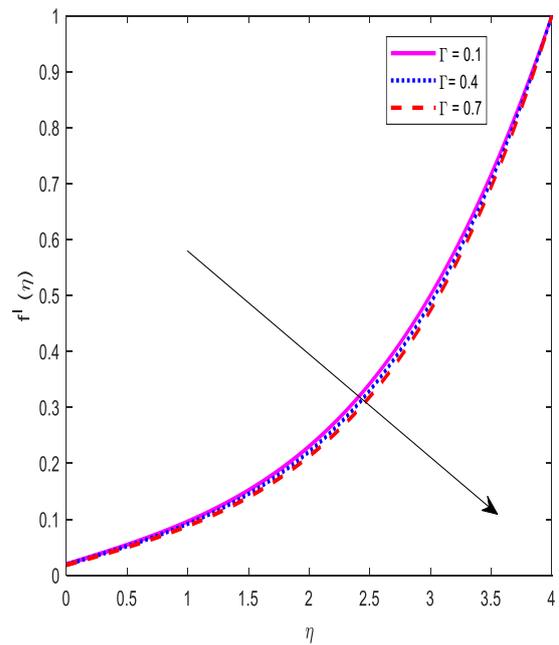


Fig. 11 Profile of $f'(\eta)$ with the impact of Γ

4 Conclusions

The purpose of this work is to examine the influence of the velocity slip parameter and viscous dissipation on the features of the flow of a radiative hybrid nanofluid (Ethylene Glycol +

Graphene + Copper) past a flat plate. Additionally, Cattaneo-Christov model is merged in the energy equation. Moreover, the momentum equation incorporates the Darcy-Brinkmann-Forchheimer model. The study's findings are briefly summarised below.

- It is observed that, at $0 \leq Mn \leq 3$, friction factor declines at a proportion of 0.07462 and the rate of increment in the friction factor is 0.385017 when ϕ_g is in the range $0 \leq \phi_g \leq 0.25$.
- Nusselt number is found to decrease by 0.85144 when Eck is adjusted to $0.1 \leq Eck \leq 0.6$, while the same increases by 0.350461 when $0 \leq \Lambda \leq 0.25$ is used.
- Temperature has been seen to rise with increases in the thermal radiation parameter.
- It has been shown that a drop in the fluid's velocity takes place whenever the porosity parameter is increased.

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