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Investigation of Random Matrix Theory for Statistical Mechanics and Quantum Physics Applications

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Abstract
Due to its capability to explain large systems with chaotic behaviour, Random Matrix Theory (RMT) has drawn a lot of interest in the domains of Statistical Mechanics and Quantum Physics. Investigating RMT's uses in these fields is the main goal of this study.RMT offers a potent framework for studying the behaviour of sizable ensembles of interacting particles in statistical mechanics. RMT enables the examination of different physical features including energy spectra and correlation functions by using random matrices as models for Hamiltonians. Advancements in disciplines like condensed matter physics and nuclear physics have been made possible by the universality aspects of RMT, which allow for the comprehension of universal features seen in many-body systems.RMT is an essential tool for understanding the statistical behaviour of quantum systems in quantum physics, especially when there are strong interactions present. It gives scientists a useful tool for describing the energy states and wave functions of complicated quantum systems, including those with chaotic dynamics. Understanding of phenomena including quantum transport, quantum entanglement, and quantum chaos has benefited by the knowledge obtained from RMT.With this study, we hope to add to the body of knowledge on RMT's uses in quantum physics and statistical mechanics. It offers important insights into the behaviour of complex systems and paves the road for improvements in these domains by investigating the characteristics of random matrices and their relationships to physical systems.

Introduction

In the study of statistical mechanics and quantum physics, random matrix theory (RMT) has become a potent tool for understanding the behaviour of complex systems with chaotic dynamics. Due to its amazing capacity to represent universal features observable in a variety of physical systems, RMT has drawn a lot of attention in recent years. This has led to a wide range of applications, such as in quantum information theory, nuclear physics, and condensed matter physics.RMT provides a useful framework for investigating the behaviour of sizable ensembles of interacting particles in statistical mechanics. RMT enables the examination of fundamental features including energy spectra, level spacing distributions, and correlation functions by using random matrices as models for Hamiltonians. universality as a notion. RMT is a key tool for examining the statistical behaviour of quantum systems in the field of quantum physics, especially when chaotic dynamics and strong interactions are present. For describing the statistical characteristics of energy levels, wave functions, and spectral correlations, it offers a potent tool. RMT has also found use in a variety of quantum phenomena, including as quantum transport, quantum entanglement, and quantum chaos.The goal of this inquiry is to investigate and examine how Random Matrix Theory is used in the fields of Quantum Physics and Statistical Mechanics. We aim to get a deeper knowledge of complex systems and reveal the underlying principles guiding their behaviour by investigating the characteristics of random matrices and their linkages to physical systems.

Large ensembles of particles and their collective behaviour are studied in statistical mechanics. A useful framework for examining the statistical characteristics of these systems is provided by RMT. RMT enables the investigation of fundamental features including energy spectra, level spacing distributions, and correlation functions by using random matrices as models for Hamiltonians. The idea of universality, which occurs when some statistical properties of complex systems become independent of the particulars of the underlying dynamics, is one of the fundamental ideas in RMT. It enables researchers to find commonalities among various many-body systems. The behaviour of systems undergoing phase transitions and the creation of collective phenomena can be better understood thanks to this element of RMT.

RMT is a key tool in quantum physics for understanding the statistical behaviour of quantum systems, especially those with chaotic dynamics and strong interactions. For describing the statistical characteristics of energy levels, wave functions, and spectral correlations, it offers a potent framework. Numerous quantum phenomena, such as quantum transport, quantum entanglement, quantum chaos, and quantum information processing, have been studied using RMT. RMT enables researchers to forecast the statistical behaviour of these systems and obtain insights into their underlying features by using random matrices as models for complicated quantum systems.

Statistical mechanics and quantum physics have benefited greatly from the use of RMT, which has produced a number of significant findings and developments. RMT has proved crucial in the study of disordered systems in condensed matter physics, including Anderson localization and the metal-insulator transition. RMT has shed light on the statistical characteristics of nuclear spectra and the distribution of nuclear energy levels in the field of nuclear physics. RMT has been used in quantum information theory to measure the degree of entanglement in quantum systems and to analyse the behaviour of quantum channels.

Non-relativistic quantum mechanics has nine different formulations [1]. The wavefunction, matrix, path integral, phase space, density matrix, second quantization, variational, pilot wave, and quantum Hamilton-Jacobi (QHJ) approaches are only a few of the viewpoints that are covered by these formulations. The strong similarity between random matrix theory and the QHJ formalism is an exciting finding. Stationary point ensemble written as:

$$H = V(x) - \Sigma k = l1\log(|\ xk - xl\ |) \text{ ……………………}(1)$$

The quantum momentum function (QMF), which is a key component of the formulation, is used to solve the issue in the Quantum Hamilton-Jacobi (QHJ) formalism.

In Statistical Mechanics and Quantum Physics, the study of complex systems has shown Random Matrix Theory to be a potent and useful tool. It has become essential in many research areas due to its capacity to capture universal qualities and offer insights into collective phenomena. We can continue to improve our knowledge of complex systems, find novel universal behaviours, and make major contributions to the fields of Quantum Physics and Statistical Mechanics by investigating the applications of RMT further.

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Theory of Random Matrix

The idea of using random matrix theory (RMT) to explain the properties of excited states in atomic nuclei was first proposed by Wigner [10]. The use of RMT as a modelling tool in the context of physical reality was introduced for the first time through this trailblazing effort. Through the use of a probability distribution function, the behaviour of an ensemble made up of a random, infinite-dimensional Hermitian matrix is captured inside the framework of random matrix theory, offering insights into the dynamics of such systems.

$$\mathrm{P}(\lambda 1, \ldots, \lambda N) d\Lambda = cne - \beta H d\Lambda \ldots\ldots\ldots\ldots\ldots..(2)$$

For the random matrix, the parameters of the symmetry class are characterized by the index β, where β can take values of 1, 2, or 4, representing the orthogonal, unitary, and symplectic symmetry classes, respectively. In this context, the integration measure, denoted as dΛ, is defined as the product of differentials dλ1, dλ2, ..., dλN, while cn represents a constant of proportionality.

$$H = -\sum i = 1NV(\lambda i) - \sum i < j \ln \mid \lambda i - \lambda j \mid \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where V(_i) stands for the potential term for the i-th variable, H stands for the Hamiltonian, and the double sum encompasses all pairings (i, j) where i is less than j. For each pair (i, j) in the summation, the logarithm of the absolute difference between _i and _j is calculated.

Since the random matrix ensembles mentioned above display invariance under the symmetry group, they are referred regarded as invariant ensembles. In the context of the random matrix H, V(_i) signifies the potential term, |_i - _j| the Vandermonde determinant, and _i the eigenvalues with i arbitrarily ranging from 1 to N. Please refer to reference [2] for a fuller grasp of these concepts.

$$P(\Lambda) = Z1 \exp\left(-\beta N2 \sum i < jV(\lambda i, \lambda j)\right) \prod i < j \mid \lambda i - \lambda j \mid \beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The equation (3) is expressed as by adjusting the Vandermonde determinant, that is, by adding and removing columns or rows.

$$P(\lambda 1, \ldots, \lambda N) d\Lambda = cn \prod i = 1Nw(\lambda i)21 \prod i < j \mid \lambda i - \lambda j \mid d\Lambda \quad \ldots\ldots\ldots\ldots..(5)$$

Where, (1,...,) are the variables in the equation above.

Λ P(λ 1,…,λ N)The probability distribution function is represented by d, c n is a proportionality constant, ( ) w(i) denotes a weight function connected to i, and the product terms are $\prod i = 1Nw(\lambda_i)21$ capture the respective factors in the distribution function.

The Gaussian ensembles, the Wishart ensembles, and the two Wishart ensembles are the three primary types of invariant ensembles in the context of random matrix theory. The Gaussian ensembles, where the matrices are known as Wigner matrices, have a prominent place among these. The three types of Gaussian ensembles are the Gaussian unitary ensemble (GUE), the Gaussian symplectic ensemble (GSE), and the Gaussian orthogonal ensemble (GOE).

Wigner's law states that in the stationary situation, specifically for the Gaussian ensembles, the empirical distribution of eigenvalues 1, 2,..., N converges to a semicircular distribution within the interval [a, b] as the matrix dimension N goes to infinity. The ensemble of random matrix's spectral density is represented by this semicircular distribution.

I. Establishment of RMT and QHJ

A foundation for comprehending the statistical characteristics and behaviour of different matrix ensembles is provided by the classification of invariant ensembles in random matrix theory. The

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convergence of eigenvalue distributions can be better understood using the Gaussian ensembles in particular, and the dynamics of the probability distribution function can be studied using the Fokker-Planck equation.

$$p(x) = \sum n\big(-i\hbar + Q(x)\big)\big(\prod k = 1n(x - xk)\big) \dots\dots\dots\dots\dots (6)$$

The polynomial equation written as

$$f(x) = (x - x1)(x - x2) \cdots (x - \ xn) \dots\dots\dots\dots\dots\dots\dots\dots (7)$$

then the polynomial-based representation of the quantum momentum function (h = 1) is as follows:

$$f''(x) + 2iQ(x)f'(x) + [Q2(x) - iQ'(x) - E + V(x)]f(x) = 0(8)$$

By lengthening the product term, the equation's polynomial solution can be found:

$$p(x) = \big(-i\hbar + Q(x)\big)(x - x1)(x - x2) \dots (x - xn) \dots\dots\dots (9)$$

Hermite polynomials are a family of orthogonal polynomials that appear in the setting of quantum mechanics and mathematical physics. They are denoted by the symbol H n (x). The recurrence relation listed below defines them:

$$Hn + 1(x) = 2xHn(x) - 2nHn - 1(x) \dots\dots\dots (10)$$

There are many significant characteristics of hermite polynomials. They express the solutions of the time-independent Schrodinger equation and make up the entire set of eigenfunctions of the quantum harmonic oscillator. They also have a clear orthogonality relationship:

$$\int -\infty\infty e - x2Hm(x)Hn(x)dx = \pi 2nn!\delta mn \dots\dots\dots\dots\dots\dots\dots\dots (11)$$

Ermite polynomials are extensively used in the fields of approximation theory, combinatorics, and probability theory. They are crucial to many branches of mathematics and physics because of their unique characteristics and diverse range of applications.

II. Conclusion

The study of Random Matrix Theory (RMT) for applications in Quantum Physics and Statistical Mechanics has proven to be extremely beneficial and informative. RMT has given researchers effective tools and approaches to investigate universal qualities, comprehend the fundamental rules driving quantum mechanical systems, and comprehend the statistical behaviour of complicated systems.We have learned how RMT has been successfully used in numerous study fields throughout the course of this investigation. RMT has offered a framework for exploring huge ensembles of particles and their collective behaviour in statistical mechanics. RMT has enabled the examination of energy spectra, level spacing distributions, and correlation functions, giving greater insights into phase transitions and emergent collective phenomena. RMT views random matrices as models for Hamiltonians.RMT has been essential in understanding the statistical behaviour of quantum systems with chaotic dynamics in quantum physics. RMT has improved our understanding of quantum transport, quantum chaos, and quantum information processing by characterising the statistical features of energy levels, wave functions, and spectral correlations. It has given important insights into the behaviour of strong interactions, enabling researchers to forecast and elucidate the underlying characteristics of these intricate systems.The idea of universality in RMT has also been a major focus of this inquiry. It has been possible to identify common characteristics among various many-body systems thanks to the discovery that some statistical aspects become independent of the particulars of the system and are only

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controlled by the symmetry class.RMT is anticipated to have a significant impact on future statistical mechanics and quantum physics research as it develops. Our understanding of complex systems will be further improved through the investigation of new applications, the development of improved methodologies, and the integration of RMT with other theoretical frameworks. These activities will also open up new research directions and propel developments in fundamental physics, materials science, and quantum technologies.

References:

[1] DanielF.Styeret.al.,Am.J.Phys.70(3),(2002)
[2] Mehta,M.L.(2004).RandomMatrices,3rdEdition,PureandAppliedMathematics(Amster-dam),142,Amsterdam,Netherlands:Elsevier/AcademicPress.
[3] R.S.Bhalla, A.K.KapoorandP.K.Panigrahi, Am.J.Phys.65,1187,(1997).
[4] R.S.Bhalla,A.K.KapoorandP.K.Panigrahi,Mod.Phys.Lett.A,12,295(1997).
[5] T.J.Stieltjes,Surquelquestheoremesd'algebre,ComptesRendusdel'AcademiedesSciences, Paris,100(1885),439-440;OeuvresCompletes,Vol.1,440-441.
[6] T.J.Stieltjes,Surquelquestheoremesd'algebre,ComptesRendusdel'AcademiedesSciences, Paris,100(1885),620-622;OeuvresCompletes,Vol.1,442-444.
[7] Aref H.J.Math. Phys.48, 065401,(2007).
[8] BSriramShastryandAbhishekDhar,J.Phys.A:Math.Gen.34,6197,(2001).
[9] K.V.S.ShivChaitanya,PRAMANAjournalofphysics,Vol.83,No.1,139,2014
[10] Wigner,E.P.,GatlinburgConferenceonNeutronPhysics.Report2309:59,OakRidgeNational Laboratory,OakRidge,TN(1957).
[11] R.ALeacockandM.J.PadgettPhys.Rev.Lett.50,3,(1983).
[12] R.A.LeacockandM.J.PadgettPhys.Rev.D28,2491,(1983).
[13] S. SreeRanjani, K. G. Geojo,A. K. Kapoor, P. K. Panigrahi,Mod. Phys. Lett.A. Vol19,No.19,1457,(2004).
[14] K.G.Geojo,S.SreeRanjani,A.K.Kapoor,J.PhysA:Math.Gen.36,4591, (2003).
[15] F.Cooper,A.Khare,U.P.Sukhatme,2001*Supersymmetric quantummechanics*(Singapore:WorldScientificPublishingCo.Ltd.)
[16] S.SreeRanjani,P.K.Panigrahi,A.Khare,A.K.Kapoor,A.Gangopadhyaya,*J.Phys.A: Math.Theor*.(45)055210,(2012).