UNIFIED APPROACH TO D-SETS IN TOPOLOGY

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Abstract
In this paper, a unified approach to the notion of D-sets in topological spaces is given. In fact new types of D-sets can be defined by replacing "open set" by a topological set. According to Tong, each pair (U,V) of open sets with U≠X, contributes a D-set . Our aim is to introduce new types of D-sets by replacing either U or V by another type of topological sets namely ω-open, pre-open, semi-open, α-open, β-open, b-open pre-ω-open, semi-ω-open, α-ω-open, β-ω-open and b-ω-open sets

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Introduction

In this paper, a unified approach to the notion of D-sets in topological spaces is given. In fact new types of D-sets can be defined by replacing "open set" by a topological set. According to Tong, each pair (U,V) of open sets with U≠X, contributes a D-set. Our aim is to introduce new types of D-sets by replacing either U or V by another type of topological sets namely ω-open, pre-open, semi-open, α-open, β-open, b-open pre-ω-open, semi-ω-open, α-ω-open, β-ω-open and b-ω-open sets. This paper is described with two sections. The notions of ω-D, pre-D, semi-D, α-D, β-D, b-D sets are introduced and studied in the first section by replacing V by a pre-open, semi-open, α-open, β-open, b-open set respectively . The second section deals with pre-ω-D, semi-ω-D, α-ω-D, β-ω-D, b-ω-D sets that are defined by replacing V by pre-ω-open, semi-ω-open, α-ω-open, β-ω-open, b-ω-open set respectively.

Throughout this paper, (X,τ) is a topological space, A and B are subsets of X. The following notations are used.

"$A\subset B$" means A is a proper subset of B.

"$A\supset B$" means A is a proper super set of B.

$\downarrow B$ = The down sets of B = The collection of all subsets of B = 2^B.

$\uparrow B$ = The up sets of B = The collection of all super sets of B.

Clearly $B\in\downarrow B$ and $B\in\uparrow B$.

"$A\subseteq B$" means $A\subset B$ or $A=B$ and "$A\supseteq B$" means $A\supset B$ or $A=B$

$A\in\downarrow B\Leftrightarrow A\subseteq B$ and $A\in\uparrow B\Leftrightarrow A\supseteq B$.

$FCount(X) = \{A\subseteq X: A \text{ is finite or countable}\}$.

$UnCount(X) = \{A\subseteq X: A \text{ is uncountable}\}$.

η-D-SETS where η∈{ ω, semi, pre, α, β, b }

Let B be a D-set in (X,τ), determined by an ordered pair (U,V) of open sets with U≠X. By varying the first coordinate U over τ and varying V over ωO(X,τ), the notion of ω-D-set is introduced. Throughout this section η∈{ ω, semi, pre, α, β, b } unless otherwise specified.

Definition 2.1.1 If A = U\V, U≠X, U∈τ and V∈ωO(X,τ), then A is called an ω-D-set.

Remark 2.1.2 An ω-D-set U\V is determined by an ordered pair (U,V) where U is open , U≠X and V is ω-open.

Notations 2.1.3

(i) D(X,τ) = the collection of all D-sets in (X,τ).
(ii) ω-D(X,τ) = the collection of all ω-D-sets in (X,τ).
(iii) LωC(X,τ) = the collection of all locallyω-closed sets in (X,τ).

Proposition 2.1.4

(i) Every proper open set U is an ω-D-set.
(ii) The whole set X is not anω-D-set.
(iii) Every D-set is an ω-D-set.
(iv) Every ω-D-set is locally ω-closed.
(v) The class ω-D(X,τ) lies properly between τ and LωC(X,τ).

Proof. Let U∈τ with U ≠X. Since U = U\∅ and since ∅ is an ω-open set, it follows that U is an ω-D-set. This proves (i). As the whole set X can not be written as X= U\V with U≠X, it is clear that X is not an ω-D-set. This proves (ii).

The set A is a D-set ⇒ A = U\V, U≠X, U∈τ, V∈τ,
⇒ A = U\V, U≠X, U∈τ, V∈τ_ω,
⇒ A is an ω-D-set. This proves (iii).
The set A is an ω-D-set ⇒ A = U\V, U≠X, U∈τ, V∈τ_ω,
⇒ A = U∩(X\V), U≠X, U∈τ, V∈τ_ω,
⇒ A = U∩F, U≠X, U∈τ, F is ω-closed ,
⇒ A is locally ω-closed. This proves (iv).

In any topological space (X,τ), X is locally ω-closed but not an ω-D-set.This together with (i) and (iv) proves (v). This completes the proof of the proposition.

The above proposition is illustrated in the next example.

Example 2.1.5 Let X= ℝ and τ= {∅, ℚ, ℝ} where ℝ is the set of all real numbers,ℚ is the set of all rational numbers and $\mathbb{Q}^c$is the set of all irrational numbers.

The $space\ R(\mathbb{Q}) = (\mathbb{R},\tau) = (\mathbb{R},\{\varnothing,\mathbb{Q},\mathbb{R}\})$. Let A be a subset of ℝ. The following results can be easily proved.

- No rational number is a condensation point of A.
- If A is finite or countable , then no irrational number is a condensation point of A.
- If A is finite or countable , then Cond(A) = ∅.
- If A is uncountable then Cond(A) = $\mathbb{Q}^c$.
- An uncountable set A is ω-closed ⇔ A⊇$\mathbb{Q}^c$.
- ωC(ℝ,τ)={A: A is a finite or countable subset of ℝ }∪{B: B⊇$\mathbb{Q}^c$ }.

={A⊆ ℝ: A is a finite or countable}∪↑$\mathbb{Q}^c$.

- ωO(ℝ, τ) ={A⊆ ℝ: A is uncountable , ℝ \A is finite or countable}∪ 2ℚ.
  = {A⊆ℝ: A is uncountable , R\A is finite or countable}∪↓ℚ.
- Clearly ℚ and $\mathbb{Q}^c$ are both ω-open and ω-closed.
- Any subset of ℚ is both ω-open and ω-closed.
- Any super set of $\mathbb{Q}^c$ is both ω-open and ω-closed.
- {1, √2 } is ω-closed but not ω-open.
- ℝ\{√2 } is ω-open but not ω-closed.

- $\{ \sqrt{3} \}$ is ω-closed but not ω-open.
- $D(\mathbb{R},\tau) = \{ U\backslash V: U\in\tau\backslash\{ \mathbb{R} \}, V\in\tau\} = \{\varnothing, \mathbb{Q}\}$
- $\omega\text{-}D(\mathbb{R},\tau) = \{ U\backslash V: U\in\tau\backslash\{ \mathbb{R} \}, V\in\tau_\omega\}$
  $= \{\varnothing, \mathbb{Q}\}\cup\{\mathbb{Q}\backslash A: A\in\downarrow \mathbb{Q} \}\cup\{\mathbb{Q}\backslash A: A\in\text{Un Count}(\mathbb{R},\tau), \mathbb{R} \backslash A \in \text{F Count}(\mathbb{R},\tau)\}$
- $L\omega C(\mathbb{R},\tau) = \omega C(\mathbb{R},\tau) \cup\{ \mathbb{Q}\cap A: A\in \text{F Count}(\mathbb{R}, \tau)\cup\uparrow\mathbb{Q}^c \}$

Proposition 2.1.6 If σ and τ are topologies on X such that σ is coarser than τ then ω-D(X,σ) is coarser than ω-D(X,τ).

Proof. $B\in\omega\text{-}D(X,\sigma) \Rightarrow B=U\backslash V$ where $U\in\sigma$, $V\in\sigma_\omega$ with $U\neq X$
$\Rightarrow B=U\backslash V$ where $U\in\tau$, $V\in\tau_\omega$ with $U\neq X$
$\Rightarrow B\in\omega\text{-}D(X,\tau)$.

Proposition 2.1.7 If σ and τ are topologies on X such that σ is finer than τ_ω then $D(X,\tau)\subseteq\omega\text{-}D(X,\tau) \subseteq\omega\text{-}D(X,\tau_\omega) \subseteq D(X,\tau_\omega) \subseteq D(X,\sigma)\subseteq\omega\text{-}D(X,\sigma)$.

Proof. Let σ and τ be topologies on X such that σ is finer than τ_ω. Then $\tau\subseteq\tau_\omega\subseteq\sigma$.
By using Proposition 2.1.6 we have, $\omega\text{-}D(X,\tau) \subseteq\omega\text{-}D(X,\tau_\omega)$ (2.1.1)
Now using Proposition 2.1.4(iv) we have $D(X,\tau)\subseteq\omega\text{-}D(X,\tau)$ (2.1.2)
Now $A\in\omega\text{-}D(X,\tau_\omega) \Rightarrow A = U\backslash V$ where $U\in\tau_\omega$, $V\in(\tau_\omega)_\omega$ with $U\neq X \Rightarrow A = U\backslash V$ where $U\in\tau_\omega$, $V\in\tau_\omega$ with $U\neq X \Rightarrow A \in D(X,\tau_\omega)$. Therefore $\omega\ D(X,\tau_\omega)\subseteq D(X,\tau_\omega)$ (2.1.3)
Again by using Proposition 2.1.6 and Proposition 2.1.4(iv) we have $D(X,\tau_\omega)\subseteq D(X,\sigma)\subseteq\omega\text{-}D(X,\sigma)$ (2.1.4)
Combining (2.1.1), (2.1.2), (2.1.3) and (2.1.4) we have $D(X,\tau)\subseteq\omega\text{-}D(X,\tau) \subseteq\omega\text{-}D(X,\tau_\omega) \subseteq D(X,\tau_\omega) \subseteq D(X,\sigma)\subseteq\omega\text{-}D(X,\sigma)$.
This proves the proposition.

Proposition 2.1.8

(i) The intersection of two ω-D-sets is an ω-D-set.
(ii) The union of two ω-D-sets is not an ω-D-set

Proof. Let $A = U_1\backslash V_1$ and $B =U_2\backslash V_2$ where U_1, U_2 are open and V_1,V_2 areω-open in (X,τ). $A \cap B = (U_1\backslash V_1) \cap (U_2\backslash V_2)$

$$= (U_1\cap (X\backslash V_1) \cap (U_2\cap (X\backslash V_2)$$
$$=(U_1\cap U_2) \cap (X\backslash V_1) \cap (X\backslash V_2)$$
$$= (U_1\cap U_2) \cap (X\backslash (V_1\cup V_2))$$
$$= (U_1\cap U_2) \backslash (V_1\cup V_2) .$$

Here $U_1\cap U_2$ is open and $V_1\cup V_2$ is ω-open in (X,τ). This shows $A\cap B$ is an ω-D-set. This proves (i).
The union of two ω-D-sets need not be an ω-D-set as shown below.
If A and B are the disjoint subsets of $\mathbb{R}$ such that $A\cup B = \mathbb{R}$ then $\{\varnothing, A, B, \mathbb{R} \}$ is a topology on $\mathbb{R}$. Clearly A and B are ω-D-sets in $\tau =(\mathbb{R}, \{\varnothing, A, B, \mathbb{R} \})$ but $A\cup B = \mathbb{R}$ is not an ω-D-set. In particular, we take $A = \mathbb{R}$ and $B= \mathbb{R} \backslash \mathbb{Z}$. Then it can be verified that $\omega O(X,\tau)= \{\varnothing, A, B, \mathbb{R}\}\cup\downarrow\mathbb{Z}\cup\uparrow(\mathbb{R}\backslash\mathbb{Z}) \cup\{B; \mathbb{Q}c\subseteq B\subseteq\mathbb{R}\backslash\mathbb{Z} \}$.
Since $\mathbb{Z} =\mathbb{Z}\backslash\varnothing$ and $\mathbb{Q}\backslash\mathbb{Z} = (\mathbb{R}\backslash \mathbb{Z}) \backslash \mathbb{Q}c$, $\mathbb{Z}$ and $\mathbb{Q}\backslash\mathbb{Z}$ are ω-D-sets but $\mathbb{Z}\cup(\mathbb{Q}\backslash\mathbb{Z}) = \mathbb{Q}$ is not an ω-D-set. This proves (ii).

Corollary 2.1.9 The intersection of a D-set with an ω-D-set is an ω-D-set.

Proof. Follows Proposition 2.1.8 and from the fact that every D-set is an ω-D-set.
The next proposition shows that the union of two ω-D-sets is an ω-D-set under some specialized conditions.

Proposition 2.1.10 If A and B are ω-D-sets, determined by the ordered pairs (U_1,V_1) and (U_2,V_2) respectively such that $U_1\cup U_2\neq X$ then $A\cup B$ is an ω-D-set.

Proof. Let $A = U_1\backslash V_1$ and $B =U_2\backslash V_2$ where U_1, U_2 are open, $U_1\cup U_2\neq X$ and V_1, V_2 are ω-open in (X, τ). Then $A\cup B = (U_1\backslash V_1) \cup (U_2\backslash V_2) =(U_1\cup U_2)\backslash V$ where $V= (U_1\cap V_1 \cap V_2) \cup (U_2 \cap V_1 \cap V_2)$.
Now $U_1\cup U_2$ is open. Since the intersection of an open set with an ω-open set is ω-open, V is ω-open in (X,τ). Then it follows that $A\cup B$ is an ω-D-set. This proves the proposition.

The next proposition shows that every element of an ω-open set contributes either a countable ω-D-set or a countable locally ω-closed set.

Proposition 2.1.11 Let V be an ω-open set in (X,τ). Then for each x in V there is a countable ω-D-set B, determined by V, such that $x\notin B$ or X is the neighbourhood of x for which $X\setminus V$ is countable.

Proof. Let V be an ω-open set in (X,τ) and $x\in V$. Then there is an open set U such that $x\in U$ and $U\setminus V$ is countable. Then $U\subset X$ or $U=X$. If $U\neq X$ then $B=U\setminus V$ is a countable D-set with $x\notin B$. If $U = X$ then $U\setminus V = X\setminus V$ is a countable locally ω-closed set. This proves the proposition.

Let B be a D-set in (X,τ), determined by a pair (U,V) of open sets with $U\neq X$. By varying the first coordinate U over τ and varying V over $\alpha O(X,\tau)$, $\beta O(X,\tau)$, $bO(X,\tau)$, $PO(X,\tau)$, $SO(X,\tau)$, the notions of α-D-set, β-D-set, b-D-set, pre-D-set and semi-D-set are introduced .

Definition 2.1.12 Let A be a subset of a topological space (X,τ) with $A = U\setminus V$, $U\neq X$, $U\in\tau$. Then A is called,

(i) an α-D-set if V is α-open,
(ii) a β-D-set if V is β-open,
(iii) a b-D-set if V is b-open,
(iv) a pre-D-set if V is pre-open,
(v) a semi-D-set if V is semi-open.

Notations 2.1.13 $\eta\text{-}D(X,\tau)$ = the collection of all η-D-sets in (X,τ).
$L\eta C(X,\tau)$ = the collection of all locallyη-closedsets in (X,τ).

Proposition 2.1.14 Let A be a subset of a topological space (X,τ) .

(i) If A is an α-D-set then it is an αD-set.
(ii) If A is a β-D-set then it is a βD-set.
(iii) If A is a pre-D-set then it is a pD-set.
(iv) If A is a semi-D-set then it is a sD-set.
(v) If A is a b-D-set then it is a bD-set.

Proof. A is an α-D-set $\Rightarrow A = U\setminus V$ where $U\in\tau$ and $V\in\alpha O(X,\tau)$.
$\Rightarrow A = U\setminus V$ where $U\in\alpha O(X,\tau)$ and $V\in\alpha O(X,\tau)$.
$\Rightarrow$A is an αD-set.
A is a β-D-set $\Rightarrow A = U\setminus V$ where $U\in\tau$ and $V\in\beta O(X,\tau)$.
$\Rightarrow A = U\setminus V$ where $U\in\beta O(X,\tau)$ and $V\in\beta O(X,\tau)$.
$\Rightarrow$ A is a βD-set.
A is a pre-D-set $\Rightarrow A = U\setminus V$ where $U\in\tau$ and $V\in PO(X,\tau)$.
$\Rightarrow A = U\setminus V$ where $U\in PO(X,\tau)$ and $V\in PO(X,\tau)$.
$\Rightarrow$ A is a pD-set.
A is a semi-D-set $\Rightarrow A = U\setminus V$ where $U\in\tau$ and $V\in SO(X,\tau)$.
$\Rightarrow A = U\setminus V$ where $U\in SO(X,\tau)$ and $V\in SO(X,\tau)$.
$\Rightarrow$ A is a sD-set.
A is a b-D-set $\Rightarrow A = U\setminus V$ where $U\in\tau$ and $V\in bO(X,\tau)$.
$\Rightarrow A = U\setminus V$ where $U\in bO(X,\tau)$ and $V\in bO(X,\tau)$.
$\Rightarrow$ A is a bD-set.
This proves the proposition.

The converses in the above proposition are not true as shown below.

Example 2.1.15 Consider the space $(\mathbb{R}, \{\emptyset, \mathbb{Q}, \mathbb{R}\})$ where $X= \mathbb{R}, \tau = \{\emptyset, \mathbb{Q}, \mathbb{R}\}$.
For any subset A of X we have the following equations.

$$Int A = \begin{cases} X & if\ A = X \\ Q & if\ Q\subseteq A\subset X, \\ \emptyset & otherwise \end{cases} \quad \text{and} \quad Cl A = \begin{cases} \emptyset & if\ A = \emptyset, \\ X\setminus Q & if\ A\subseteq X\setminus Q, \\ R & otherwise, \end{cases} \qquad (2.1.5)$$

$$Cl\, Int A = \begin{cases} X & if\ Q\subseteq A\subseteq X, \\ \emptyset & otherwise \end{cases} \quad \text{and} \quad Int Cl A = \begin{cases} \emptyset & if\ A\subseteq X\setminus Q, \\ X & otherwise \end{cases} \qquad (2.1.6)$$

$IntClInt$A = $\begin{cases} X & if\ Q\subseteq A\subseteq X, \\ \emptyset & otherwise \end{cases}$ and $ClIntCl$A = $\begin{cases} \emptyset & if\ A\subseteq X\setminus Q, \\ X & otherwise \end{cases}$ (2.1.7)

From (2.1.6) we have SO(X,τ) =$\uparrow\mathbb{Q}\cup\{\emptyset\}$and PO(X,τ) = $(2^X \setminus 2^{X\setminus Q})\cup\{\emptyset\}$= bO(X,τ).

From (2.1.7) we have αO(X,τ) =$\uparrow\mathbb{Q}\cup\{\emptyset\}$and βO(X,τ) = $(2^X \setminus 2^{X\setminus Q})\cup\{\emptyset\}$

Therefore α-D(X,τ) = { U\V: U∈τ \{X}, V∈αO(X,τ) }
= {∅, $\mathbb{Q}$}∪{$\mathbb{Q}$\V: V∈$\uparrow\mathbb{Q}$ } .
= {∅, $\mathbb{Q}$}∪{∅ } = {∅, $\mathbb{Q}$}= semi-D(X,τ).

αD(X,τ) = { U\V: U∈αO(X,τ) \{X}, V∈αO(X,τ)} .
= {∅} ∪ ($\uparrow\mathbb{Q}$\{X}) ∪{ U\V: U∈$\uparrow\mathbb{Q}$ \{X}, V∈$\uparrow\mathbb{Q}$ }.
=sD(X,τ).

Therefore semi-D(X,τ) is properly contained in sD(X,τ) and α-D(X,τ) is properly contained in αD(X,τ). Similarly we can prove that pre-D(X,τ) is properly contained in pD(X,τ), b-D(X,τ) is properly contained in bD(X,τ) and β-D(X,τ) is properly contained in βD(X,τ). In particular, $\mathbb{Q}\cup\{\sqrt{2}\}$ is both an αD-set and a sD-set but it is neither an α-D-set nor a semi-D-set. Also $\mathbb{Q}^c\cup\{2\}$ is a pD-set, a bD-set and a βD-set. But it is not a pre-D-set, not a b-D-set and not a β-D-set.

Example 2.1.16 Let X ={a,b,c,d} and τ= { ∅, {d}, {a,b}, {a,b, d},X}. Then the following can be proved.

(i) semi-D(X,τ) = { {∅, {d}, {a,b}, {a,b, d}} and sD(X,τ) = { ∅, {c}, {d}, {c, d}, {a,b}, {a,b,c}, {a,b, d}}.

Therefore the sets {c}, {d}, {c, d} and {a,b,c} are sD-sets but not semi-D-sets.

(ii) pre-D(X,τ) = { { ∅, {a}, {b}, {d}, {a,b}, {a,d}, {b,d},{a,b, d}}= b-D(X,τ)= β-D(X,τ) and pD(X,τ) = $2^X \setminus \{X\}$ = bD(X,τ)= βD(X,τ) .

This shows that {c}, {a,c}, {b,c}, {c,d}, {a,b,c}, {a,c,d} and {b,c,d} are all pD-sets , bD-sets and βD-sets but none of them is a pre-D-set, a b-D-set and a β-D-set.

Proposition 2.1.17

(i) Every proper open set U is an α-D-set.
(ii) The whole set X is not anα-D-set.
(iii) Every D-set is an α-D-set.
(iv) Every α-D-set is locally α-closed.
(v) The class α-D(X,τ) properly lies betweenτ and LαC(X,τ).

Proof. Let U∈τ with U ≠X. Since U = U\∅ and since ∅ is an α-open set, it follows that U is an α-D-set. This proves (i). As the whole set X can not be written as X= U\V with U≠X, it is clear that X is not an α-D-set. This proves (ii).

A is a D-set ⇒ A = U\V where U∈τ , U≠X and V∈τ
⇒ A = U\V where U∈τ and V∈αO(X, τ)
⇒ A is an α-D-set.This proves (iii).

A is an α-D-set ⇒ A = U\V where U∈τ , U≠X and V∈αO(X, τ)
⇒ A = U∩(X\V) where U∈τ and X\V∈αC(X, τ)
⇒ A is locally α-closed.

This proves (iv). The assertion (v) follows from (i) and (iv).

Proposition 2.1.18 (i) Every proper open set U is a pre-D-set.
(ii) The whole set X is not a pre-D-set.
(iii) Every α-D-set is a pre-D-set.
(iv) Every pre-D-set is locally pre-closed.
(v) The class pre-D(X,τ) lies between τ and LpreC(X,τ).

Proof. The assertions (i) and (ii) follow easily.

A is an α-D-set ⇒ A = U\V where U∈τ , U≠X and V∈αO(X,τ)
⇒ A = U\V where U∈τ and V∈PO(X,τ)
⇒ A is a pre-D-set. This proves (iii).

A is a pre-D-set ⇒ A = U\V where U∈τ , U≠X and V∈PO(X,τ)

$\Rightarrow$ A = U$\cap$(X\V) where U$\in\tau$ and X\V$\in$PC(X,τ)

$\Rightarrow$ A is locally pre-closed .

This proves (iv). The assertion (v) follows from (i) and (iv).

Proposition 2.1.19 (i) Every proper open set U is a semi-D-set.

(ii) The whole set X is not a semi-D-set.

(iii) Every α-D-set is a semi-D-set.

(iv) Every semi-D-set is locally semi-closed.

(v) The class semi-D(X,τ) lies between τ and LsemiC(X,τ).

Proof. Analogous to Proposition 2.1.18.

Proposition 2.1.20 (i) Every proper open set U is a b-D-set.

(ii) The whole set X is not a b-D-set.

(iii) Every pre-D-set is a b-D-set.

(iv) Every semi-D-set is a b-D-set.

(v) Every b-D-set is locally b-closed.

(vi) The class b-D(X,τ) lies between τ and LbC(X,τ).

Proof. The assertions (i) and (ii) follow easily.

A is a pre-D-set or semi-D-set $\Rightarrow$ A = U\V where U$\in\tau$, U$\neq$X and V$\in$PO(X,τ)$\cup$SO(X,τ)

$\Rightarrow$ A = U\V where U$\in\tau$ and V$\in$bO(X,τ)

$\Rightarrow$ A is a b-D-set. This proves (iii).and (iv)

A is a b-D-set $\Rightarrow$ A = U\V where U$\in\tau$, U$\neq$X and V$\in$bO(X,τ)

$\Rightarrow$ A = U$\cap$(X\V) where U$\in\tau$ and X\V$\in$bC(X,τ)

$\Rightarrow$ A is locally b-closed . This proves (v).

The assertion (vi) follows from (i) and (v).

Proposition 2.1.21 (i) Every proper open set U is a β-D-set.

(ii) The whole set X is not aβ-D-set.

(iii) Every b-D-set is a β-D-set.

(iv) Every β-D-set is locally β-closed.

(v) The class β-D(X,τ) lies between τ and LβC(X,τ)..

Proof. The assertions (i) and (ii) follow easily.

A is a b-D-set $\Rightarrow$ A = U\V where U$\in\tau$, U$\neq$X and V$\in$bO(X,τ)

$\Rightarrow$ A = U\V where U$\in\tau$ and V$\in\beta$O(X,τ)

$\Rightarrow$ A is a β-D-set.This proves (iii).

A is a β-D-set $\Rightarrow$ A = U\V where U$\in\tau$, U$\neq$X and V$\in\beta$O(X,τ)

$\Rightarrow$ A = U$\cap$(X\V) where U$\in\tau$ and X\V$\in\beta$C(X,τ)

$\Rightarrow$ A is locally β-closed .This proves (iv).

The assertion (v) follows from (i) and (iv).

Remark 2.1.22 (i) The null set $\varnothing$ is an η-D-set in every topological space.

(ii) The null set $\varnothing$ is the only η-D-set in the indiscrete space.

(iii) Every proper subset of the discrete space is an η-D-set.

(iv) The whole set can never be an η-D-set in any topological space.

The discussion about D-sets in this section leads to following implications on various types of difference sets**.**

Diagram 2.1.23

D(X,τ)
↧
α-D(X,τ)
pre-D(X,τ)↲ ↳ semi-D(X,τ)
↘ ↙
b-D(X,τ)
↧

β-D(X,τ)

Proposition 2.1.24 The intersection of two η-D-sets is anη-D-set
Proof. Let $A = U_1\backslash V_1$ and $B = U_2\backslash V_2$ where U_1, U_2 are open and $V_1, V_2 \in \alpha O(X,\tau)$.
$A \cap B = (U_1\backslash V_1) \cap (U_2\backslash V_2) = (U_1 \cap (X\backslash V_1) \cap (U_2 \cap (X\backslash V_2)$.
$=(U_1 \cap U_2) \cap (X\backslash V_1) \cap (X\backslash V_2)$.
$= (U_1 \cap U_2) \cap (X\backslash (V_1 \cup V_2))$.
$= (U_1 \cap U_2) \backslash (V_1 \cup V_2)$.
Here $U_1 \cap U_2$ is open and $V_1 \cup V_2$ is α-open in (X,τ). This shows $A \cap B$ is an α-D-set. This proves the proposition for the case $\eta = \alpha$. The proof for the rest is analogous.
Corollary 2.1.25 The intersection of a D-set with an η-D-set is anη-D-set.
Proof. Follows from Proposition 2.1.24 and Diagram 2.1.23.
Proposition 2.1.26 If A and B are α-D-sets, determined by the ordered pairs (U_1, V_1) and (U_2, V_2) respectively such that $U_1 \cup U_2 \neq X$ then $A \cup B$ is an α-D-set .
Proof. Let $A = U_1\backslash V_1$ and $B = U_2\backslash V_2$ where U_1, U_2 are open, $U_1 \cup U_2 \neq X$ and V_1, V_2 are α-open in (X, τ). Then $A \cup B = (U_1\backslash V_1) \cup (U_2\backslash V_2) = (U_1 \cup U_2) \backslash V$ where $V = (U_1 \cap V_1 \cap V_2) \cup (U_2 \cap V_1 \cap V_2)$.

Since $\alpha O(X, \tau)$ is a topology and since every open set is α-open, V is α-open. Also since $U_1 \cup U_2$ is open, $U_1 \cup U_2 \neq X$ and V is α-open in (X, τ), it follows that $A \cup B$ is an α-D-set. This proves the proposition.
Proposition 2.1.27 If A and B are D-set and b-D-set, determined by the ordered pairs (U_1, V_1) and (U_2, V_2) respectively such that $U_1 \cup U_2 \neq X$ then $A \cup B$ is a b-D-set .
Proof. Since the intersection of an open set with a b-open set is b-open , the proof is analogous to Proposition 2.1.26.
The union of two η-D-sets, determined by a same η-open set is a η-D-set where $\eta \in \{\beta$, pre, semi ,b$\}$ as shown in the next proposition.
Proposition 2.1.28 If A and B are η-D-sets, determined by the ordered pairs (U_1, V) and (U_2, V) respectively such that $U_1 \cup U_2 \neq X$ then $A \cup B$ is anη-D-set where $\eta \in \{\beta$, b, pre, semi $\}$.
Proof. Let $A = U_1\backslash V$ and $B = U_2\backslash V$ where U_1, U_2 are open and V is η-open in (X,τ) .
$A \cup B = (U_1\backslash V) \cup (U_2\backslash V) = (U_1 \cap (X\backslash V) \cup (U_2 \cap (X\backslash V)$.
$=(U_1 \cup U_2) \cap (X\backslash V)$.
$= (U_1 \cup U_2) \backslash V$
Here $U_1 \cap U_2$ is open and $V_1 \cup V_2$ is η-open in (X,τ). This shows $A \cup B$ is a η-D-set. This proves the proposition.
Proposition 2.1.29 Suppose$\beta O(X,\tau)$, $bO(X,\tau)$, $PO(X,\tau)$ and $SO(X,\tau)$ are topologies on X. If A and B are η-D-sets, determined by the ordered pairs (U_1,V_1) and (U_2,V_2) respectively such that $U_1 \cup U_2 \neq X$ then $A \cup B$ is anη-D-set where $\eta \in \{\beta$, b, pre, semi$\}$.
Proof. Analogous to Proposition 2.1.26.

2.3 D-η-sets where $\eta \in \{\omega$, semi, pre, α, β, b$\}$

Let B be a D-set in (X,τ), determined by a pair (U,V) of open sets with $U \neq X$. By varying the second coordinate V over τ and varying U over $\omega O(X$, the notion of a D-ω-set is introduced. Throughout this section $\eta \in \{\omega$, semi, pre, α, β, b $\}$ unless otherwise specified.
Definition 2.3.1 Let A be a subset of a topological space (X,τ) with $A = U\backslash V$, $U \neq X$. Then A is called a D-ω-set if U is ω-open and V is open.
Notations 2.3.2 (i) D-$\omega(X,\tau)$ = the collection of all D-ω-sets in (X,τ).
(ii) $\omega LC(X,\tau)$ = the collection of all ω-locally closed sets in (X,τ).
Proposition 2.3.3 (i) Every proper ω-open set U is a D-ω-set.
(ii) The whole set X is not a D-ω-set.
(iii) Every D-set is a D-ω-set. .
(iv) Every D-ω-set is ω-locally closed.

(v) The class D-$\omega(X,\tau)$ properly lies between τ_ω and $\omega LC(X,\tau)$.

Proof. The assertions (i) and (ii) can be proved easily.

The set A is a D-set $\Rightarrow$ A= U\V where $U\neq X$, $U\in\tau$ and $V\in\tau$.

$\Rightarrow$ A= U\V where $U\neq X$, $U\in\omega O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A= U\V where $U\neq X$, $U\in\omega O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A is a D-ω-set This proves (iii).

A is a D-ω-set $\Rightarrow$ A= U\V where $U\neq X$, $U\in\omega O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A= $U\cap(X\backslash V)$ where $U\neq X$, $U\in\omega O(X,\tau)$ and X\V is closed.

$\Rightarrow$ A is ω-locally closed. This proves (iv).

The assertion (v) follows from (i) and (iv).

Example 2.3.4 Consider the point inclusion topology on $\mathbb{R}$. The point may be taken as 2. Then [0, 2] is open and [-1, 1] is ω-open but not open. Therefore [-1, 1]\ [0, 2] = [-1, 0) is a D-ω-set but not a D-set.

Proposition 2.3.5 If σ and τ are topologies on X such that σ is coarser than τ then D-$\omega(X,\sigma)$ is coarser than D-$\omega(X,\tau)$.

Proof. $B\in$D-$\omega(X,\sigma)$ $\Rightarrow$ B=U\V where $U\in\sigma_\omega$, $V\in\sigma$ with $U\neq X$.

$\Rightarrow$ B=U\V where $U\in\tau_\omega$, $V\in\tau$ with $U\neq X$.

$\Rightarrow$ $B\in$D-$\omega(X,\tau)$.

Proposition 2.3.6 Ifσ and τ are topologies on X such that σ is finer than τ_ω then $D(X,\tau)\subseteq$D-$\omega(X,\tau)\subseteq$ D-$\omega(X,\tau_\omega)$ = $D(X,\tau_\omega)\subseteq$ D-$\omega(X,\sigma)$.

Proof. Let σ and τ be topologies on X such that σ is finer than τ_ω. Then $\tau\subseteq\tau_\omega\subseteq\sigma$.

By using Proposition 2.3.3 we have, D-$\omega(X,\tau)\subseteq$ D-$\omega(X,\tau_\omega)$ = $D(X,\tau_\omega)$ that implies $D(X,\tau)\subseteq$D-$\omega(X,\tau)\subseteq$ D-$\omega(X,\tau_\omega)$ = $D(X,\tau_\omega)\subseteq D(X,\sigma)\subseteq$ D-$\omega(X,\sigma)$.

Let B be a D-set in (X,τ), determined by a pair (U, V) of open sets with $U\neq X$. By varying the second coordinate V over τ and varying U over $\alpha O(X,\tau)$, $\beta O(X,\tau)$, $bO(X,\tau)$, $PO(X,\tau)$, $SO(X,\tau)$ the notions of D-α-set, D-β-set, D-b-set, D-pre-set, D-semi-set are introduced .

Definition 2.3.7 Let A be a subset of a topological space (X,τ) with A = U\V, $U\neq X$. Then A is called a D-η-set if U is η-open and $V\in\tau$ where $\eta\in\{\alpha,\beta$, pre, semi, b$\}$

Notations 2.3.8 D-$\eta(X,\tau)$ = the collection of all D-η-sets in (X,τ) where $\eta\in\{\alpha,\beta$, b, pre, semi$\}$ η-$LC(X,\tau)$= the collection of all η-locally closed sets in (X,τ).

Proposition 2.3.9 (i) Every proper α-open set U is a D-α-set.

(ii) The whole set X is not a D-α-set.

(iii) Every D-set is a D-α-set.

(iv) Every D-α-set is α-locally closed.

(v) The class of all D-α-sets properly les between $\alpha O(X,\tau)$ and α-$LC(X,\tau)$.

Proof. The first two assertions can be easily shown.

The set A is a D-set $\Rightarrow$ A= U\V where $U\neq X$, $U\in\tau$ and $V\in\tau$.

$\Rightarrow$ A= U\V where $U\neq X$, $U\in\alpha O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A is a D-α-set . This proves (iii).

A is a D-α-set $\Rightarrow$ A= U\V where $U\neq X$, $U\in\alpha O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A= $U\cap(X\backslash V)$ where $U\neq X$, $U\in\alpha O(X,\tau)$ and X\V is closed.

$\Rightarrow$ A is α-locally closed. This proves (iv).

The assertion (v) follows from (i), (ii) and (iv).

Example 2.3.10 Consider the topology $\tau=\{\emptyset,\{a\},X\}$ where X = {a,b,c}. Then it can be verified that {a , b} is a D-α-set but not a D-set.

Proposition 2.3.11 A set B is a D-α-set $\Leftrightarrow$ it is an αD-set

Proof. The set B is a D-α-set $\Rightarrow$ B= U\V where $U\neq X$, $U\in\alpha O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ B= U\V where $U\neq X$, $U\in\alpha O(X,\tau)$ and $V\in\alpha O(X,\tau)$.

$\Rightarrow$ B is an αD-set.

Conversely let B be anαD-set. Then B= U\V where $U\neq X$, $U\in\alpha O(X,\tau)$ and $V\in\alpha O(X,\tau)$.

Since V isα-open , V $\subseteq$ *Int Cl Int* V. We take W= *Int Cl Int* V. Therefore U\W is a D-α-set.

ClearlyU\W = U\ *Int Cl Int* V $\subseteq$ U\V = B.

Now B = U\V = U$\cap$(X\V) . Since X\V is α-closed and since everyα-closed set is semi-closed , we have B = U$\cap$(X\V) = U$\cap$ *Int Cl* (X\V) $\subseteq$ U$\cap$ *Cl Int Cl* (X\V).

= U$\cap$(X\ *Int ClInt* V).

= U$\cap$(X\W) = U\W.

This proves that B = U\W which is a D-α-set.

Corollary 2.3.12 Every α-D-set is a D-α-set.

Proof. The set A is an α-D-set $\Rightarrow$ A= U\V where U$\neq$X, U$\in\tau$ and V$\in\alpha$O(X,τ).

$\Rightarrow$ A= U\V where U$\neq$X, U$\in\alpha$O(X, τ) and V$\in\alpha$O(X,τ).

$\Rightarrow$ A is an αD-set.

$\Rightarrow$ A is a D-α-set, by using Proposition 2.3.11.

The converse of the above proposition is not true as {a, c} is a D-α-set in (X,τ) but not an α-D-set where X = {a,b,c} and τ= {$\emptyset$, {a}, X}.

Proposition 2.3.13 (i) Every proper pre-open set U is a D-pre-set.

(ii) The whole set X is not a D-pre-set.

(iii) Every D-set is a D-pre-set.

(iv) Every D-pre-set is a pD-set.

(v) Every D-α-set is a D-pre-set.

(vi) Every D-pre-set is pre-locally closed.

(vii) The class of all D-pre-sets properly lies between PO(X,τ) and pre-LC(X,τ).

Proof. The first two assertions can be easily shown.

The set A is a D-set $\Rightarrow$ A= U\V where U$\neq$X, U$\in\tau$ and V$\in\tau$.

$\Rightarrow$ A= U\V where U$\neq$X, U$\in$PO(X,τ) and V$\in\tau$.

$\Rightarrow$ A is a D-pre-set .This proves (iii).

The set A is a D-pre-set $\Rightarrow$ A= U\V where U$\neq$X, U$\in$PO(X, τ) and V$\in\tau$.

$\Rightarrow$ A= U\V where U$\neq$X, U$\in$PO(X, τ) and V$\in$PO(X,τ).

$\Rightarrow$ A is an pD-set. This proves (iv).

A is a D-α-set $\Rightarrow$ A= U\V where U$\neq$X, U$\in\alpha$O(X,τ) and V$\in\tau$.

$\Rightarrow$ A= U\V where U$\neq$X, U$\in$PO(X,τ) and V$\in\tau$.

$\Rightarrow$A is a D-pre-set. This proves (v).

A is a D-pre-set $\Rightarrow$ A= U\V where U$\neq$X, U$\in$PO(X,τ) and V$\in\tau$.

$\Rightarrow$ A= U$\cap$(X\V) where U$\neq$X, U$\in$PO(X,τ) and X\V is closed.

$\Rightarrow$ A is pre-locally closed. This proves (vi).

The assertion (vii) follows from (i), (ii) and (vi).

Example 2.3.14 The set {a,d} is a D-pre-set in (X,τ) but not a D-set where X={a,b,c,d} and τ= {$\emptyset$, {d}, {a,b}, {a,b,d}, X }.

Proposition 2.3.15 (i) Every proper semi-open set U is a D-semi-set.

(ii) The whole set X is not a D-semi-set.

(iii) Every D-set is a D-semi-set.

(iv) Every D-semi-set is a sD-set.

(v) Every D-α-set is a D-semi-set.

(vi) Every D-semi-set is semi-locally closed.

(vii) The class of all D-semi-sets lies between SO(X,τ)and semi-LC(X,τ).

Proof. Analogous to Proposition 2.3.13.

Proposition 2.3.16 (i) Every proper b-open set U is a D-b-set.

(ii) The whole set X is not a D-b-set.

(iii) Every D-set is a D-b-set

(iv) Every D-b-set is a bD-set.

(v) Every D-pre-set or D-semi-set is a D-b-set.

(vi) Every D-b-set is b-locally closed.

(vii) The class of all D-b-sets properly lies between $bO(X,\tau)$ and $b\text{-}LC(X,\tau)$.

Proof. The first two assertions can be easily shown.

The set A is a D-set $\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in\tau$ and $V\in\tau$.

$\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in bO(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A is a D-b-set . This proves (iii).

The set A is a D-b-set $\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in bO(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in bO(X,\tau)$ and $V\in bO(X,\tau)$.

$\Rightarrow$ A is an bD-set .This proves (iv).

A is a D-pre-set or D-semi-set $\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in PO(X,\tau)\cup SO(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in bO(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A is a D-b -set. This proves (v).

A is a D-b- set $\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in bO(X,\tau)$ and $V\in\tau$

$\Rightarrow$ A= $U\cap(X\backslash V)$ where $U\neq X$, $U\in bO(X,\tau)$ and $X\backslash V$ is closed.

$\Rightarrow$ A is b-locally closed. This proves (vi). The assertion (vii) follows from (i) , (ii) and (vi).

Proposition 2.3.17 (i) Every proper β-open set U is a D-β-set.

(ii) The whole set X is not a D-β-set.

(iii) Every D-set is a D-β-set

(iv) Every D-β-set is a βD-set.

(v) Every D-b-set is a D-β-set

(vi) Every D-β-set is β-locally closed.

(vii) The class of all D-β-sets properly lies between $\beta O(X,\tau)$ and $\beta\text{-}LC(X,\tau)$.

Proof. The first two assertions can be easily shown.

The set A is a D-set $\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in\tau$ and $V\in\tau$.

$\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in\beta O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A is a D-β-set .This proves (iii).

The set A is a D-β-set $\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in\beta O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in\beta O(X,\tau)$ and $V\in\beta O(X,\tau)$.

$\Rightarrow$ A is an βD-set . This proves (iv).

A is a D-b-set $\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in bO(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in\beta O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$A is a D-β -set. This proves (v).

A is a D-β- set $\Rightarrow$ A= $U\backslash V$ where $U\neq X$, $U\in\beta O(X,\tau)$ and $V\in\tau$.

$\Rightarrow$ A= $U\cap(X\backslash V)$ where $U\neq X$, $U\in\beta O(X,\tau)$ and $X\backslash V$ is closed.

$\Rightarrow$ A is β-locally closed. This proves (vi).

The assertion (vii) follows from (i), (ii) and (vi).

Remark 2.3.18 (i) The null set ∅ is a D-η-set in every topological space.

(ii) The null set ∅is the only D-η-set in the indiscrete space.

(iii) Every proper subset of the discrete space is a D-η-set.

(iv) The whole set can never be a D-η-set in any topological space.

The discussion in this section leads to following implications on various types of difference sets

Diagram 2.3.19

D-set
↧
D-β-set
D-pre-set↲ ↳D-semi-set
↘ ↙
D-b-set

↧
D-β-set

Proposition 2.3.20 Let $\eta\in\{\omega, \alpha, \beta, \text{pre}, \text{semi}, b\}$.
(i) If $U\backslash V$ with $V\neq X$ is an η-D-set then $V\backslash U$ is a D-η-set.
(ii) If $U\backslash V$ with $V\neq X$ is a D-η-set then $V\backslash U$ is an η-D-set.
Proof. Let (U, V) be a pair of proper subsets of X.
Suppose $A = U\backslash V$ is an ω-D-set . Then $U\in\tau$ and $V\in\omega O(X,\tau)$
Therefore $B= V\backslash U$ is D-ω-set. The converse is also true. This proves the proposition for the case $\eta=\omega$. The other cases can analogously discussed.

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