Minimal and Maximal Sets uses.

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Abstract: The aim of this paper is to introduce new types of sets maximal open set (M-open) and maximal closed set(M-closed) and minimal open set(m-open) and minimal closed set(m-closed) with examples and theorems The relationships between them and other will be studied like M-compact ,M-compact sub space ,continuity function

Keywords:m-open,m-closed,M-open,M-closed,M compact.

Definition(1.1). [1]: Let X be a topological space. A proper nonempty open subset

U of X is said to be:

a m- open set if any open set which is contained in U is ϕ or U, and

a m-closed setif any closed set which is contained in U is ϕ or U.

Definition(1.2). [2]: Let X be a topological space. A proper nonempty open subset

U of X is said to be:

a M- open set if any open set which contains U is X or U, and

M- closed set if any closed set which contains U is X or U.

Example (1.3): [4] Let X={1,2,3,4} with a topology

τ= {ø,X,{1},{1,2},{3,4},{1,3,4}} , then the set {3,4} is m- open and the set{1,3,4} is M-open .Also the set {2,3,4} is M-closed and the set {2} is m-closed.

Theorem(1.3). [1] Let X be a topological space and U ⊆ X. Then, U is m- open set if and only if X\ **U** M- closed set.

Theorem(1.4). [2] Let X be a topological space and U ⊆ X. Then, U is m- closed set if and only if X\ **U** M- open set.

Corollary (1.5) [4]. Let X be a topological space with a, b ∈ X. Then we have the following:

(1) if {a}is an open set in X, then {a}is a m- open set and so X\ {**a**} is a M- closed set.

(2) if {b}is a closed set, then{b} is a m- closed set and so X\{b} is a M- open set.

Lemma 1**.6. [1]** Let (X,Ʈ) be a topological space.

(1) If U is a m- open set and W is an open set such that U ∩ W≠ ϕ, then U ⊆ W.

(2) If U and V are m- open sets such that U∩ V≠ ϕ, then U = V .

Lemma (1.7). [1] Let (X,Ʈ) be a topological space.

(1) If U is a M-open set and W is an open set such that U∩W≠X, then W ⊆ U.

(2) If U and V are M- open sets such that U∩VX≠, then U = V .

Theorem(1.8) : Let Y open in X , if U M-open in X ,then U∩Y M-open in Y.

Proof: Let U M-open in Y. Let U∩Y⊆ **W**

W ∋ open in Y , U ⊆ **W** ∪ **U**

∵ U M-open in X either U= **W** ∪ **U** or **W** ∪ **U** =X

If U∩Y= (**W** ∪ **U**) ∩Y

=(W∩Y) ∪ (U∩Y)

W ∪ (U∩Y) =W

Or

Y∩(**W** ∪ **U**) =Y

(Y∩W) ∪ (**Y** ∩ **U**) = **Y**

W ∪ (**Y** ∩ **U**) = **Y**

W=Y , then **U ∩ Y = W or U ∩ Y = Y**

Theorem (1.9) : if U M-open in Y ,then U is not M-open in X

Example : l et X={a,b,c,d,e}

$\boldsymbol{\tau}$= $\{\emptyset, \mathbf{X}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}\}$

Y={a,b,c,d}

$\tau_{\mathbf{y}}$= $\{\emptyset, \mathbf{X}\ \{\mathbf{a}, \mathbf{b}, \mathbf{d}\},\}$

$\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$ $\mathbf{M-open\ in\ Y}$, but $\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$ not M- open in X

Definition(2.1) : Cover & Finite Cover &M- Open (resp.,M- Closed) Cover

Let {A□}□∈□□be a family of subsets of the space (X, □). We called the family {A□}□∈□

cover of X iff X equal the union of elements of the family {A□}□∈□□□.

(i.e., X = ∪□∈□□A□)

If {A□}□∈□□is finite and cover X, then {A□}□∈□□is called a **finite cover** of **X.**

If each A□, □∈□, is M-open (resp.,M- closed) in X and {A□}□∈□□cover X, then {A□}□∈□□is

called an M-**open (resp.,M- closed) cover** of X.

Definition(2.2) :[9] Sub cover

Let C = {A□}□∈□□be a cover of X and {Bi}i∈□□be a sub family of C and

cover X, then

{Bi}i∈□□is called **sub cover** from C.

Definition(2.3) : M- Compact Space

A space X is called M-**compact** iff each M- open cover of X has a finite sub cover for X.

i.e.,

X is M- compact □□□□C = {U□}□∈□□; U□□∈ □□□□□□□□□X = ∪□∈□ U□

□□□□□1, □2, … , □n ; X = $\cup_{i=1}^{n} U_{\alpha i}$

X is not M-compact □□□□C = {U□}□∈□□; U□∈ □□□□□□□□□X = ∪□∈□ □∈□□U□

□□∄ □1, □2, … , □n ; X =$\cup_{i=1}^{n} U_{\alpha i}$□□.

Example(2.4) : If X is infinite set **.** Show that (X, □cof) is M- compact space

Solution : Let $\mathcal{G}$={U□}□∈□ be an M- open cover for X

T_{cof} = {ф} ∪{ A⊆ X:A^c is finite}

Chose any sub set of $\mathcal{G}$

Let $U_{t0} \subseteq \mathcal{G}$ is M-open set of, Then $(U_{t0}{}^{c})$ is finite $\ni (U_{t0}{}^{c})$

$=\{b_1, b_2 \ \ldots . b_n\}$

$\because X \subseteq \cup_{i\in I} U_i$

Since $U_{t0} \cap (U_{t0}{}^{c}) = \phi$, $U_{t0} \cup (U_{t0}{}^{c}) = X$

$\because b_k \in X$, k= 1,…,n

$b_k \in \cup_{i\in I} U_i$, then there exist

$U_{t1}\ U_{t2} \ldots . U_{tn}$, $b_k \in U_{tk}$ k= 1,….,n

Let $\mathcal{G}^*=\{U_{t1}\ U_{t2} \ldots . U_{tn}\}$

$\mathcal{G}^* = \{U_{tk}, k = 1, \ldots ., n\}$ is M-open cover of X

$\because \mathcal{G}^* \subseteq \mathcal{G}$ and $\mathcal{G}^*$ has finite members of X ,then $\mathcal{G}$ has finite sub cover of X

Hence (X, □cof) is M- compact space.

Remark (2.5) Every M-open cover is an open cover ,but convers is not true

Definition(2.6) : M-Compact Subspace

Let (X, □) be a topological space and W be a subspace of X. We called a space W is **M-compact space** iff every M- open cover from X cover W has a finite sub cover. i.e.,

W is M- compact □□□□{U□}□∈□□; U□□∈ □□□□□□□□□W □□∪□∈□□U□ □□□□□1, □2, … , □n ; W □□$\cup_{i=1}^{n} U_{\alpha i}$

Theorem (2.7): Every compact space is M-compact

Proof: let X be compact space

Let {U□}□ ∈ □□; U□∈ ҭ be □□□open cover of X

(since every M-open cover is an open cover)

Then {U□}□ ∈ □ is an open cover of X

Since X is compact ,then has finite sub cover

$\{U_{\alpha i} : \alpha \in \wedge \ , i = 1, \ldots , n\}$

Thus every M-open cover has finite sub cover

Hence X is M-compact

Theorem(2.7) : If A and B are M- compact sets in a space (X, □), then A ∪ B is M- compact set.

Proof : Let C = {U□}□ ∈ □□; U□∈ ट □□□□□□□□open cover of A ∪ B

To prove C has a finite sub cover

∵ A ∪ B □□∪□∈□□U□□□□A □□∪□∈□□U□□□□B □∪□∈□□U□

(since A □□A ∪ B , B □□A∪ B)

□□C cover of A and B, but A and B are M- compact

□□□□□1, … , □n ; A □□$\cup_{i=1}^{n} U\alpha_i$

and □□□1, … , □m ; B □□$\cup_{j=1}^{m} U\alpha_j$

□□A ∪ B □□$\cup_{k=1}^{n+m} U\alpha_k$

□□C has finite sub cover for A∪B □□A ∪ B compact set.

Remark(2.8) : If A and B are M- compact sets in a space (X, □), then A ∩B is not necessary M- compact set.

Solution :

ट={X,ø,[0,1), (0,1],(0,1)}∪ {($\frac{1}{n}$, 1), n∈ N}

(X,ट) be a topological space

A=[0,1) ,(0,1] are M-compact

But A ∩B =(0,1) is not M-compact

Let c={($\frac{1}{n}$, 1), n∈ N} is an M-open cover of A ∩B

Since n→ ∞ , ($\frac{1}{n}$, 1) → (0,1)

$\nexists$ $\alpha_{1,}, \alpha_{2,} \ldots\ldots \alpha_n$, then has no finite sub cover

A ∩B $\nsubseteq$ $\cup_{i=1}^{n} U\alpha_i$ ∵ A ∩B is not M-compact

Theorem(2.9) : A space (X, □) is compact iff every family of m- closed subsets of X satisfy

With the finite interaction property has a non- empty interaction .

Proof : (□) Suppose that X be M-compact space and F= { C_α:□□∈□}□□be a family of m-closed sets of M-compact with finite interaction property

To prove $\cap_{\alpha\in\Lambda} C_\alpha$□□□□.

Suppose $\cap_{\alpha\in\Lambda}\ C_\alpha$□□□

□ Let □$\mathcal{G}$ ={O_α: X \ C_α :α ∈ Λ}

$\mathcal{G}$ is family of M-open set in X

$\cup_{\alpha\in\Lambda} O_\alpha$=X

□□$\mathcal{G}$ is M- open cover of X , since X is M-compact then must be have

finite sub cover

□□X= $\cup_{i=1}^{n} O_{\alpha i}$= ∪{ X \ C_α} = X \$\cap_{i=1}^{n}\ C_{\alpha i}$

I.e

$\cap_{i=1}^{n}\ C_{\alpha i}$ must be empty

This is contradiction the fact that F has finite interaction property

Thus F has the finite members of F must to be non-empty

(□) every family m-closed subset of X with finite interaction

Let $\mathcal{G}$= {U_α : α ∈ Λ} be an M-open cover of X, so that

X= $\cup_{\alpha\in\Lambda} U_\alpha$

Taking complements

X-X=X-($\cup_{\alpha\in\Lambda} U_\alpha$)

ϕ =∩{ X-(U_α : α ∈ Λ}

Thus { X-(U_α : α ∈ Λ} is family of m-closed sets with empty interaction

Suppose this family does not have a finite interaction property

There exist a finite number of sets

X= $U_{\alpha i}$ i=1….,n ∋ $\mathcal{G}$=∩{ X-$U_{\alpha i}$: α ∈ Λ :i=1…n }

X-$\mathcal{G}$= X-∩{ X-$U_{\alpha i}$: α ∈ Λ :i=1…n }

Hence X is M-compact

Theorem (2.10) Each m-closed subset of a M-compact space is M- compact.

Proof: Let A be a m- closed subset of the M- compact space X and

let C = {U□}□ ∈ □□; U□∈ ℧ be M-open cover of A

let { U_α : α ∈Λ }∪{X\ A} be M-open cover of X

. Since X is M- compact

$\exists\ \{U_{\alpha i} : \alpha \in \wedge\ , i = 1, \dots , n\} \cup \{X \setminus A\}$ be a finite sub cover of X

Then $\{U_{\alpha i} \cap A : \alpha \in \wedge\ , i = 1, \dots , n\}$ be a finite sub cover of A

Hence A is M-compact

Remark (2.11) M-compact $\nrightarrow$ compact

Example if X infinite set ,a∈ X

τ={A: a∈ A}∪{ϕ}

Solution :

X is M-compact ,since every M-open cover has finite sub cove,

$\{a, X_{\alpha,} X_{\beta}, \dots\} \setminus \{X_i\}$

$\cup \{a, X_{\alpha,} X_{\beta}, \dots\} \setminus \{X_j\}\ \ni j \neq i$, finite sub cover for every M-open cover of X ,but X is not compact since X is infinite set, then has no finite

sub cover

theorem(2.12) if X is finite set and T is a topology on X ,then (X,τ) is M-compact .

proof: let X=$\{\mathbf{X_1}, \mathbf{X_2}, \dots . \mathbf{X_n}\}$

let $\mathcal{G}$= $\{U_i$, i ∈ N $\}$ is a M-open cover of X ,then

$\forall x_j \in$ X there exist $U_{ij} \in \mathcal{G} \ni x_j \in U_{ij}$, then

$\mathcal{G}^*=\{U_{ij}\}_{i=1}^{n}$ is finite sub cover of X

Hence X is M- compact

Definition (3.1) [10]_a function f :(X,τ) → (Y, Ω) between two topological space is called continuous if for every open set V⊆ Y, it is $f^{-1}(V)$ is open in X.

Definition (3.2):function f :(X,τ) → (Y, Ω) between two topological space is called M-continuous if for every M- open set V⊆ Y, it is $f^{-1}(V)$ is open in X.

Definition (3.3):function f :(X,τ) → (Y, Ω) between two topological space is called M^* -continuous if for every M- open set V⊆ Y, it is $f^{-1}(V)$ is M- open in X.

Definition (3.4):function f :(X,τ) → (Y, Ω) between two topological space is called M^{**}-continuous if for every open set V⊆ Y, it is $f^{-1}(V)$ is M- open in X.

Theorem(.) :[8] The continuous image of compact space is compact. i.e.,

If f : (X, □) □□(Y, □') is continuous function and X is compact space, then f(X) is

compact.

<u>Theorem(3.5)</u>The image compact space is M- compact. i.e.,

If f : (X, □) □□(Y, □') is M- continuous function and X is compact space then f(X) is M-compact

Proof : Let f : (X, □) □□(Y, □') be M- continuous and X compact space.

To prove, f(X) M- compact in Y

Let C = {V□}□∈□□□□□open cover for f(X)

□□f(X) □$\cup_{\alpha}$□ ∈ □□V□□; V□□∈ □' □□□□∈ □

Since f is M- continuous ,we know that each of $\mathbf{f^{-1}}$(V□) is open in X .since X is compact there are exist finite sub cover □1, … , □n

$\cup_{i=1}^{n} f^{-1}(V_{\alpha i}) \Longrightarrow$ f(x)□□f($\cup_{i=1}^{n} f^{-1}(V_{\alpha i})$) X □

□ $\cup_{i=1}^{n} f(f^{-1}(V_{\alpha i}))$

$\Longrightarrow$ f(x)□ $\cup_{i=1}^{n} (V_{\alpha i})$

f(x) is M-compact

<u>Theorem(3.6)</u>The image M- compact space is M- compact. i.e.,

If f : (X, □) □□(Y, □') is M^*- continuous function and X is M-compact space then f(X) is M-compact

Proof : Let f : (X, □) □□(Y, □') be M^*- continuous and X M- compact space.

To prove, f(X) M- compact in Y

Let C = {V□}□∈□□□□□open cover for f(X)

□f(X) □$\cup_{\alpha}$□ ∈ □□V□□; V□□∈ □' □□□□∈ □

Since f is $\mathbf{M}^*$- continuous ,we know that each of $\mathbf{f^{-1}}$(V□) isM- open in X .since X is M-compact there are exist finite sub cover □1, … , □n

$\cup_{i=1}^{n} f^{-1}(V_{\alpha i}) \Longrightarrow$ f(x)□□f($\cup_{i=1}^{n} f^{-1}(V_{\alpha i})$) X □

□ $\cup_{i=1}^{n} f(f^{-1}(V_{\alpha i}))$

$\Longrightarrow$ f(x)□ $\cup_{i=1}^{n} (V_{\alpha i})$

f(x) is M-compact

<u>Theorem(3.7)</u>The image compact space is M- compact. i.e.,

If f : (X, □) □□(Y, □') is M- continuous function and X is compact space then f(X) is M-compact

Proof : Let f : (X, □) □□(Y, □') be M- continuous and X compact space.

To prove, f(X) M- compact in Y

Let C = {V□}□∈□□□□□open cover for f(X)

□□f(X) □$\cup_{\alpha}$□ ∈ □□V□□; V□□∈ □' □□□□∈ □

Since f is M- continuous ,we know that each of $\mathbf{f^{-1}}$(V□) is open in X .since X is compact there are exist finite sub cover □1, … , □n

$\cup_{i=1}^{n} f^{-1}(V_{\alpha i}) \Longrightarrow$ f(x)□□f($\cup_{i=1}^{n} f^{-1}(V_{\alpha i})$) X □

□ $\cup_{i=1}^{n}$ f($f^{-1}(V_{\alpha i})$)

$\Longrightarrow$ f(x)□ $\cup_{i=1}^{n} (V_{\alpha i})$

f(x) is M-compact

<u>**Theorem(3.8)**</u>The image M- compact space is compact. i.e.,

If f : (X, □) □□(Y, □') is M^{**}- continuous function and X is M-compact space then f(X) is compact

Proof : Let f : (X, □) □□(Y, □') be M^{**}- continuous and X M- compact space.

To prove, f(X) compact in Y

Let C = {V□}□∈□□□open cover for f(X)

□□f(X) □$\cup_{\alpha}$□ ∈ □□V□□; V□□∈ □' □□□□∈ □

Since f is $\mathbf{M}^{**}$- continuous ,we know that each of $\mathbf{f^{-1}}$(V□) is M- open in X .since X is M-compact there are exist finite sub cover □1, … , □n

$\cup_{i=1}^{n} f^{-1}(V_{\alpha i}) \Longrightarrow$ f(x)□□f($\cup_{i=1}^{n} f^{-1}(V_{\alpha i})$) X □

□ $\cup_{i=1}^{n}$ f($f^{-1}(V_{\alpha i})$)

$\Longrightarrow$ f(x)□ $\cup_{i=1}^{n} (V_{\alpha i})$

f(x) is compact

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