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New Stability Result in Random Normed Space

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Abstract

We investigated the equation's stability.

3 ₩(𝔁 +3 ݕ)-₩(3 𝔁 + ݕ)=12 [₩(𝔁 + ݕ)+₩(𝔁 - ݕ)]+80 ₩(ݕ)-48 ₩(𝔁) (0.1)

with some related results and solve a good example that we using lukasiewicz t-norm.

Keywords: cubic mapping, random normed space (ṘṄ −space), direct method.

1.Introduction

Mathematicians use the random norm in cases where the usual norm is not appropriate. Therefore, generalizing the normed space to random normed space is of special importance. The subject of stability, which was inquiry from S.M.Ulam [10] in1940 expanded greatly in multiple spaces and using different equations. Mathematicians scientists have reached good result in different ways, including direct and fixed point method [1,2,3,4,6,7,8,9].

We discuss the stability of the cubic functional equation(0.1) ṘṄ −space. We reached good results and confirmed the solution with a good example that we use lukasiewicz t-norm.

2. Preliminaries

Definition (2.1)[5]: " A mapping ₱: $[0, 1]^2 \rightarrow [0, 1]$ is called a triangular norm, if ₱ the following four axioms are satisfies :

((1) ₱(𝑥 , ݕ) = ₱(ݕ, 𝑥);

(2) ₱ (𝑥, ₱(ݕ, ẓ)= ₱ (₱(𝑥, ݕ), ẓ) ;

(3) ₱ (𝑥, 1)=𝑥 , ∀𝑥, ݕ∈ [0, 1];

(4) ₱ (𝑥, ݕ) ≤ ₱ (ẓ, Ŝ) when 𝑥 ≤ ẓ where ݕ ≤ Ŝ, ∀ 𝑥, ݕ, ẓ, Ŝ ∈ [0, 1]"

Definition (2.2)[5]:" A random normed space (briefly, ṘṄ −space) is a triple (ℵ, ℘, ₱), , where ℵ is a vector space. ℘is a continuous t−norm and ℘ is a mapping from ℵ into D^+ satisfying the following conditions :

(1) $\wp_{x}$ (r̈) = ε_0 (r̈) ∀ r̈ > 0 ⟺ x = 0; ∀ x ∈ ℵ

(2) $\wp_{\lambda x}$(r̈) = $\wp_{x}\left(\frac{\ddot{r}}{|\lambda|}\right)$ ∀ x ∈ ℵ, and λ ≠ 0;

(3) $\wp_{x+y}$(r̈ + k) ≥ ₱ ($\wp_{x}$(r̈,), $\wp_{y}$(k)) ∀ x, y ∈ ℵ and r̈, k ≥ 0".

Definition (2.3)[5]:" Let (ℵ, ℘, ₱) be an R̈Ṅ −space.

(1) A sequence {x_n }in ℵ is said to be convergent to a point x ∈ ℵ if, for any ε > 0 and λ > 0, there exists a positive integer N such that $\wp_{x_n - x}$ (ε) > 1 − λ , ∀n > N.

(2) A sequence {x_n }in ℵis called a Cauchy sequence if, for any ε > 0 and λ > 0, there exists a positive integer N such that $\wp_{x_n - x_m}$ (ε) > 1 − λ , ∀ n ≥ m ≥ N.

(3) An RN−space (ℵ, ℘, Ŧ) is said to be complete, if every Cauchy sequence in ℵ is convergent to a point in x."

3-Main Results

Theorem 3.1 Let ℵ be a real linear space, (ϒ, ℘, ₱) be a complete(R̈Ṅ −space) *where* ₩: ℵ → y when ₩(O)=O where there is Ꝫ :X^2 → D^+ Ꝫ(x, y) is know as $Ꝫ_{x,y}$concerning the asset:

$$\wp_{D_S\, ₩(x,y)}(\ddot{r}) \geq Ꝫ_{x,y}(\ddot{r}) \quad (3.1)$$

∀ x, y ∈ ℵ where r̈ > 0. If

$$\lim_{J\to\infty} ₱_{I=1}^{\infty} (Ꝫ_{3^{I+J-1}x,0}(3^{3I+3J}\ddot{r}) = 1 \quad (3.\ 2)$$

and $$\lim_{m\to\infty} (Ꝫ_{3^m x, 3^m y}(3^{3m}\ddot{r}) = 1 \quad (3.3)$$

∀ x, y ∈ ℵ , r̈ > 0. Where there is a single cubic function

S: ℵ → y where $\wp_{₩(x)-S(x)}(\ddot{r}) \geq$ $₱_{I=1}^{\infty}\left(Ꝫ_{3^{I-1}x,0}(3^{3I}\ddot{r})\right)$. (3.4)

∀ x ϵ ℵ and r̈ > 0.

proof. Putting y = 0 in (3.1), where $\wp_{₩(x)-\frac{₩(3x)}{3^3}}(\ddot{r}) \geq Ꝫ_{x,0}(3^3\ddot{r})$ (3.5)

∀ x, y ∈ ℵ ,r̈ >0. Therefore,

$$\wp_{\frac{₩(3^k x)}{3^{3k}} - \frac{₩(3^{k+1}x)}{3^{3+3k}}}\left(\frac{\ddot{r}}{3^{3K}}\right) \geq Ꝫ_{3^K x,0}(3^3\ddot{r}) \quad (3.6)$$

$\forall\, x \in \aleph$,k $\in$ **N** and $\ddot{r} > 0$

$$\wp_{\frac{₩(3^{k}x)}{3^{3k}}-\frac{₩(3^{k+1}x)}{3^{3+3k}}}(\ddot{r}) \geq \mathbb{Q}_{3^{K}x,0}\left(3^{3+3k}\ddot{r}\right) \tag{3.7}$$

$\forall x\in\ \aleph$, k $\in$ N and $\ddot{r}\ >\ 0$. Since $1 > \frac{1}{3}+\cdots+\frac{1}{3^{n}}$, as a result,

$$\wp_{\frac{₩(3^{k}x)}{3^{3k}}-₩(x)}(\ddot{r}) \geq Ᵽ_{k=0}^{n-1}\left(\wp_{\frac{₩(3^{k}x)}{3^{3k}}-\frac{₩(3^{k+1}x)}{3^{3+3k}}}\left(\sum_{k=0}^{n-1}\frac{\ddot{r}}{3^{k+1}}\right)\right)$$

$$\geq Ᵽ_{k=0}^{n-1}\left(\wp_{\frac{₩(3^{k}x)}{3^{3k}}-\frac{₩(3^{k+1}x)}{3^{3+3k}}}\left(\frac{\ddot{r}}{3^{k+1}}\right)\right) \geq Ᵽ_{k=0}^{n-1}\left(\mathbb{Q}_{3^{K}x,0}\left(3^{3+3k}\ddot{r}\right)\right)$$

$$\geq Ᵽ_{I=1}^{n}\left(\mathbb{Q}_{3^{I-1}x,0}\left(3^{3I}\ddot{r}\right)\right) \tag{3.8}$$

$\forall\, x \in\aleph$,$\ddot{r} > 0$.

Now show convergence of the sequence$\left\{\frac{₩(3^{J}x)}{3^{3J}}\right\}$ x is changed to $3^{J}x$ in place of (3.8), for all j, k $\in$ **N**,

$$\wp_{\frac{₩(3^{J+K}x)}{3^{3k+3j}}-\frac{₩(3^{J}x)}{3^{3J}}}(\ddot{r}) \geq \quad Ᵽ_{I=1}^{n-1}\left(\mathbb{Q}_{3^{I-1+1}x,0}\left(3^{3I+3J}\ddot{r}\right)\right) \tag{3.9}$$

$\forall\, x \in \aleph$ and $\ddot{r}>0$. The inequality's right-hand side tends to have a value of 1,

as $j,k\to\infty$, $\left\{\frac{₩(3^{J}x)}{3^{3J}}\right\}$ is a Cauchy sequence know as $S(x) = \lim_{j\to\infty}\left\{\frac{₩(3^{J}x)}{3^{3J}}\right\}$ $\forall\, x \in \aleph$. changing x, y with $3^{m}\, x$ and $3^{m}\, y$, in(3.1) when the right side is multiplied by$\frac{3^{3m}}{3^{3m}}$, as a result

$$\wp_{\frac{1}{3^{3m}}D_{S}₩(3^{m}x,3^{m}y)}(\ddot{r}) \geq \mathbb{Q}_{3^{m}x,3^{m}y}\left(3^{3m}\ddot{r}\right) \tag{3.10}$$

$\forall\, x, y \in \aleph$, m $\in$ N and $\ddot{r} > 0$. Taking m$\rightarrow\infty$, we show S is satisfying. (0.1)

$\forall\, x$,y $\in \aleph$ which is, S cubic function.Toprove (3.4) taking m $\rightarrow\infty$ in(3.8), we obtain (3.4)

At last, to show the cubic function singularity S topic to (3.4) let us suppose there is z a cubic function is satisfying.(3.4).

$$\wp_{z(x)-s(x)}(\ddot{r}) = \wp_{z(x)-\frac{₩(3^J x)}{3^{3J}}+\frac{₩(3^J x)}{3^{3J}}-S(x)}(\ddot{r})$$

$$\geq ₱\left(\wp_{z(x)-\frac{₩(3^J x)}{3^{3J}}}\left(\frac{\ddot{r}}{2}\right), \wp_{\frac{₩(3^J x)}{3^{3J}}-s(x)}\left(\frac{\ddot{r}}{2}\right)\right)$$

By letting J→ ∞ we show that z=s. This completes the proof .

Corollary (3.2)

suppose ℵ real linear space , (ƴ, ℘,₱) be complete ($\ddot{R}\dot{N}$ −space) . When ₱=$₱_M$ or ₱ = $₱_P$ and ₩: ℵ → ƴ is a function satisfies

$\wp_{D_S ₩(x,y)}(\ddot{r}) \geq \frac{\ddot{r}}{\ddot{r}+\varepsilon\|x_0\|}$,$x_0 \in ℵ$ and $\ddot{r} > 0$. In this case a single cubic function S exists andS: ℵ → ƴ satisfying (0.1) also

$$\wp_{₩(x)-s(x)}(\ddot{r}) \geq ₱_{I=1}^{\infty}\left(\frac{\ddot{r}}{\ddot{r}+\frac{\varepsilon\|x_0\|}{3^{3I}}}\right).$$

Proof. We will arrive at the result directly, if put $Ꝍ_{x,y}(\ddot{r}) = \frac{\ddot{r}}{\ddot{r}+\varepsilon\|x\|}$

$\forall x, y \in ℵ$ where $\ddot{r} > 0$, in Theorem (3.1).

Corollary (3.3)

suppose ℵ real linear space , (ƴ, ℘,₱) be complete ($\ddot{R}\dot{N}$ −space) when ₱=$₱_M$ or ₱ = $₱_P$ and ₩: ℵ → ƴ is a function satisfies

$\wp_{D_S ₩(x,y)}(\ddot{r}) \geq \frac{\ddot{r}}{\ddot{r}+V(\|x\|^p+\|y\|^p)}$,V≥ 0, where $\ddot{r} > 0$, p∊ IR. In this case a single cubic function S exists andS: ℵ → ƴ satisfying (0.1) also

$$\wp_{₩(x)-s(x)}(\ddot{r}) \geq ₱_{I=1}^{\infty}\left(\frac{\ddot{r}}{\ddot{r}+V3^{I(P-3)-P}\|x\|^P}\right), \forall x, y \in ℵ \text{ where } p < 3,$$

Proof. We will arrive at the result directly, if put $Ꝍ_{x,y}(\ddot{r}) = \frac{\ddot{r}}{\ddot{r}+V(\|x\|^p+\|y\|^p)}$, ∀ x ,y∊ ℵ ,

$\ddot{r} > 0$, $p < 3$, in Theorem (3.1) .

Corollary (3.4)

suppose ℵ real linear space where (ƴ, ℘,₱) be complete ($\ddot{R}\dot{N}$ −space) where ₱=$₱_M$ or ₱ = $₱_P$ and ₩: ℵ → ƴ is a function satisfies

$\wp_{D_S ₩(x,y)}(\ddot{r}) \geq 1 - \frac{\|x\|}{\ddot{r}+\|x\|}$, and $\ddot{r} > 0$. In this case a single cubic function S exists*and* S: ℵ → ƴ satisfying (0.1) also

$\wp_{₩(x)-s(x)}$ (r̃) $\geq Ᵽ_{I=1}^{\infty}\left(1-\frac{\|x\|}{3^{2I+1}+\|x\|}\right)$.

Proof. We will arrive at the result directly, if put $Ɋ_{x,y}$ (r̃) $= 1-\frac{\|x\|}{r̃+\|x\|}$, ∀ x ,y∈ ℵ where r̃ > 0 , in Theorem (3.1) .

Example 3.5 suppose (ℵ , ‖.‖) be a Banach algebra where

$$\wp_x(r̃) = \begin{cases} \max\left\{1-\frac{\|x\|}{r̃}, 0\right\}, \text{if } r̃ > 0 \\ o \quad\quad\quad\quad \text{if } r̃ \leq 0 \end{cases}$$

∀x , y ∈ ℵ , r̃ > 0,let

$$Ɋ_{x,y}(r̃) = \max\left\{1-\frac{120(\|x\|+\|y\|)}{r̃}, 0\right\}$$

and $Ɋ_{x,y}$(r̃) = 0 if r̃ ≤ 0 , $Ɋ_{x,y}$(r̃)is distribution function

$$\lim_{m\to\infty} 3^m x, 3^m y(3^{3m}r̃) = 1$$

∀x , y ∈ ℵ ,r̃ > 0, it simple to prove that (ℵ, ℘, $Ᵽ_L$) is (R̈Ṅ −space). And

$$\wp_x(r̃) = 1 \Rightarrow \frac{\|x\|}{r̃} = 1 \Leftrightarrow x = 0$$

∀x , y ∈ ℵ ,r̃ > 0 and, obviously,($\wp_{\lambda x}$(r̃) $= \wp_x(\frac{r̃}{\lambda})$) ∀x , y ∈ ℵ where r̃ > 0 .

Next, we have

$$\wp_{x+y}(r̃ + n) = \max\left\{1-\frac{\|x+y\|}{r̃ + n}, 0\right\}$$

$$= \max\left\{1-\left\|\frac{x+y}{r̃ + n}\right\|, 0\right\}$$

$$= \max\left\{1-\left\|\frac{x}{r̃ + n}+\frac{y}{r̃ + n}\right\|, 0\right\}$$

$$\geq \max\left\{1-\left\|\frac{x}{r̃}\right\|-\left\|\frac{y}{n}\right\|, 0\right\}$$

$$= Ᵽ_L(\wp_x(r̃), \wp_y(n))$$

S0 (ℵ, ℘, $Ᵽ_L$) is complete *since* $\wp_{x-y}$(r̃) $\geq 1-\frac{\|x-y\|}{r̃}$

∀x , y ∈ ℵ , r̃ > 0and (ℵ , ‖.‖) is complete

$\| 3₩(x+3y) - ₩(3x+y) - 12₩(x+y) - 12₩(x-y) - 80₩(y) + 48₩(x) \|$

=|3‖ $x + 3y$ ‖ −‖ $3x + y$ ‖ −12 ‖ $x + y$ ‖ -12 ‖ $x - y$ ‖ −80 ‖ y ‖ +48 ‖ x ‖ | ‖ x_0 ‖

$\leq 120(\| x \| + \| y \|)$

$\forall\, x, y \in \aleph$ hence

$\wp_{D_S ₩(x,y)}(\mathring{r}) \geq \; \mathbb{Q}_{x,y}(\mathring{r})$

$\forall\, x, y \in \aleph$ where $\mathring{r} > 0$. Now

$$(₱_L)_{I=1}^{\infty}(\mathbb{Q}_{3^{I+J-1}x,0}\left(3^{3I+3J}(\mathring{r})\right) = \max\{\sum_{I=1}^{\infty}(\mathbb{Q}_{3^{I+J-1}x,0}(3^{3I+3J}\mathring{r}) - 1) + 1,0\}$$

$$=\max\left\{\sum_{I=1}^{\infty}\left(1 - \frac{120\|3^{I+J-1}x\|}{3^{2I+2J+1}\mathring{r}} - 1\right) + 1,0\right\} = \max\left\{1 - \frac{120\|x\|}{8\,3^{2J+1}\mathring{r}}, 0\right\} \;\forall\, x, y \in \aleph,\ \mathring{r} > 0$$

$$\text{And } \lim_{J\to\infty}(₱_L)_{I=1}^{\infty} = \left(\mathbb{Q}_{3^{I+J-1},0}, (3^{3I+3J}\mathring{r})\right) = 1$$

$\forall\, x, y \in \aleph$ where $\mathring{r} > 0$, all the axiom of Theorem (3.1) verified Since

$$(₱_L)_{I=1}^{\infty}\left(\mathbb{Q}_{3^{I-1}x,0,}(3^{3I}\mathring{r})\right) = \max\{\sum_{I=1}^{\infty}\left(\mathbb{Q}_{3^{I-1}x,0,}(3^{3I}\mathring{r}) - 1\right) + 1,0\} = \max\left\{\mathbf{1} - \frac{5\|x\|}{(\mathring{r})}, 0\right\}$$

$\forall\, x, y \in \aleph$ where $\mathring{r} > 0$, Inferring that $\mathbb{Q}(x) = x^3$ is the single cubic function

$\mathbb{Q} : \aleph \longrightarrow \aleph$ where

$$\wp_{₩(x)-\mathbf{Q}(x)}(\mathring{r}) \geq \max\left\{\mathbf{1} - \frac{5\|x\|}{(\mathring{r})}, \mathbf{0}\right\}$$

$\forall x \in \aleph$ and $\mathring{r} > 0$.

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