Near and Closer Relations Via ω-Open Sets

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ABSTRACT: This papeer is divided into two sections. The concepts of near and closer relations that are defined respectively using the interior and closure operators in omega topology, are respectively discussed in the first and second sections.

1. Introduction

This paper starts by introducing the notions of ω-near and ω-closer relations on the subsets of a topological space. Some of the recent concepts that are available in the literature of topology and its omega topology are characterized using the above relations.

2. Prelimeneries

Result 2.1

Let A and B be any two subsets of a topological space (X,τ). The following relations on the interior and closure operators will be useful.

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$Int\mathrm{A} \subseteq IntClInt\mathrm{A} \subseteq ClInt\mathrm{A} \subseteq Cl\,IntCl\,\mathrm{A} \subseteq Cl\mathrm{A}$.

$Int\mathrm{A} \subseteq IntClInt\mathrm{A} \subseteq IntCl\mathrm{A} \subseteq Cl\,IntCl\mathrm{A} \subseteq Cl\mathrm{A}$.

$IntCl\,(\mathrm{A}\cap\mathrm{B}) \subseteq (IntCl\mathrm{A}) \cap (IntCl\mathrm{B})$.

$ClInt\,(\mathrm{A}\cap\mathrm{B}) \subseteq (ClInt\mathrm{A}) \cap (ClInt\mathrm{B})$.

$(IntCl\mathrm{A}) \cup (IntCl\mathrm{B}) \subseteq IntCl(\mathrm{A}\cup\mathrm{B})$.

$(ClIntA) \cup (ClIntB) \subseteq ClInt(A \cup B)$.

$ClIntClIntA = ClIntA$.

$IntClInt ClA = IntClA$.

Lemma 2.2

(i) If B is open then $(ClA) \cap B \subseteq Cl(A \cap B)$.
(ii) If B is closed then $Int(A \cup B) \subseteq (IntA) \cup B$.

Lemma 2.3

(i) $ClInt Cl(A \cup B) = ClIntClA \cup ClIn ClB$.

(ii) $IntClInt(A \cap B) = IntClIntA \cap IntClIntB$.

Definition 2.4 The set A is called

(i) regular open if $A = Int ClA$,
(ii) semi-open if $A \subseteq Cl IntA$,
(iii) pre-open if $A \subseteq IntClA$,
(iv) b-open if $A \subseteq Cl IntA \cup IntClA$,
(v) *b-open if $A \subseteq Cl IntA \cap IntClA$,
(vi) $b^{\#}$-open if $A = ClIntA \cup IntClA$,

Definition 2.5 The set A is called

(i) a p-set if $ClIntA \subseteq IntClA$,
(ii) a q-set if $IntClA \subseteq ClIntA$,
(iii) a Q-set if $IntClA \subseteq ClIntA$,
(iv) a t-set if $IntA = IntClA$,
(v) a t*-set if $ClA = ClIntA$.

Definition 2.6 The set A is called

(i) α-open if $A \subseteq IntClIntA$.
(ii) β-open if $A \subseteq ClIntClA$.

Definition 2.7 The set A is called

(i) regular closed $\Leftrightarrow A = ClIntA$,
(ii) semi-closed $\Leftrightarrow IntClA \subseteq A$,
(iii) pre-closed $\Leftrightarrow ClIntA \subseteq A$,
(iv) b-closed $\Leftrightarrow Cl IntA \cap IntClA \subseteq A$,
(v) *b-closed $\Leftrightarrow ClIntA \cup IntClA \subseteq A$,
(vi) $b^{\#}$-closed $\Leftrightarrow Cl IntA \cap IntClA \subseteq A$,
(vii) α-closed $\Leftrightarrow Cl IntClA \subseteq A$,

(viii) β-closed ⇔ $Int Cl Int A \subseteq A$.

Lemma 2.8 The set A is

(i) regular open ⇔ $A = Int Cl Int\ A$,

(ii) regular closed ⇔ $A = Cl Int Cl\ A$,

(iii) semi-open ⇔ $Cl A = Cl Int A$,

(iv) semi-closed ⇔ $Int A = Int Cl A$,

(v) **β**-open ⇔ $Cl A = Cl Int Cl A$,

(vi) **β**-closed ⇔ $Int A = Int Cl Int A$.

Lemma 2.9

(i) If A or B is semi-open then $Int Cl A \cap Int Cl B = Int Cl(A \cap B)$.

(ii) If A or B is semi-closed then $Cl Int(A \cup B) = Cl Int A \cup Cl Int B$.

Definition 2. 10 Let A and B be any two subsets of a space (X,τ). We say that

(i) A is near to B in (X,τ) if $Int A = Int B$
(ii) A is closer to B in (X,τ) if $Cl A = Cl B$.
(iii) A is almost near to B in (X,τ) if $Int Cl A = Int Cl B$.
(iv) A is almost closer to B in (X,τ) if $Cl Int\ A = Int Cl B$.

Definition 2.11 A function $f : (X,\tau) \rightarrow (Y, \sigma)$ is called

(i) regular continuous if $f^{-1}(V)$ is regular open in X for each $V \in \sigma$,
(ii) regular irresolute if $f^{-1}(V)$ is regular open in X for each $V \in RO(Y,\sigma)$.
Other types of continuity and irresoluteness can be analogously defined.

Definition 2.12 By a neighourhood (briefly nbd) of a point x in a space X we mean an open set containing x.

Definition 2.13 A space X is locally countable if the space has a base consisting of countable sets and is anti locally countable if every non-empty open set in X is uncountable.

Definition 2.14 For every open neighbourhood U of A,

(i) if $Cl A \subseteq U$ then A is g-closed,

(ii) if $Cl\ Int A \subseteq U$ then A is wg-closed,

(iii) if $\alpha Cl A \subseteq U$ then A is αg-closed,

(iv) if $s Cl A \subseteq U$ then A is gs-closed,

(v) if $pCl A \subseteq U$ then A is gp-closed and

(vi) if $\beta Cl A \subseteq U$ then A is gβ-closed.

Definition 2.15

(i) If $A \subseteq V$, V is regular open $\Rightarrow Cl A \subseteq V$ then A is rg-closed.

(ii) If $A \subseteq V$, V is regular open $\Rightarrow pCl A \subseteq V$ then A is gpr-closed.

(iii) If $A \subseteq V$, V is α-open $\Rightarrow \alpha Cl A \subseteq V$ then A is gα-closed.

Definition 2.16 A point x of X is said to be a condensation point of A if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable.

Clearly every condensation point of A is its limit point. Let Cond(A)={x:x is a condensation point of A} and Limit(A) ={x:x is a limit point of A. Obviously Limit(A)$\supseteq$Cond(A).

Definition 2.17 A subset B of X is said to be ω-closed in (X,τ) if $B \supseteq$ Cond(B).

It is easy to see that every closed set is ω-closed. The complement of an ω-closed set is ω-open. Khalid Y.Al.Zoubi, Al.Nashef established that the collection of all ω-open sets in (X,τ) is a topology on X denoted by τ_ω which is finer than τ. Let $Cl_\omega(\ )$ and $Int_\omega(\ )$ denote the closure and interior operators in (X, τ_ω).

Lemma 2.18 A subset B of X is ω-open in (X,τ) if and only if for each $x \in B$ there exists $U \in \tau$ such that U\B is countable. Equivalently $x \in Int_\omega B$if and only if there exists $U \in \tau$ such that U\B is countable.

3. ρ-ω*- OPEN SETS where $\rho \in$ {semi, pre, α, β, b}

3. ω-NEAR RELATION

There are distinct subsets of a topological space having the same ω-interior. For instance consider the topology $\tau = \{\emptyset, Q, R\}$ where R is the set of real numbers and Q is the set of rational numbers. It is easy to see that every subset of Q is ω-open in (R, τ). Let A be a non empty subset of Q. If x and y are any two distinct irrational numbers then $A_x = A \cup \{x\}$ and $A_y = A \cup \{y\}$ have the same ω-interior in (R, τ). That is $Int_\omega A_x = Int_\omega A_y = A$. This motivates us to have the following definition.

Definition 3. 1 The set A is ω-near to B if $Int_\omega A = Int_\omega B$.

Example 3.2 Let (R, τ) be the topological space where $\tau = \{\emptyset, Q, R\}$. It is easy to see that every subset of Q is ω-open in (R, τ). Let N, W and Z respectively denote the set of all natural numbers, whole numbers and integer. If A and B are disjoint finite or countable subsets of Q^c then $Int_\omega (N \cup A) = Int_\omega(N \cup B) = N$, $Int_\omega (W \cup A) = Int_\omega(W \cup B) = W$,

Int_ω (Z∪A) = Int_ω(Z∪B) =Z and Int_ω (Q∪A) = Int_ω(Q∪B) =Q so that N∪A is ω - near to N∪B, W∪A is ω - near to W∪B, Z∪A is ω - near to Z∪B.

Proposition 3. 3 A is ω-near to B ⇒A is near to B. The converse need not be true.

Proof. A is ω-near to B ⇒Int_ω A = Int_ω B.

⇒*Int*A⊆Int_ω A =Int_ω B ⊆B.

⇒*Int* A⊆B ⇒*Int* A ⊆*Int* B.

Again A is ω-near to B⇒Int_ω B = Int_ω A.

⇒*Int*B⊆Int_ω B = Int_ω A ⊆A.

⇒*Int* B⊆A ⇒*Int* B ⊆*Int* A.

Therefore, *Int*A = *Int*B that implies A is near to B. However the reverse implication need not true as shown below. As seen from Example 3. 2,

Int_ω (N∪A) = Int_ω(N∪B) =N and

Int (N∪A) = *Int*(N∪B) =∅ that implies N∪A is ω-near to N∪B and N∪A is near to N∪B respectively. This example shows that near ⇒ω - near.

It is easy to check that Q is ω-closed and Q^c is ω-open in the real line with standard topology. It is easy to check that *Int* Q = *Int* Q^c =∅ but Int_ωQ =∅ and $Int_\omega Q^c$ = Q^c so that Q is near to Q^c but Q is not ω-near to Q^c which shows that near relation does not imply ω-near relation.

Lemma 3. 4 Let (X,τ) be a topological space. The relation " is ω-near to" is an equivalence relation on the power set of X.

Proof. Let A, B, C be the subsets of X. Since Int_ω A = Int_ω A , A is ω-near to A so that the relation is reflexive.

A is ω-near to B ⇒Int_ω A = Int_ω B ⇒Int_ω B = Int_ω A ⇒ B is ω-near to A.

A is ω-near to B and B is ω-near to C ⇒Int_ω A = Int_ω B and Int_ω B = Int_ω C

⇒Int_ωA = Int_ω C .

⇒ A is ω-near to C.

The equivalence classes of the relation "is ω-near to" are called the ω-near classes of the subsets of X. If A is a subset of X , then the ω-near class of A = ω-near[A] = {B: A is ω-near to B}.

Proposition 3. 5 There is an one-to-one correspondence between ωO(X, τ) and the collection of ω-near classes in X,

Proof. For any ω-open set O in (X,τ), a subset A of X is ω-near to O if and only if O= $Int_\omega A$. Conversely, every subset A of X is ω-near to some ω-open set in (X,τ). This proves the proposition .

Corollary 3. 6 The set $B \in \omega$-near[A] $\Leftrightarrow A \in \omega$-near[B]

Example 3.7 Let X = R, the set of all real numbers. Fix $t \in R$. Then $\tau = \{ \emptyset, \{t\}, R \}$ is a topology on R. $\tau' = \{ \emptyset, R\setminus\{ t\}, R \}$ = the set of all closed sets in (R, τ). Let A, B be subsets of R with $t \in A$ and $t \notin B$. We compute the condensation points of A and B.

$x \in Cond(A) \Leftrightarrow$ for every $U \in \tau$ with $x \in A$, $U \cap A$ is uncountable.

$x \notin Cond(A) \Leftrightarrow$ there exists $U \in \tau$ with $x \in A$, $U \cap A$ is not uncountable.

By taking U ={t},we see that tis neither a condensation point of Anor a condensation point of B. So let $x \in R$ and $x \neq t$. The only open set containing x is R.

Since $R \cap A = A$, it is clear that $R \cap A$ is uncountable if and only if A is uncountable,

$R \cap A$ is countable if and only if A is countable ,

$R \cap A$ is finite if and only if A is finite.

Therefore, Cond(A) = Cond(B) = $\emptyset$ if and only if A and B are finite or countable.

Cond(A) = Cond(B) = $R\setminus\{ t\}$ if and only if A, B are uncountable.

Thus,if A and B are finite or countable then they are ω-closed. If A and B are uncountable, then $A \cup (R\setminus\{t\}) = R$ and $B \cup (R\setminus\{ t\}) = R\setminus\{ t\}$ are ω -closed sets.

Therefore, $\omega C(R, \tau) = \{A:$ A is a finite or countable subset of R$\} \cup \{ \emptyset, R\setminus\{ t\}, R \}$.

$\omega O(R, \tau) = \{A:$ A is an uncountable subset of R, $R\setminus A$ is finite or countable$\} \cup \{ \emptyset, \{ t\}, R \}$.

This shows that ω-topology of (R, τ) is strictly finer than τ.

Clearly,{t}, $R\setminus\{t\}$ are both ω-closed and ω-open in (R, τ) and {t} is the only non empty finite set which is ω-open.

In the following three cases, we assume $A \subseteq R$, $t \in A$, $B \subseteq R$, $t \notin B$, $B \neq \emptyset$.

Case-1: Suppose A and B are finite or countable.

$Int A = \{t\} = Int_\omega A$ and $Int\, B = \emptyset$, $Int_\omega B = \emptyset$. This shows that

A is neither ω-near to B nor near to B.

Case-2: Suppose A and B are uncountable such that $R\setminus B$, $R\setminus A$ are finite or countable.

$Int_\omega A = A$, $Int\, A = \{t\}$ and $Int_\omega B = B$ and $Int\, B = \emptyset$.

This shows that A is neither ω-near to B nor near to B.

Case-3: Suppose A and B are uncountable such that R\A and R\B are uncountable.

$Int_{\omega}A = \{t\}$, $Int\,A = \{t\}$ and $Int_{\omega}B = \varnothing$, $Int\,B = \varnothing$.

This shows that A is neither ω-near to B nor near to B.

In the following three cases, we assume $A \subseteq R$, $B \subseteq R$, $t \in A$, $t \in B$, $B \neq A$.

Case-4: Suppose A and B are finite or countable.

$IntA = \{t\} = Int_{\omega}A$ and $IntB = \{t\} = Int_{\omega}B$. This shows that

A is ω-near to B and A is near to B.

Case-5: Suppose A and B are uncountable such that R\B , R\A are finite or countable.

$Int_{\omega}A = A$, $Int\,A = \{t\}$ and $Int_{\omega}B = B$, $Int\,B = \{t\}$.

This shows that A is not ω-near to B but A is near to B.

Case-6: Suppose A and B are uncountable such that R\A and R\B are uncountable.

$Int_{\omega}A = \{t\}$, $Int\,A = \{t\}$ and $Int_{\omega}B = \{t\}$, $Int\,B = \{t\}$.

This shows that A is ω-near to B and is near to B.

In the following three cases,we assume $\varnothing \neq A \subseteq R$, $\varnothing \neq B \subseteq R$, $t \notin A$, $t \notin B$, $B \neq A$.

Case-7: Suppose A and B are finite or countable.

$IntA = \varnothing$, $Int_{\omega}A = \varnothing$ and $IntB = \varnothing = Int_{\omega}B$. This shows that

A isω-near to B and A is near to B.

Case-8: Suppose A and B are uncountable such that R\B, R\A are finite or countable.

$Int_{\omega}A = A$, $IntA = \varnothing$ and $Int_{\omega}B = B$ and $Int\,B = \varnothing$.

This shows that A is not ω-near to B but A is near to B.

Case-9: Suppose A and B are uncountable such that R\A and R\B are uncountable.

$Int_{\omega}A = \varnothing$, $Int\,A = \varnothing$ and $Int_{\omega}B = \varnothing$, $Int\,B = \varnothing$..

This shows that A is ω-near to B and is near to B.

The proper subsets of R will be classified in the following ways.

$PR_1 = \{A: A \subset R$, A is finite or countable$\}$

$PR_2 = \{A: A \subset R$, A is uncountable with finite or countable complement $\}$

PR_3 = {A: A$\subset$ R, A is uncountable with uncountable complement }

The above discussion leads to the following table which compares the near and ω-near classes

Table 3. 8 Comparison of near and ω-near classes.

Open/ω-open set A	Near[A] where $A\in\tau$	ω-near[A] where $A\in\tau_\omega$
$\varnothing$	{B: B$\subset$R, t$\notin$ B}	{B: B$\in PR_1\cup PR_3$, t$\notin$ B}
{t}	{B: B$\subset$R, t$\in$ B}	{B: B$\in PR_1\cup PR_3$, t$\in$B}
R	{R}	{R}
A $\in PR_2$	Not applicable	{A}

Proposition 3. 9 Every regular ω*-open set is near to a regular ω-closed set.

Proof. Let A be a regular ω*-open set. We have .

A= $IntCl_\omega Int$A that implies IntA= $IntCl_\omega Int$A = Int($Cl_\omega Int$A) which shows that A is near $Cl_\omega Int$A. Since $Cl_\omega Int$A is regular ω-closed , it follows that A is near to a a regular ω-closed set.

Proposition 3. 10 Let A be a subset of X.

(i) A is semi-ω-closed $\Leftrightarrow$ A is near to Cl_ωA.

(ii) A is β-ω-closed in (X,τ) $\Leftrightarrow$ A is near to a regular ω-closedset.

Proof. Suppose A is semi-ω-closed in (X,τ). Then IntA = $IntCl_\omega$A that implies A is near to Cl_ωA. The converse part is obvious. This proves (i). Suppose A is β-ω-closed in (X,τ). Then $IntCl_\omega Int$ A$\subseteq$A that implies IntA$\subseteq$ $IntCl_\omega Int$ A$\subseteq$ IntAso that IntA = $IntCl_\omega Int$A that proves that A is near to $Cl_\omega Int$ A. The converse part is trivial. This proves (ii).

Proposition 3. 11

(i) If A is regular ω-open then A is ω-near to ClA.
(ii) If A is α-ω-closed then A is near to Cl_ωA.
(iii) IfA is pre-ω-closed or b-ω-closed or $b^{\#}$-ω-closed or *b-ω-closed, then A is near to a regular ω-closedset.

Proof .A is regular ω-open $\Rightarrow$ A = $Int_\omega Cl$A $\Rightarrow Int_\omega$A = $Int_\omega Cl$A

$\Rightarrow$ A is ω-near to ClA

A is α-ω-closed⇒$Cl_\omega IntCl_\omega$A ⊆A ⇒$IntCl_\omega$A ⊆ A

⇒$IntCl_\omega$A ⊆IntA

⇒ IntA ⊆$IntCl_\omega$A ⊆IntA

⇒ IntA = $IntCl_\omega$A

⇒ A is near to Cl_ωA

The set A is pre-ω-closed⇒ b-ω-closed ⇒β-ω-closed

⇒A is near to $Cl_\omega Int$ A.

The set A is *b-ω-closed ⇒ b-ω-closed ⇒β-ω-closed

⇒A is near to $Cl_\omega Int$ A.

The set A is $b^{\#}$-ω-closed⇒ b-ω-closed ⇒β-ω-closed

⇒A is near to $Cl_\omega Int$ A.

Proposition 3. 12 Let A be a subset of X.

(i) A is semi-ω*-closed ⇔ A is ω-near to ClA.

(ii) A is β-ω*-closed in (X,τ) ⇔ A is ω-near to a regular ω*-closed set.

Proof. Suppose A is semi-ω*-closed in (X,τ). Then Int_ωA = $Int_\omega Cl$A that implies A is ω-near to ClA. The convers part is obvious. This proves (i). Suppose A is β-ω*-closed in (X,τ). Then Int_ωA = $Int_\omega ClInt_\omega$A that proves that A is ω-near to $ClInt_\omega$A. The converse part is trivial. This proves (ii)

Proposition 3. 13

(i) If A is regular ω*-open then A is near to Cl_ωA.
(ii) If A is α-ω*-closed then A is ω-near to ClA.
(iii) IfA is pre-ω*-closed or b-ω*-closed or $b^{\#}$-ω*-closed or *b-ω*- closed, then A is ω-near to a regular ω*-closed set.

Proof. A is regular ω*-open ⇒ A = $IntCl_\omega$A ⇒IntA = $IntCl_\omega$A

⇒ A is near to Cl_ωA

A is α-ω*-closed ⇒$ClInt_\omega Cl$A ⊆ A ⇒$Int_\omega Cl$A ⊆ A

⇒$Int_\omega Cl$A ⊆Int_ωA

⇒ Int_ωA ⊆$Int_\omega Cl$A ⊆Int_ωA

⇒ Int_ωA = $Int_\omega Cl$A

$\Rightarrow$ A is ω-near to *Cl*A

The set A is pre-ω*-closed $\Rightarrow$ b-ω*-closed $\Rightarrow$β-ω*-closed

$\Rightarrow$A is ω-near to $ClInt_{\omega}$A.

The set A is *b-ω*-closed $\Rightarrow$ b-ω*-closed $\Rightarrow$β-ω*-closed

$\Rightarrow$A is ω-near to $ClInt_{\omega}$A.

The set A is $b^{\#}$-ω*-closed $\Rightarrow$ b-ω*-closed $\Rightarrow$β-ω*-closed

$\Rightarrow$A is ω-near to $ClInt_{\omega}$A.

Proposition 3. 14 Let A be ω-near to B and C be ω-near to D. Then

(i) A∩C is ω-near to B∩D

(ii) A∩D is ω-near to B∩C

Proof. Suppose A is ω-near to B and C is ω-near to D. Then Int_{ω}A= Int_{ω}B and Int_{ω}C = Int_{ω}D. Now Int_{ω}(A∩C) =Int_{ω}A ∩Int_{ω} C= Int_{ω}B∩Int_{ω} D = Int_{ω} (B∩D) that implies A∩C is ω-near to B∩ D. This proves (i) and the proof for (ii) is analog.

Definition 3. 15 Let B∈ω-near[A]. If B ⊆ A then, B is called a ω-near subset of A and if B⊇ A then B is called a ω-near superset of A in X.

Proposition 3. 16 The set B is a ω-near subset of A ⇔ A is a ω-near super set of B.

Proof. The set B is a ω-near subset of A ⇔ B∈ω-near[A] and B⊆A

⇔ A∈ω-near[B] and A⊇ B

⇔ A is a ω-near super set of B.

Proposition 3. 17 Every ω-near subset of an ω-open set is ω-open.

Proof. Let B be a ω- near subset of A and A be ω-open. Then

B⊆A = Int_{ω}A = Int_{ω}B that implies B = Int_{ω}B is ω-open.

Proposition 3. 18 Let B be a ω-near subset of A. The set B is semi-ω-open or α-ω-open according as A is semi-ω-open or α-ω-open.

Proof. Suppose A is semi-ω-open. Then A⊆$ClInt_{\omega}$A. Since A is ω-near to B, Int_{ω}A = Int_{ω}B that implies B⊆A⊆$ClInt_{\omega}$A=$Cl\ Int_{\omega}$B. This proves that B is semi-ω-open. If A is α-ω-open,then A⊆$Int_{\omega}\ Cl\ Int_{\omega}$A that implies B⊆A ⊆$Int_{\omega}\ ClInt_{\omega}$A = $Int_{\omega}ClInt_{\omega}$B , proving that B is α-ω-open. This proves (i).

Proposition 3. 19 Let B be a near subset of A. The set B is semi-ω-open or α-ω*-open according as A is semi-ω*-open orα-ω*-open.

Proof. Suppose A is semi-ω*-open. Then $A \subseteq Cl_{\omega} Int A$. Since A is near to B, $Int A = Int B$ that implies $B \subseteq A \subseteq Cl_{\omega} Int A = Cl_{\omega} Int B$. This proves that B is semi-ω*-open. If A is α-ω*-open then $A \subseteq Int\, Cl_{\omega}\, Int A$ that implies $B \subseteq A \subseteq Int\, Cl_{\omega} Int A = Int\, Cl_{\omega} Int B$, proving that B is α-ω*-open.

Corollary 3. 20

(i) A is semi-ω-open ⇔ every ω-near subset of A is semi-ω-open.
(ii) A is α-ω-open ⇔ every ω-near subset of A is α-ω-open.
(iii) A is semi-ω*-open ⇔ every near subset of A is semi-ω*-open.
(iv) A is α-ω*-open ⇔ every near subset of A is α-ω*-open.

Proposition 3. 21 If A is an ω-t-set then

(i) A is near to ClA and Cl_{ω}A and
(ii) ClA is near to and ω-near to Cl_{ω}A .

Proof. Let A be an ω-t-set. We have

$Int A = Int Cl A = Int_{\omega} Cl A = Int\, Cl_{\omega} A = Int_{\omega} Cl_{\omega} A$.

$Int A = Int Cl A = Int\, Cl_{\omega} A \Rightarrow$ A is near to ClA and Cl_{ω}A. This proves (i)

$Int Cl A = Int Cl_{\omega} A \Rightarrow Cl A$ is near to Cl_{ω}A.

$Int_{\omega} Cl A = Int_{\omega} Cl_{\omega} A \Rightarrow Cl A$ is ω-near to Cl_{ω}A. This proves (ii).

Proposition 3. 22 Let A and B be any two subsets of an anti locally countable space (X, τ). Then, the following results always hold.

(i) If A and B are closed sets then A is near to B if and only if A is ω-near B.
(ii) Cl A is near to Cl B⇔Cl A is ω -near Cl B.
(iii) Cl_{ω}A is ω-near to Cl_{ω}B ⇔Cl A is near Cl B.

Proof. Let (X, τ) be an anti locally countable space. Let A and B be any two closed sets. Then using we have$Int A = Int_{\omega} A$ and $Int B = Int_{\omega} B$ that implies $Int A = Int B \Leftrightarrow Int_{\omega} A = Int_{\omega} B$ that proves that A is near to B ⇔ A is ω -near B.

This proves (i).

Now let A and B be any two subsets of X. Then, using

$Int Cl A = Int_{\omega} Cl A$ and $Int Cl B = Int_{\omega} Cl B$.

$Int_{\omega} Cl_{\omega} A = Int Cl_{\omega} A$ and $Int_{\omega} Cl_{\omega} B = Int Cl_{\omega} B$.

$\Rightarrow IntCl\text{A} = IntCl\text{B} \Leftrightarrow Int_\omega Cl\text{A} = Int_\omega Cl\text{B}$

$\Rightarrow Cl$ A is near to Cl B $\Leftrightarrow Cl$ A is ω-near Cl B.

$\Rightarrow Int_\omega Cl_\omega\text{A} = Int_\omega Cl_\omega\text{B} \Leftrightarrow IntCl_\omega\text{A} = IntCl_\omega\text{B}$

$\Rightarrow Cl_\omega$A is ω-near to Cl_ωB $\Leftrightarrow Cl$ A is near Cl B.

Proposition 3. 23 In an anti locally countable space ,

(i) every regular ω-closed set is near to a regular closed set.
(ii) every regular ω-closed set is ω-near to a regular closed set.
(iii) every regular closed set in (X, τ_ω) is near to a regular ω*-closed set.
(iv) every regular closed set in (X, τ_ω) is ω-near to a regular ω*-closed set.

Proof. Let X be an anti locally countable space and A be regular ω-closed. we have

$Int\ Cl_\omega Int\text{A} = Int\ ClInt\text{A}$.

$Int_\omega Cl_\omega Int\text{A} = Int_\omega ClInt\text{A}$.

$Int\ Cl_\omega Int_\omega\text{A} = Int\ ClInt_\omega\text{A}$.

$Int_\omega Cl_\omega Int_\omega\text{A} = Int_\omega ClInt_\omega\text{A}$.

$\Rightarrow$ every regular ω-closed set is near to a regular closed set.

$\Rightarrow$ every regular ω-closed set is ω-near to a regular closed set.

$\Rightarrow$ every regular closed set in (X, τ_ω) is near to a regular ω*-closed set.

$\Rightarrow$ every regular closed set in (X, τ_ω) is ω-near to a regular ω*-closed set.

Proposition 3. 24

(i) If A is a Qω-set, then $ClInt_\omega$A is both ω-near and near to ClA .
(ii) If A is a Qω-set in an anti locally countable space, then $Cl_\omega Int_\omega$A and Cl Int_ωA are near to ClA.
(iii) If B is a Qω*-set in an anti locally countable space, then $Cl\ Int$B is ω-near to Cl_ωB *and* also near Cl_ωB.

Proof. Let A be a Qω-set. Then $ClInt_\omega\text{A} = Int_\omega\ Cl\text{A}$ that implies $Int_\omega ClInt_\omega\text{A} = Int_\omega\ Cl\text{A}$ so that $ClInt_\omega$A is ω-near ClA.

Also $ClInt_\omega\text{A} = Int_\omega\ Cl\text{A} \Rightarrow Int\ ClInt_\omega\text{A} = Int\ Int_\omega Cl\text{A} = IntCl\text{A}$

$\Rightarrow ClInt_\omega$A is near ClA. This proves (i).

Now,let A be a Qω-set and B be a Qω*-set in an anti locally countable space. Since A is a Qω-set , we have

$Cl_{\omega}Int_{\omega}A = IntClA = ClInt_{\omega}A$

$\Rightarrow IntCl_{\omega}Int_{\omega}A = IntClA = IntCl\ Int_{\omega}A$ that implies both $Cl_{\omega}Int_{\omega}A$ and $ClInt_{\omega}A$ are near to ClA.

Now since B is a Qω*-set, we have

$Cl\ IntB = Int_{\omega}Cl_{\omega}B = IntCl_{\omega}B$.

$\Rightarrow IntCl\ IntB = IntInt_{\omega}Cl_{\omega}B = IntIntCl_{\omega}\ B$

$\Rightarrow IntCl\ IntB = IntCl_{\omega}B = IntCl_{\omega}B$

$\Rightarrow Cl\ Int$B is near to Cl_{ω}B.

Also $\Rightarrow Int_{\omega}Cl\ IntB = Int_{\omega}Int_{\omega}Cl_{\omega}B = Int_{\omega}IntCl_{\omega}B$

$\Rightarrow Int_{\omega}Cl\ IntB = Int_{\omega}Cl_{\omega}B = IntCl_{\omega}B = Int_{\omega}Cl_{\omega}B$

(since the space is anti locally finite)

$\Rightarrow Cl\ Int$B is ω-near to Cl_{ω}B.

REFERENCES

1. Andrijevic D, 'Semi-pre-open sets', *Math.Vesnik, vol.* 38(1986), pp.24-32.
2. Andrijevic D, 'On b-open sets', *Mat. Vesnik, vol.* 48(1996), pp. 59-64.
3. Arya SP and Nour TM,'Characterizations of s-normal Spaces', *Indian. J. Pure. Appl. Math.*, vol.19(1988), pp.42-50.
4. Chattopadhyay C and Bandyopadhyay C, 'On structure of δ-sets', *Bulletin of Calcutta Mathematical Society*, vol.83(1991), pp.281-290.
5. Chattopadhyay C and Roy UK, 'δ-sets, irresolvable and resolvable spaces' , *Mathematica Slovaca*,vol.42(1992), pp.371-378.
6. Chiralekha R, Anitha M and Meena N, ' Near and closer relations in topology' *Malaya Journal of Metematik*, vol.8, no.4 (2020), pp.2169-2172.
7. Chiralekha R, ' On b-open sets, near and closer relations in topology and related concepts' *Ph.D thesis, Manonmniam Sundaranar University*, India.(2021),
8. Dontchev J,'On generalizing semi-pre-open sets', *Mem. Fac. Sci. Kochi. Univ. Ser. A. Math.*, vol 16(1995),, pp.36 - 48.
9. Kumar, S. A. S., Naveen, R., Dhabliya, D., Shankar, B. M., & Rajesh, B. N. (2020). Electronic currency note sterilizer machine. Materials Today: Proceedings, 37(Part 2), 1442-1444. doi:10.1016/j.matpr.2020.07.064
10. Mandal, D., Shukla, A., Ghosh, A., Gupta, A., & Dhabliya, D. (2022). Molecular dynamics simulation for serial and parallel computation using leaf frog algorithm. Paper presented at the PDGC 2022 - 2022 7th International Conference on Parallel, Distributed and Grid Computing, 552-557. doi:10.1109/PDGC56933.2022.10053161 Retrieved from www.scopus.com

11. Mehraj, H., Jayadevappa, D., Haleem, S. L. A., Parveen, R., Madduri, A., Ayyagari, M. R., & Dhabliya, D. (2021). Protection motivation theory using multi-factor authentication for providing security over social networking sites. Pattern Recognition Letters, 152, 218-224. doi:10.1016/j.patrec.2021.10.002
12. Raghavendra, S., Dhabliya, D., Mondal, D., Omarov, B., Sankaran, K. S., Dhablia, A., . . . Shabaz, M. (2022). Development of intrusion detection system using machine learning for the analytics of internet of things enabled enterprises. IET Communications, doi:10.1049/cmu2.12530
13. Gnanambal Y,'On generalized pre regular closed sets in topological spaces', *Indian J. Pure. Appl. Math.*, vol.28(1997), pp.351 - 360.
14. Indira T & Rekha K, ' Applications of *b-open sets and **b-open sets in topological spaces' *Annals of Pure and Applied Mathematics*, vol.1, no.1(2012), pp.44-56.
15. Indira T & Rekha K,'Decomposition of Continuity via *b-open Set', *Acta Ciencia Indica, Vol.39* M, no.1(2013), pp. 73-85.
16. Levine N, 'On the commutativity of the interior and closure operators in topological spaces', A*mer.Math.Monthly*, vol.68(1961), pp.474-477.
17. Levine N,'Semiopen Sets and Semi Continuity in Topological Spaces', A*mer.Math.Monthly,* vol.70(1963), pp. 36-41.
18. Levine N, 'Generalized closed sets in topology', *Rend. Circ. Mat. Palermo,* vol.19, no.2(1970), pp. 89-96.
19. Maki H, Umehara J and NoiriT,'Every topological space is pre- $T_{1/2}$', *Mem. Fac. Sci. Kochi. Univ. Ser. A. Math.* vol.17(1996), pp.33 - 42.
20. Maki H, Balachandran K and Devi R,'Generalized α-closed sets in topology', *Bull. Fukuoka Univ. Ed part III,* vol.42(1993), pp.13 - 21.
21. Maki H, Balachandran K and Devi R,'Associated topologies of generalized α-closed sets and α-generalized closed sets', *Mem. Fac. Sci. Kochi. Univ. Ser. A. Math.*, vol.15 (1994), pp.57 - 63.
22. Mashhour AS, Abd El-Monsef ME & El-Deeb SN,'On Pre-Continuous and Weak Pre-Continuous Functions', *Proc. Math. Phys. Soc. Egypt,* vol. 53(1982), pp. 47-53.
23. Murugesan S, ' On Rω-open sets' , *Journal of Advanced Studies in Topology*, vol.5, no.3(2014), pp.24-27.
24. Nagaveni N,'Studies on Generalizations of Homeomorphisms in Topological spaces', *Ph.D. Thesis, Bharathiar University,* Coimbatore(1999),.
25. Palaniappan N and Rao KC,'Regular generalized closed sets', *Kyungpook Math. J.*, vol.33(1993), pp. 211 - 219.
26. Samer Al Ghour, Souad Al-Zoubi, 'A new class between theta open sets and theta omega open sets' , *Heliyon*, vol.7(2021), pp. 1-11.
27. Stone MH, 'Application of the Theory of Boolean Rings to the General Topology', *Trans. A.M.S.* ,vol. 41(1937), pp. 375-481.
28. Takashi Noiri, 'Super Continuity and Some Strong Forms of Continuity', *Indian J. Pure Appl.Math.*,vol.15, no. 3(1984), pp. 241-250.
29. Thamizharasi G, 'Studies in Bitopological Spaces', *Ph.D Thesis, Manonmaniam Sundaranar University*, Tirunelveli , TN, India.(2010),

30. Thangavelu P & Rao KC, 'p-sets in Topological Spaces', *Bulletion of Pure and Applied Sciences*, vol.21E, no. 2(2002),, pp. 341-355.
31. Thangavelu P & Rao KC,' q-sets in Topological Spaces', *Prog.of Maths*, vol.36, no. 1&2(2002),, pp. 159-165.
32. Usha Parameswari R,'A Study on Generalization of b-Open Sets and Related Concepts in Topology', *Ph.D Thesis*, *Manonmaniam Sundaranar University*, Tirunelveli- 12, India.(2015),
33. Usha Parameswari R& Thangavelu P, 'On $b^{\#}$-open Sets', *International Journal of Mathematics Trends and Technology*, vol. 5, no. 3(2014), pp. 202-218.