Multicriteria Decision Aid: Some Remarks on the Behaviour of PROMETHEE Methods

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Abstract
In this paper, we study the sensitivity of PROMETHEE family methods using different versions of Independence property. In particular, we analyse the strangeness of each type of relation (ie. Indifference, Preference and Incomparability) between two given alternatives in the overall ranking. On this basis, we establish relevant rules and mathematical conditions upon which PROMETHEE family methods keep their original ranking. In case of change, we propose post-optimality study and enquiries allowing to expect the new relation between the alternatives while avoiding to reapply the method used. In addition, the intervals of stability of each relation are extracted accordingly, this can provide a sharp overview of the eventual action to undertake and its impact upon the overall ranking. The efficiency of the results is showed using numerical data among what two are real-life cases. Indeed, the first one describes the project of ranking alternatives for the selection of electric buses while the second one defines the problem of scheduling decision making within mechanical workshop for tracks construction. (Abstract)

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Introduction

In the case of a decision rule based upon aggregation function that places us in the Arrow's theorem context ([1], [3]), it is not possible to check some mathematical properties simultaneously. Indeed, Arrow has shown that, it is not possible to construct a method verifying the following properties together: Non-dictatorship, Unanimity, Universality, Transitivity and Independence ([3], [4], [16]). In order to define a method using the same concepts of properties, we cannot abandon the principles of Unanimity, Universality and Non-dictatorship. This is why, the only two principles between what we have to choose are Transitivity and Independence. In other words, it is impossible to build an ordinal method verifying the transitivity and the independence simultaneously, this subject was largely discussed in literature ([1], [4], [15], [16], [19]). In addition, a thorough analysis has shown the impossibility to find and/or define MCDM methods ([4], [16]) satisfying some derived versions of independence [1].

PROMETHEE I is one of MCDM methods which cannot verify the Independence property and some of its variants ([2], [6], [7], [8], [13], [14]). Indeed, the used data plays a crucial role in the stability of the original ranking, likewise after its modification as well as the delete of some of its components (Alternatives or criteria) ([5], [7], [9], [11], [12], [20]). In this paper, we study the sensitivity of PROMETHEE I to the use of four different versions of independence. Some rules and mathematical conditions related to the verification of these versions is introduced. In other words, we define the intervals of stability under which PROMETHEE I remains robust to any change of the data, these conditions are data independent. In each case, a demonstrative example is presented, some of them are taken from real-life situations.

This paper is partitioned into five main sections, starting by an Introduction about the subject of this study. After that we introduce the second section, denoted: Generality and notations, in which we review the procedure of PROMETHEE methods. In section 3, we define the four versions of Independence analyzed in this study. Section four is devoted to the principal results about the sensitivity of PROMETHEE methods as well as the stability intervals. We achieve this paper by a conclusion and some perspectives.

Generality and Notations

In this paper, a sensitivity study of PROMETHEE methods is undertaken, it aims to define the intervals in which the application of one of the derived versions of Independence (see section 3) does not lead to any change in the initial ranking of alternatives. In other words, the intervals in which PROMETHEE I method is stable to the application of any of these versions or doesn't verify them.

For this purpose, we introduce some mathematical notations that will be recalled along this document. In addition, the steps describing the procedure of PROMETHEE methods is reviewed hereafter.

Let us consider a decision problem with a set $A = \{A_1, A_2, \ldots, A_m\}$ of m actions (that is the subject of the decision), and a criteria family F of cardinality n. We define for each criterion j a numerical function in the set of real numbers R, such that: $g_j(A_i) = a_{ij}, i = \overline{1,m}$, $j = \overline{1,n}$, representing the evaluations of the alternatives (Actions). For each criterion j, we evaluate a weight w_j which increases with the importance of the related criterion.

Furthermore, let be A_k and A_l two arbitrary actions in A. Consider as well another action A_p, different from A_k and A_l and can take several positions in the overall ranking as we can see in the following sections.

Remember that the application of PROMETHEE requires the introduction of generalized criteria ([5], [6], [12], [17]). For each criterion j of the decision problem, a generalized criterion of the same type is associated.

Many types were defined in the literature, they are partitioned into two major groups: qualitative and quantitative generalized criteria. In our research work we study the case of the two most used types: True criteria and Quasi criteria.

The pairwise comparison (partial ranking of alternatives) is determined using the following preferences index:

$$\pi(A_k, A_l) = \Sigma_{j=1}^{n} w_j p_j(A_k, A_l)$$

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where $p_j(A_k, A_l)$ is a function which measures the preference degree of A_k compared to A_l and according to the criterion j Its value is determined by the generalized criterion chosen for this criterion.

Constructing the overall ranking of alternatives requires the calculation of the outgoing flow:

$$\Phi^+(A_k) = \frac{1}{m-1} \sum_{a \in A \setminus \{A_k\}} \pi(A_k, a)$$

and the incoming flow:

$$\Phi^-(A_k) = \frac{1}{m-1} \sum_{a \in A \setminus \{A_k\}} \pi(a, A_k)$$

This step should be iterated as many times as the number of alternatives in the decision problem.

PROMETHEE I, the subject of our study, allows only to obtain a partial ranking, so A_k outranks A_l, if and only if $\Phi^+(A_k) \geq \Phi^+(A_l)$ and $\Phi^+(A_k) \leq \Phi^+(A_l)$ with at least one strict inequality. This kind of comparisons provide in some cases the creation of Incomparability relations.

All results in the following study are subject to an easy adaptation to PROMETHEE II. Remember that, this latest is a derived version allowing to determine a total ranking using the outgoing and incoming flow introduced in steps of PROMETHEE I above. Formally:

A_k outranks A_l, if and only if, $\Phi(A_k) > \Phi(A_l)$ where $\Phi(A_k) = \Phi^+(A_k) - \Phi^-(A_k)$ and $\Phi(A_l) = \Phi^+(A_l) - \Phi^-(A_l)$

Obviously, analyzing the case of PROMETHEE I will be more complicated than PROMETHEE II. For this reason, it will be wiser to analyze only the case of PROMETHEE I.

Some derived properties of Independence

Independence property is the less verified version by the most of multicriteria methods. PROMETHEE, as the other methods uses the all of all alternatives ranking which affect directly the verification of this property [4]. In the present section, we summarize four versions of Independence. Each version has its own framework and impact on the overall ranking:

<u>Version 1:</u>
The principle of the first version leads to replace a non-optimal action by another one less better ([1], [18], [19]). PROMETHEE I verifies this version, if and only if, it stays stable to this change. In other words, the initial overall ranking stays the same after changing.
<u>Version 2:</u>{The independence of non-discriminating element }
PROMETHEE I verifies this version, whether the deleting of a given alternative cannot affect the initial overall ranking [18].
<u>Version 3:</u>{Independence of the best or the worst ranked elements:}
PROMETHEE I verifies this version, if deleting the best (resp. the worst) alternative does not change the overall ranking [18].

Version 4:{Independence of the best or the worst set of ranked elements:}
PROMETHEE I verifies this version, if deleting the group of best (resp. worst) alternatives B does not change the overall ranking [18.]

Sensitivity study of PROMETHEE I to different versions of Independence property
In this section, we analyze the performance of PROMETHEE I under the four derived versions of Independence:

Version 1

Let us replace a non-optimal action A_p by another one less good, noted A'_p, such that: $\Delta_j = a_{pj} - a'_{pj} \geq 0, j = \overline{1,n}$, with at list one strict inequality.

By analyzing the preference computational steps of a given alternative A_k, we note that if the new alternative A'_p makes a change at least in one value $p_j(A_k, A'_p)$ (resp. $p_j(A'_p, A_k)$) in the preference indices $\pi(A_k, A'_p)$ (resp. $\pi(A'_p, A_k)$), the initial ranking may change. That is because, as shown in the mathematical formula of outgoing flow (2) and (3), all the alternatives of the problem are taken into account when computing the flow.

In the same context, an indifference relation between two actions A_k and A_l is kept [1], if and only if,

$$D_k^+ = D_l^+ \ \& \ D_k^- = D_l^-$$

Where, $D_i^+ = \pi(A_i, A'_p) - \pi(A_i, A_p), i = k, l$ and $D_i^- = \pi(A_p, A_i) - \pi(A'_p, A_i), i = k, l$ [1].

On the other hand, A_k is preferred to A_l in the initial ranking, this relation is conserved, if and only if, the following inequalities are verified:

$$D_k^+ \geq D_l^+ - (m-1)[\Phi^+(A_k) - \Phi^+(A_l)]$$

And

$$D_k^- \leq D_l^- - (m-1)[\Phi^-(A_l) - \Phi^-(A_k)]$$

with at least one strict inequality.

In order to prove these inequalities, we suppose that A_k is preferred to A_l in the initial ranking, this gives that $\Phi^+(A_k) > \Phi^+(A_l)$.
Let be, $D_k^+ \geq D_l^+ - (m-1)[\Phi^+(A_k) - \Phi^+(A_l)]$,
Then $\pi(A_k, A'_p) - \pi(A_k, A_p) \geq \pi(A_l, A'_p) - \pi(A_l, A_p) - (m-1)[\Phi^+(A_k) - \Phi^+(A_l)]$;
This implies that: $\Phi^+_{A'_p}(A_k) \geq \Phi^+_{A'_p}(A_l)$.

We proceed by the same manner to demonstrate that $\Phi^-_{A'_p}(A_k) \leq \Phi^-_{A'_p}(A_l)$, hence A_k is preferred to A_l in the new ranking.

Now, we consider that A_k is preferred to A_l in the new ranking, this implies that $\Phi^+_{A'_p}(A_k) \geq \Phi^+_{A'_p}(A_l)$, However, $\Phi^+_{A'_p}(A_k) + \frac{1}{m-1}\left(\pi\left(A_k, A_p\right) - \pi\left(A_k, A_p\right)\right) \geq \Phi^+_{A'_p}(A_l) + \frac{1}{m-1}\left(\pi\left(A_l, A_p\right) - \pi\left(A_l, A_p\right)\right)$;
Then, $\pi\left(A_k, A'_p\right) - \pi\left(A_k, A_p\right) \geq \backslash pi\left(A_l, A'_p\right) - \pi\left(A_l, A_p\right) - (m-1)[\Phi^+(A_k) - \Phi^+(A_l)]$;
Hence, $D_k^+ \geq D_l^+ - (m-1)[\Phi^+(A_k) - \Phi^+(A_l)]$. By analogy, we prove the second inequality of the formula.
The stability intervals for a preference relation are as follow:

$$D_k^+ \in [D_l^+ - (m-1)[\Phi^+(A_k) - \Phi^+(A_l)], 1]$$

and

$$D_k^- \in \left[0, D_l^- - (m-1)[\Phi^-(A_l) - \Phi^-(A_k)]\right]$$

Both of them are included in [0,1].

For the Incomparability relation, this relation is conserved, if and only if, the following inequalities are hold:

$$D_k^+ > (resp. <) D_l^+ - (m-1)[\Phi^+(A_k) - \Phi^+(A_l)]$$

And

$$D_k^- > (resp. <) D_l^- - (m-1)[\backslash Phi^-(A_l) - \backslash Phi^-(A_k)]$$

Through these latest results, we can easily see that Indifference is the most sensitive relation to this change. Yet, it is possible that some changes can not affect the overall ranking. The following numerical example (Table 1) illustrates a preference relation case, this relation is changed after the introduction of a new alternative at the place of another non-optimal alternative:

	C1	**C2**	**C3**	**C4**
A_1	3	2	1	2
A_2	2	1	2	1
A_3	3	2	3	1

Table 1: Performance matrix.

Weights are considered equal and the criteria are to maximize in the following examples only if it is indicated. The resolution using PROMETHEE I gives the overall ranking: $A_3 \succ A_1 \succ A_2$($\succ$ denotes outranks).

This ranking is resulted from the comparison between actions in the table hereafter:

	A_1	A_2	A_3	$\Phi^+(A_.)$
A_1	-	**3/4**	1/4	1/2
A_2	**1/4**	-	**0**	1/8
A_3	1/4	**3/4**	-	1/2
$\Phi^-(A_.)$	1/4	3/4	1/8	-

Table 2: Flows and Global preferences.

Values in cells indicate the global preferences.

The replacement of A_2 by a new alternative $A_2' = (2,1,0,1)$ which is less good, gives a new overall ranking (see table 3) where: $A_1 \succ A_3 \succ A_2'$.

	A_1	A_2	A_3	$\Phi^+(A_.)$
A_1	-	**1**	1/4	5/8
A_2	**0**	-	**0**	0
A_3	1/4	**3/4**	-	1/2
$\Phi^-(A_.)$	1/8	7/8	1/8	-

Table 3: Flows and Global preferences after change.

PROMETHEE I is sensitive to this modification. Note that this modification brought a change to the outgoing flow values of A_1 which is more important than the change provided to A_3, this can be explained by different values of the characteristic quantities in Table 4:

D_3^+	$D_1^+ - (m-1)[\Phi^+(A_3) - \Phi^+(A_1)]$
0	1/4
D_3^-	$D_1^- - (m-1)[\Phi^-(A_1) - \Phi^-(A_3)]$
0	0

Table 4: Characteristic quantities

Additionally, the impact on the global ranking depends mainly on the type of generalized criterion used. According to the above data, the stability intervals for the preference relation between A_3 and A_1 are given as follow:

$$D_3^+ \in [1/4,1] \; and \; D_3^- = 0$$

Version 2:

PROMETHEE I is found not immune from the delete of a given alternative A_{p}. Indeed, the procedure of this method which uses all the information contained in the whole set of alternatives makes the non-verification of this version of Independence a non-avoided fact. However, as in the above case, what are the conditions causing the change of a given relation?

According to the indifference relation, it can be conserved if and only if: $\pi(A_k, A_p) = \pi(A_l, A_p)$ and $\pi(A_p, A_k) = \pi(A_p, A_l)$, where A_k and A_l are indifferent in the initial ranking (before the delete of A_p).

Consider that A_k is preferred than A_l in the initial ranking, the preference relation keeps unchanged if and only if,

$$\Phi^+_{A\setminus A_p}(A_k) \geq \Phi^+_{A\setminus A_p}(A_l)$$

and

$$\Phi^-_{A\setminus A_p}(A_k) \leq \Phi^-_{A\setminus A_p}(A_l)$$

or

$$\pi(A_k, A_p) \leq \pi(A_l, A_p) - (m-1)[\Phi^+(A_l) - \Phi^+(A_k)]$$

and

$$\pi(A_p, A_k) \geq \pi(A_p, A_l) - (m-1)[\Phi^-(A_l) - \Phi^-(A_k)]$$

with at least one strict inequality. Indeed, Let us consider $\Phi^+_{A\setminus A_p}(A_k) \geq \Phi^+_{A\setminus A_p}(A_l)$, then: $\frac{1}{m-1}\left(\pi(A_k, A_p) - \pi(A_k, A_p)\right) + \Phi^+_{A\setminus A_p}(A_k) \geq \Phi^+_{A\setminus A_p}(A_l) + \frac{1}{m-1}\left(\pi(A_l, A_p) - \pi(A_l, A_p)\right)$. Also, $\frac{1}{m-1}\pi(A_k, A_p) + \Phi^+(A_k) \geq \Phi^+(A_l) - \frac{1}{m-1}\pi(A_l, A_p)$. This implies that, $\pi(A_k, A_p) \leq \pi(A_l, A_p) - (m-1)\left(\Phi^+(A_l) - \Phi^+(A_k)\right)$. By analogy we can easily prove the second inequality.

The stability intervals in this case are as fellow:

$$\pi(A_k, A_p) \in \left[0, \pi(A_l, A_p) - (m-1)[\Phi^+(A_l) - \Phi^+(A_k)]\right]$$

And

$$\pi(A_p, A_k) \in \left[\pi(A_p, A_l) - (m-1)[\Phi^-(A_l) - \Phi^-(A_k)], 1\right]$$

In case of change with equality in both last conditions, the preference relation becomes indifference.

According to Incomparability relation. First, we assume that A_k is incomparable to A_l in the initial overall ranking, this relation is kept by deleting a given alternative A_p, if and only if, $\Phi^+_{A\setminus A_p}(A_k) > (resp. <)\Phi^+_{A\setminus A_p}(A_l)$ And $\Phi^-_{A\setminus A_p}(A_k) > (resp. <)\Phi^-_{A\setminus A_p}(A_l)$ or $\pi(A_k, A_p) < (resp. >)\pi(A_l, A_p) - (m-1)[\Phi^+(A_l) - \Phi^+(A_k)]$ and $\pi(A_p, A_k) < (resp. >)\pi(A_p, A_l) - (m-1)[\Phi^-(A_l) - \Phi^-(A_k)]$

An example (see table. 5) is presented in order to illustrate the Incomparability case, it is composed of four alternatives and three quasi-criteria, such that the thresholds of Indifference and Preference are, $q = 0.2$ and $p = 0.6$. Weights are considered equal:

	C1	C2	C3
A_1	1	1	1
A_2	3	3	2
A_3	3	2,6	2
A_4	3	2	3

Table 5: Performance matrix.

PROMETHEE I provides an Incomparability relation between A_2 and A_4(for the computation results, see table 6):

	A_1	A_2	A_3	A_4	$\Phi^+(A_.)$
A_1	-	0	0	0	0
A_2	1	-	1/6	1/3	9/18
A_3	1	0	-	1/3	4/9
A_4	1	1/3	1/3	-	5/9
$\Phi^-(A_.)$	1	1/9	3/18	2/9	-

Table 6: Flows and Global preferences.

Deleting the alternative A_3 replaces the relation between A_2 and A_4 by an indifference relation (table 7):

	A_1	A_2	A_4	$\Phi^+(A_.)$
A_1	-	0	0	0
A_2	1	-	1/3	4/6
A_4	1	1/3	-	4/6
$\Phi^-(A_.)$	2/3	1/6	1/6	-

Table 7: Flows and Global preferences after change.

According to the condition of preference relation above, the characteristic quantities are defined (see table 8):

$\pi(A_2, A_3)$	$\pi(A_4, A_3) - 3[\Phi^+(A_4) - \Phi^+(A_2)]$
0	0
$\pi(A_3, A_2)$	$\pi(A_3, A_4) - 3[\Phi^-(A_4) - \Phi^-(A_2)]$
0	0

Table 8: Characteristic quantities

Since the inequalities of preference relation are not verified for at least one of them, the preference relation cannot be conserved. furthermore, remark that the quantities in the table are equal which satisfies the indifference relation between A_2 and A_4 in the new ranking.

Version 3:

Remember that in this version we examine whether PROMETHEE I is sensitive to the delete of best (resp. worst) alternative. Formally, this version has same mathematical conditions as version 2. The only difference resides in the impact of the deleted alternative on the overall ranking. Indeed, removing the best alternative in PROMETHEE I makes preference relations stranger than before and grow the chance to the appearance of new other preference relations. In the example bellow (table 9), we illustrate the creation of new preference relation from an Incomparability relation:

	C1	**C2**	**C3**
A_1	1	1	3
A_2	3	3	1
A_3	3	2	1
A_4	3	2	2

Table 9: Performance matrix.

In the overall ranking, A_4 and A_2 are the best alternatives, they are indifferent and preferred to the other alternatives. A_1 and A_3 are incomparable, the computation results are mentioned in table 10:

	A_1	A_2	A_3	A_4	$\Phi^+(A_.)$
A_1	-	1/3	1/3	1/3	1/3
A_2	2/3	-	1/3	1/3	4/9
A_3	2/3	0	-	0	2/9
A_4	2/3	1/3	1/3	-	4/9
$\Phi^-(A_.)$	6/9	2/9	3/9	2/9	-

Table 10: Flows and Global preferences.

Deleting the best alternative A_2 transforms the relation between A_1 and A_3 to a preference relation. In the table 11 bellow, the first inequality of the incomparability condition is not checked:

$\pi(A_1, A_2)$	$\pi(A_3, A_2) - 3[\Phi^+(A_3) - \Phi^+(A_1)]$
1/3	1/3
$\pi(A_2, A_1)$	$\pi(A_2, A_3) - 3[\Phi^-(A_3) - \Phi^-(A_1)]$
2/3	4/3

Table 11: Characteristic quantities

The characteristic quantities show that the incomparability relation is not hold which is impacted by the delete of A_2. Besides, A_3 is preferred to A_1.

In this second example which is inspired from a real-life case, an electric buses selection for urban mass transportation is analyzed [10]. The proposed structure included six criteria:

-- Speed (C1): Fast transportation and mobility are preferable to residents; therefore, the speed of electric buses is an essential factor when it comes to alternatives vehicles.

-- Passenger capacity (C2): This criterion is a critical factor for planners and active transportation. City managers would like to serve more residents and decrease the number of private vehicles on roads.

-- Range (C3): Electric vehicles have limited range; therefore, this factor is a critical specific feature. Longer range means greater network area involvement.

-- Maximum Power (C4): This feature deals with climbing capacity but does depend on the electric motor.

-- Battery capacity (C5): The capacity of batteries like fossil fuels ensures greater range and time efficiency.

-- Charging time (C6): Short charging times or nocturnal charging is essential for the continuation of bus services.

And six alternatives (kind of electric buses). Table 12 shows the corresponding data (evaluations):

	C1	**C2**	**C3**	**C4**	**C5**	**C6**
A_1	72	50	200	360	360	2
A_2	68.4	58	200	360	394	1.25
A_3	90	50	280	103	170	7
A_4	80	57	50	200	200	2
A_5	75	90	280	250	230	5
A_6	75	136	300	250	346	7

Table 12: Performance matrix.

The weights vector $W = (0.0710, 0.1196, 0.1529, 0.1014, 0.3428, 0.2123)$. In this example, the criteria are taken as usual the only one to minimize is criterion 6 (**C6).**

Using PROMETHEE-GAIA software, the overall ranking in figure 1 is defined.

Deleting the best alternative A_2 changes the relation between A_6 and A_1 from incomparability to preference. the illustration from PROMETHEE-GAIA tool in figure 2 shows the new overall ranking:

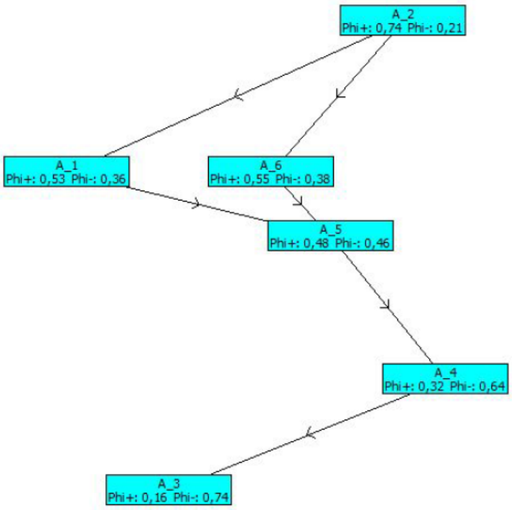

Figure 1: Global ranking.

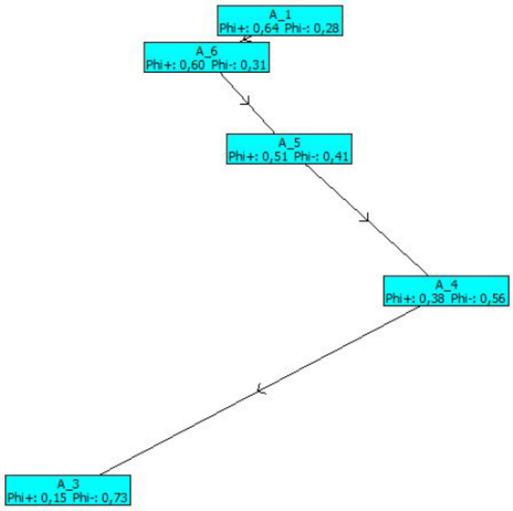

Figure 2: Global ranking after delete.

The sensitivity conditions introduced for this version are not verified which justifies the creation of a new relation (see Table 13):

$\pi(A_1, A_2)$	$\pi(A_6, A_2) - 5[\Phi^+(A_6) - \Phi^+(A_1)]$
0.071	0.2455
$\pi(A_2, A_1)$	$\pi(A_2, A_6) - 5[\Phi^-(A_6) - \Phi^-(A_1)]$
0.6747	0.553

Table 13: Characteristic quantities

In both examples, deleting the best alternatives creates a preference relation at each case. Indeed, as it is mentioned before, this action of delete enhance the chance to the appearance of new preference relations in the overall ranking.

Version 4:

In this section, we study the case of deleting a set of best alternatives, let's be B. A generalization of conditions in version 2 is possible. Let us assume that A_k and A_l are two indifferent alternatives in the initial ranking and do not belong to B. This relation is conserved after delete of B the set of best alternatives, if and only if,

$$\sum_{A_i \in B} \pi(A_k, A_i) = \sum_{A_i \in B} \pi(A_l, A_i)$$

and

$$\sum_{A_i \in B} \pi(A_i, A_k) = \sum_{A_i \in B} \pi(A_i, A_l)$$

.

Relatively to the preference relation, let be A_k preferred to A_l in the initial ranking, such that, A_k and A_l are not in B. This relation remains unchanged if and only if:

$$\sum_{A_i \in B} \pi(A_k, A_i) \leq \sum_{A_i \in B} \pi(A_l, A_i) - (m-1)[\Phi^+(A_l) - \Phi^+(A_k)]$$

and

$$\sum_{A_i \in B} \pi(A_i, A_k) \geq \sum_{A_i \in B} \pi(A_i, A_l) - (m-1)[\Phi^-(A_l) - \Phi^-(A_k)]$$

The stability intervals are as follow:

$$\sum_{A_i \in B} \pi(A_k, A_i) \in \left[0, \sum_{A_i \in B} \pi(A_l, A_i) - (m-1)[\Phi^+(A_l) - \Phi^+(A_k)]\right]$$

and

$$\sum_{A_i \in B} \pi(A_i, A_k) \in \left[\sum_{A_i \in B} \pi(A_i, A_l) - (m-1)[\Phi^-(A_l) - \Phi^-(A_k)], |B|\right]$$

In Incomparability case,

$$\sum_{A_i \in B} \pi(A_k, A_i) < (resp. >) \sum_{A_i \in B} \pi(A_l, A_i) - (m-1)[\Phi^+(A_l) - \Phi^+(A_k)]$$

and

$$\sum_{A_i \in B} \pi(A_i, A_k) < (resp. >) \sum_{A_i \in B} \pi(A_i, A_l) - (m-1)[\Phi^-(A_l) - \Phi^-(A_k)]$$

.

In mechanical engineering workshop, it is highly recommended to optimize the production time function. An optimal scheduling constitutes one of the most sought solutions allowing to improve the service quality. In this real-life case, a problem of decision making within an Algerian workshop for trucks fabrication is illustrated, two types of pieces are constructed (Bride 180 and Spools with different diameters). At one step of their process, they should use the same drill machine. In order to avoid the creation of choke machine, minimize the technical unemployment and respecting the due date of each piece, a decision function is defined, it uses three minimized criteria with the same priority value:

-- C1: processing duration,

-- C2: due date,

-- C3: achievement date from the previous machine.

Six alternatives are considered, the evaluation (by minutes) of each alternative according to the criteria are given by the following table (Table 14):

	C1	C2	C3
A_1	9.2	360	315.87
A_2	12.9	320	275.77
A_3	15.3	270	203.60
A_4	12.88	250	218.42
A_5	30	300	259.31
A_6	15.17	200	155.32

Table 14: Performance matrix.

Using PROMETHEE I, the overall ranking is as follow: $A_6 \succ A_4 \succ A_3 \succ A_1 = A_2 \succ A_5$. Since the machine cannot process more than one piece at the same time, two ranking are possibles depending to the position of A_1 over A_2. The best one is the one securing a minimum of Makespan with the other machines.

Let's consider the set $B = \{A_4, A_6\}$ containing the best alternatives, by deleting B from the performance matrix (Table 14), a new relation between A_1, A_2 and A_5 is defined (table 15):

$\sum_{A_i \in B} \pi(A_1, A_i)$	$\sum_{A_i \in B} \pi(A_5, A_i) - (m-1)[\Phi^+(A_5) - \Phi^+(A_1)]$
2/3	1/3
$\sum_{A_i \in B} \pi(A_i, A_1)$	$\sum_{A_i \in B} \pi(A_i, A_1) - (m-1)[\Phi^+(A_5) - \Phi^+(A_1)]$
4/3	5/3

Table 15: Characteristic quantities

The quantities show that the relation between A_5 and A_1 is changed. Indeed, A_5 moved from the queue to a better position where A_1 is moved back, then the overall ranking is: $A\_3 \succ A\_5 = A\_2 \succ A\_1$. Hence, PROMETHEE cannot conserve its ranking by deleting a set of good alternatives.

An illustration to the modification of incomparability relation is possible using the example in version 3 with $B = \{A_4, A_2\}$.

Numerical study and comparison

In order to study the impact of changing data upon the overall ranking of alternatives a numerical study is carried out. it consists in studying the effect of deleting (resp. altering) a given alternative A_k upon the relations between the other alternatives. Three positions of this alternative in the global ranking are taken into consideration: A_k is the best alternative, A_k is the worst alternative and A_k is an alternative other than the best/worst alternative. In each case, a comparison of the results, between deleting A_k and changing it by a worst one, is established. For this purpose, a software is written, it aims to generate decision matrices and resolve them by PROMETHEE 1. For each fixed number of alternatives $(3 \dots 20)$ and criteria $(3 ... 10)$, 3000 instances are randomly generated. The graphes bellow illustrate the obtained results (see figure3):

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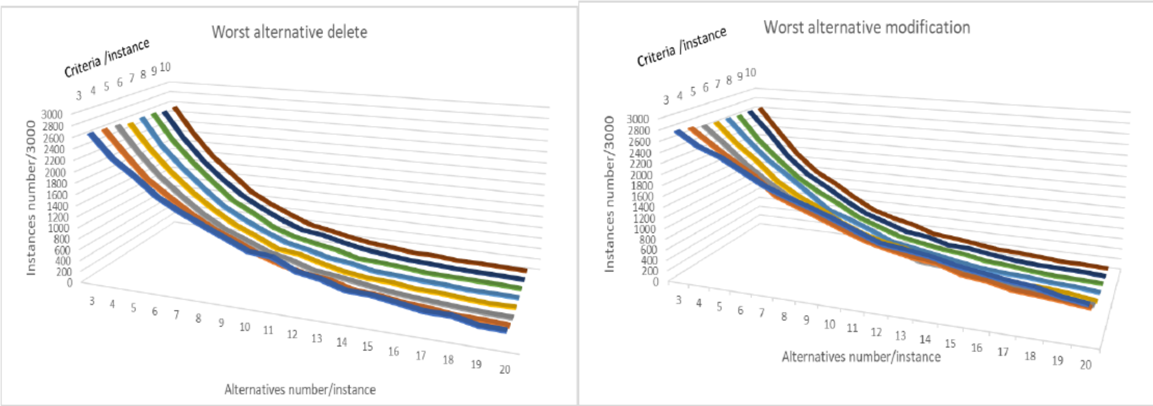
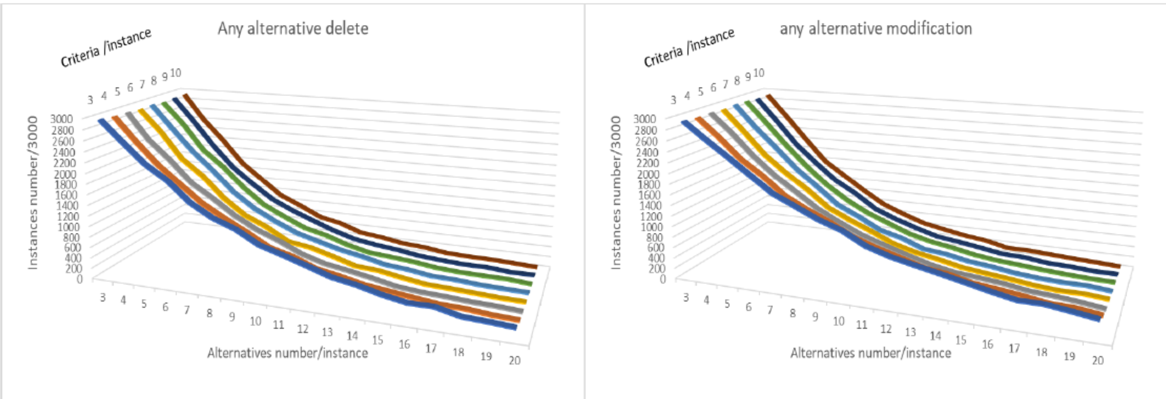
Graph1.



Graph 2.



Graph 3.

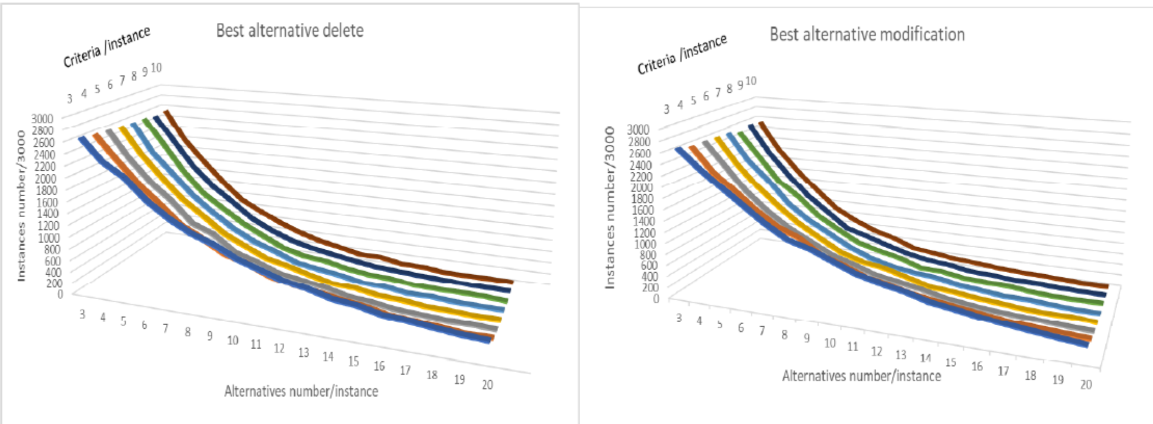


Figure 3: Comparative study

The vertical axis in the three graphs shows the number of the instances with the same ranking as the initial one. Clearly, modifying an alternative, whatever its position, has slightly less impact than deleting it. In addition, delete/modification of the best (resp. worst) alternative affect the other relations of the overall ranking more than the modification of any other alternative.

Conclusion

The sensitivity study of MCDM methods allows to explore the possible features and limitations of these methods. In addition, it can bring an immediate information about the impact of eventual changes of the data which can easily occur after ranking.

PROMETHEE family is a composition of MCDA methods which are well-known by their efficiency and stability. Yet, a post-optimal study still to be required in order to understand and illustrate the strangeness of the data used as well as the relations provided.

In this paper, we analyze the sensitivity of PROMETHEE I to the use of four versions of independence property. We introduce for the first time, rules and conditions regarding the impact of data change upon the three types of relation in the overall ranking. At each case we illustrate the obtained results by numerical examples showing the sensitivity of the method as well as their stability intervals. All kind of relations is explored in this paper and some cases of real-life situation are introduced. This paper is elaborated in order to provide a numerical interpretation to the results in [18] as well.

This study allows to expect the type of the new relation between two given alternatives, the impact of change, without being able to process all the data.

As a perspective to this study, we propose to determine sensitivity study of Independence and transitivity properties using other MCDM methods. Analyze the case of sensitive study while using different types of generalized criteria. Furthermore, in order to preserve the purpose of this study, it will be wise to include these results in a software which provides the overall ranking and the intervals of stability of PROMETHEE I and other MCDA methods.

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