Finite Element Examination of Stress-Strain Fields Near Ductile Crack Tip

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Abstract

We are considering a centre crack in ductile steel plate of finite dimensions. Modeling and analysis of the crack is performed in ANSYS. Non-linear stress-strain data of steel is used in the software. A suitable value of far field tensile stress (pressure) is chosen such that the applied stress intensity parameter is less than the fracture toughness value. Plastic stress-strain fields are obtained near the crack tip in EPFM. Desired values are noted. Areas of high stress and high strain are identified. Validation of void nucleation taking place ahead of crack tip and not exactly at the crack tip and coalescence of voids happening at the crack tip is confirmed with the results. Plane strain case is considered. Plots between the distance of desired location from the crack tip and load line stresses and strains are drawn. The plots are in accordance with the expected ones.

Keywords: Process zone, Plastic zone, Elastic zone, LEFM, EPFM, void nucleation, coalescence.

Nomenclature:

- $A$: Half crack length
- $P$: Applied pressure
- $E$: Young’s Modulus of elasticity
- $K_fK_I$: Stress intensity factor
- $K_{Ic}K_{Ic}$: Fracture toughness
- $\nu, \nu$: Poisson’s ratio
- $W$: Width of plate
- $\sigma_{yy}, \sigma_{yy}$: Stress in Y-direction
- $\epsilon_{\sigma}, \epsilon_{\sigma}$: Equivalent plastic strain
- $T_i$: Traction vector
- $J$: Applied J-integral
- $\sigma_{\sigma}, \sigma_{\sigma}$: Reference stress value equal to yield strength
- $\epsilon_{\sigma}, \epsilon_{\sigma}$: Yield strain
- $\alpha$: Dimensionless constant
- $n$: Strain hardening exponent
- $F(\alpha)F(\alpha)$: Configuration factor
1. Introduction

Fracture is the separation or fragmentation of solid in two or more parts under the action of imposed stress. Ductile fracture is caused mainly by micro-mechanical processes of nucleation, growth and coalescence of voids. Voids nucleate due to cracking of inclusions or interfacial de-cohesion of inclusions from parent material which then grow by plastic deformation of the surrounding material. Fig. 1[1] illustrates micro-void initiation, growth and coalescence at the tip of pre-existing crack. As the crack structure is loaded, local strains and stress at the crack tip become sufficient to nucleate voids. These voids grow as crack blunts and they eventually link with the main crack causing crack growth.

![Fig. 1 Stages of ductile crack growth][1]

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$r_p$ Radius of plastic zone  
SSY Small scale yielding  
LEFM Linear elastic fracture mechanics  
EPFM Elastic-plastic fracture mechanics  
UTS Ultimate tensile strength  
$L$ Uncracked ligament length  
$\Gamma$ Counter-clockwise path around the crack tip  
$r$ Distance of node from crack tip  
$\omega$ Strain energy density  
$u_i u_i$ Displacement vector components  
$ds/ds$ Length increment along the contour  
LSY Large scale yielding  
HRR Hutchinson-Rice-Rosengren  
$L_p$ Length of plate  
$\sigma_{ij}\sigma_{ij}$ Stress tensor  
$\varepsilon_{ij}\varepsilon_{ij}$ Strain tensor
The strain at blunted crack tip exhibits a singularity while the stress reaches the peak value at approximately two times the value of crack tip opening displacement (CTOD) ahead of crack tip. As such, there is sufficient stress elevation ahead of crack tip for void nucleation while most of the void growth and coalescence occurs much closer to the crack tip where strains are high. Characteristics of ductile crack tip zone in terms of stress and strain evoke lot of interest. The paper therefore presents finite element analysis of ductile cracked plate to obtain stress and strain fields near the crack tip and to predict the crack growth features.

2. Problem definition: Details of 1018 hot rolled steel plate are as follows: (Refer Fig. 2)
Plate dimensions:
w = 50 cm, L = 50 cm, 2a = 6 cm
Material Properties: E = 200 GPa, σ_y = 290 N/mm^2, UTS = 430 N/mm^2, ν = 0.3, elongation = 36%
Type of mode: Mode I (opening)
Type of crack: Through centre crack
Applied pressure: P = 150 MPa.

![Fig 2: Centre crack configuration](image)

The Ramberg-Osgood equation applied to plastic stress-strain data of the steel is given by [1]

\[
\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n = \frac{\sigma}{\sigma_y} + \frac{\sigma}{\sigma_0} \left(\frac{\sigma}{\sigma_0}\right)^n = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n
\]

where, ε_0 = (σ_y/E)ε_0 = (σ_y/E), σ_0 = σ_{ys}

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http://philstat.org.ph
from above equation considering different values of stress & strain, we obtain dimensionless constant as

\[ n = 6.02 \quad \alpha = 9.054. \]

Stress intensity factor, \( K_I K_I \) is calculated by,

\[ K_I = \sigma \sqrt{\pi a} \times F(\alpha)K_I = \sigma \sqrt{\pi a} \times F(\alpha) = \sigma \sqrt{\pi a} \times F(\alpha), \]

where \( \alpha = \left( \frac{a}{W} \right)^2 = 0.12 \)

for center crack \( F(\alpha)F(\alpha) \) is given by,

\[ \mathcal{F}(\alpha) = 1.0 + 0.128\alpha - 0.288\alpha^2 + 1.53\alpha^3, \]

\[ \mathcal{F}(\alpha) = 1.0 + 0.128\alpha - 0.288\alpha^2 + 1.53\alpha^3 = 1.013 \]

\[ K_I = 46.648 \text{MPa}\sqrt{\text{m}}, \quad K_I = 46.648 \text{MPa}\sqrt{\text{m}}, \quad K_I = 46.648 \text{MPa}\sqrt{\text{m}} \]

we have for steel is

\[ K_{IC} = 50 \text{MPa}\sqrt{\text{m}}, K_I = 50 \text{MPa}\sqrt{\text{m}}, K_{IC} = 50 \text{MPa}\sqrt{\text{m}} \]

Since, \( K_I < K_{IC} \quad K_I < K_{IC} \quad K_I < K_{IC} \) crack is assumed to be safe.

3. Analysis

The cracked plate is modeled in ANSYS. Half plate is only modelled due to the condition of symmetry about the crack axis. The mesh model of plate is shown in Fig. 4.

8-node quadrilateral elements (PLANE 82) are used. Number of nodes are 63285 and number of elements are 20932. The far field tensile pressure in \( y \)-direction is 150MPa and displacement in \( y \)-direction is 0 for uncracked ligaments. Plane strain condition is employed.

![Stress-Strain curve for 1018 hot rolled steel](image)

Fig 3 : Stress-Strain curve for 1018 hot rolled steel

4. Results

Nodal and elemental stress and strain plots are shown in Fig. 5. Process, plastic and elastic zones are also displayed.
1. Nodal solution (stress): View at ‘A’

![Stress solution image]

3. Nodal solution (strain):

![Strain solution image]

Fig. 5 Stress and strain solutions

The size of plastic zone in small scale yielding (SSY) under plane strain condition is theoretically obtained as [2]

\[ r_p = 2.71 \text{ mm} \]

But the value of \( r_p \) obtained from finite element analysis is 7.53 mm which is almost 3 times (approx.) greater than theoretically \( r_p \) value for SSY. Thus, it is concluded that crack tip does not undergo SSY.
2. Elemental solution (stress):

and elastic-plastic regime (EPFM) is valid

\[ r_p = \frac{1}{3\pi} \left( \frac{K_I}{\sigma_{YS}} \right)^2 \]

The \( J \) contour integral [1] has enjoyed great success as a fracture characterizing parameter in EPFM. Rice applied deformation plasticity (i.e. nonlinear elasticity) to the analysis of a crack in EPFM. He showed that the nonlinear energy release rate \( J \), could be written as a path-independent line integral as follows:

\[ J = \int \left( w d y - T_i \frac{\partial u_i}{\partial x} \right) ds \]

Where \( T_i = \sigma_{ij}n_i \) \& \( w = \int \sigma_{ij}d\varepsilon_{ij} \int \sigma_{ij}d\varepsilon_{ij} \)

\( J \) is obtained as 9.9 N/mm

The equivalent plastic strain responsible for void coalescence is given as [3]

\[ \varepsilon_p = \sqrt{\frac{2}{3} (\varepsilon_{r1})^2 + (\varepsilon_{r2})^2 + (\varepsilon_{r3})^2 + (\varepsilon_{r4})^2 + (\varepsilon_{r5})^2 + (\varepsilon_{r6})^2} \]

Plots of \( \frac{\sigma_{yy}}{\sigma_0}, \frac{\sigma_{yy}}{r_0}, \log \sigma_{yy}, \sigma_{yy} \) v.s. \( \log \left( \frac{r}{r_0} \right) \) & \( \log \left( \frac{K_I}{K_f} \right) \) v.s. \( \log \left( \frac{r}{r_0} \right) \) along crack axis are displayed in Fig. 6, 7 and 8 respectively.
Fig. 6. $\frac{\sigma_{yy}}{\sigma_0}$ v.s. $\frac{r\sigma_0}{f}$

Fig. 7. $\log \sigma_{yy}$ v.s. $\log \left(\frac{f}{T}\right)$ & $\log \frac{K_T}{\sqrt{2\pi r}}$ v.s. $\log \left(\frac{f}{T}\right)$

Fig. 8. $\varepsilon_{eq}$ v.s. $\frac{r\sigma_0}{f}$
5. Discussion

Plot between $\frac{\sigma_{yy}}{\sigma_0}$ v.s. $\frac{r}{a} \frac{\sigma_{yy}}{\sigma_0}$ shows that stress in y direction (At $\theta=0^0$) at the crack tip is less than the value of stress ahead of crack tip in the process zone. The stress then gradually decreases.

Plot between log $\sigma_{yy}$ v.s. log $\frac{r}{a}$ & log $\frac{K_i}{\sqrt{2\pi r}}$ v.s. log $\frac{r}{a}$ shows that the void nucleation starts ahead of crack tip but not at the crack tip. It is because of high stress value obtained ahead of crack tip. Stresses in process zone follows large scale yielding (LSY) and stresses in plastic zone follows HRR solution. But stresses in elastic zone, due to EPFM regime, don’t follow LEFM solution that is K dominated. Hence numerical values and theoretical LEFM values based on K exhibit deviation.

Plot between $\epsilon_{eq} v.s. \epsilon_{eq} \frac{r}{a}$ shows the growth and coalescence zone of voids takes place at the crack tip where strain value is maximum. Plastic strain cause the voids to grow and eventually coalesce with the parent crack tip.

6. Conclusion

Stress and strain fields ahead of ductile crack tip have been obtained numerically. Void nucleation taking place ahead of crack tip and not exactly at the crack tip and coalescence of voids happening at the crack tip is confirmed with the results. Results can further be refined by modeling the bluntness at the crack tip.

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