A Research on Bipolar Valued Vague Normal Subrings of A Ring

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Abstract. In this paper, bipolar valued vague normal subring of a ring is introduced and some properties are discussed. bipolar valued vague normal subring of a ring is a generalized form of vague normal subring of the ring, vague normal subring of the ring is a generalized form of fuzzy normal subring of the ring and fuzzy normal subring of the ring is a generalized form of ring.

Key Words. Fuzzy subset, vague subset, bipolar valued fuzzy subset, bipolar valued vague subset, bipolar valued vague subring, bipolar valued vague normal subring, intersection, product, strongest, height.

Introduction:

In 1965, Zadeh [15] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets etc. Grattan-Guiness [8] discussed about fuzzy membership mapped onto interval and many valued quantities. Vague set is an extension of fuzzy set and it is appeared as a unique case of context dependent fuzzy sets. The vague set was introduced by W.L.Gau and D.J.Buehrer [7]. Lee [9] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [9, 10]. Fuzzy subgroup was introduced by Azriel Rosenfeld [2]. RanjitBiswas [12] introduced the vague groups. Cicily Flora. S and Arockiarani.I [4] have introduced a new class of generalized bipolar vague sets. Anitha.M.S., et.al.[1] defined as bipolar valued fuzzy subgroups of a group and Balasubramanian.A et.al[3] have defined the bipolar interval valued fuzzy subgroups of a group. K.Murugalingam and K.Arjunan[11] have discussed about interval valued fuzzy subsemiring of a semiring and Somasundra Moorthy.M.G.,[13] gave a idea about the fuzzy ring. Bipolar valued multi fuzzy subsemirings of a semiring have been introduced by Yasodara.B and KE.Sathappan[14]. Deepa.B., et.al.[5] defined as bipolar valued vague subrings of a ring. Here, the concept of bipolar valued vague subring of a ring is introduced and estaiblished some results. Particularly, bipolar valued vague normal subring of a ring is introduced in this paper.

1.Preliminaries.

Definition 1.1. [15] Let X be any nonempty set. A mapping M : X $\rightarrow$ [0, 1] is called a fuzzy subset of X.

Definition 1.2. [7] A vague set A in the universe of discourse U is a pair $[t_A, 1-f_A]$,where t_A : U$\rightarrow$ [0, 1] and f_A : U$\rightarrow$ [0, 1] are mappings, they are called truth membership function and false membership function respectively. Here t_A(x) is a lower bound of the grade of membership of x derived from the evidence for x and f_A(x) is a lower bound on the negation of x derived from the evidence against x and $t_A(x) + f_A(x) \leq 1$, for all x$\in$U.

Definition 1.3. [7] The interval [$t_A(x)$, $1-f_A(x)$] is called the vague value of x in A and it is denoted by V_A(x), i.e., $V_A(x) = [\, t_A(x), 1-f_A(x)\,]$.

Example 1.4. A = { < a, [0.5, 0.6] >, < b, [0.7, 0.8] >, < c, [0.4, 0.9] >} is a vague subset of X = {a, b, c }.

Definition 1.5. [9] A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = { < x, $A^+(x)$, $A^-(x)$ >/ x$\in$X}, where A^+ : X$\rightarrow$ [0, 1] and A^- : X$\rightarrow$ [–1, 0]. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.6. A = { < a, 0.5, –0.3 >, < b, 0.4, –0.6 >, < c, 0.4, –0.7 >} is a bipolar valued fuzzy subset of X = {a, b, c }.

Definition 1.7. [5] A bipolar valued vague subset A in X is defined as an object of the form A = { $\langle$ x, [$t_A^+(x), 1-f_A^+(x)$], [$-1-f_A^-(x), t_A^-(x)$] $\rangle$ / x$\in$X }, where t_A^+: X$\rightarrow$ [0, 1], f_A^+ : X$\rightarrow$ [0, 1], t_A^- : X$\rightarrow$ [–1, 0] and f_A^- : X$\rightarrow$ [–1, 0] are mapping such that $t_A(x) + f_A(x) \leq 1$ and $-1 \leq t_A^- + f_A^-$. The positive interval membership degree [$t_A^+(x), 1-f_A^+(x)$] denotes the satisfaction region of an element x to the property corresponding to a bipolar valued vague subset A and the negative interval membership degree [$-1-f_A^-(x), t_A^-(x)$] denotes the satisfaction region of an element x to some implicit counter-property corresponding to a bipolar valued vague subset A. Bipolar valued vague subset A is denoted as A = { $\langle$ x, $V_A^+(x)$, $V_A^-(x)$ $\rangle$ / x$\in$X }, where $V_A^+(x)$,= [$t_A^+(x), 1-f_A^+(x)$] and $V_A^-(x)$ = [$-1-f_A^-(x), t_A^-(x)$].

Note that. [0] = [0, 0], [1] = [1, 1] and [–1] = [–1, –1].

Example 1.8. [A] = { < a, [0.3, 0,6], [–0.6, –0.2] >, < b, [0.3, 0.4], [–0.5, –0.3] >, < c, [0.2, 0.6], [–0.5, –0.2] > } is a bipolar valued vague subset of X = {a, b, c }.

Definition 1.9. [5] Let A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ be two bipolar valued vague subsets of a set X. We define the following relations and operations:

(i) [A] $\subset$ [B] if and only if V_A^+(u) $\leq$ V_B^+(u) and V_A^-(u) $\geq$ V_B^-(u), $\forall$ u$\in$X.

(ii) [A] = [B] if and only if V_A^+(u) = V_B^+(u) and V_A^-(u) = V_B^-(u), $\forall$ u$\in$X.

(iii) [A]$\cap$[B] = { $\langle$ u, rmin (V_A^+(u), V_B^+(u)), rmax (V_A^-(u), V_B^-(u)) $\rangle$ / u$\in$X }.

(iv) [A]$\cup$[B] = { $\langle$ u, rmax (V_A^+(u), V_B^+(u)), rmin (V_A^-(u), V_B^-(u)) $\rangle$ / u$\in$X }. Here rmin (V_A^+(u), V_B^+(u)) = [min { $t_A^+(x), t_B^+(x)$ }, min {$1-f_A^+(x)$, $1-f_B^+(x)$}], rmax (V_A^+(u), V_B^+(u)) = [max { $t_A^+(x), t_B^+(x)$ }, max {$1-f_A^+(x)$, $1-f_B^+(x)$ }], rmin (V_A^-(u), V_B^-(u)) = [min {$-1-f_A^-(x)$, $-1-f_B^-(x)$}, min { $t_A^-(x), t_B^-(x)$ }], rmax (V_A^-(u), V_B^-(u)) = [max {$-1-f_A^-(x)$, $-1-f_B^-(x)$ }, max { $t_A^-(x), t_B^-(x)$ }].

Definition 1.10. [5] Let R be a ring. A bipolar valued vague subset A of R is said to be a bipolar valued vague subring of R (BVVSR) if the following conditions are satisfied,

(i) V_A^+(x-y) $\geq$ rmin{ V_A^+(x), V_A^+(y) }

(ii) V_A^+(xy) $\geq$ rmin{ V_A^+(x), V_A^+(y) }

(iii) V_A^-(x-y) $\leq$ rmax{ V_A^-(x), V_A^-(y) }

(iv) V_A^-(xy) $\leq$ rmax{ V_A^-(x), V_A^-(y) } for all x and y in R.

Example 1.11. Let R = Z_3 = { 0, 1, 2 } be a ring with respect to the ordinary addition and multiplication. Then A = { < 0, [0.5, 0.7], [– 0.8, – 0.5] >, < 1, [0.4, 0.6], [–0.7, –0.4] >, <2, [0.4, 0.6], [–0.7, –0.4] > } is a BVVSR of R.

Definition 1.12. [5] Let A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ be any two bipolar valued vague subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as A×B = { $\langle$ (x, y), $V_{A\times B}^+$(x, y), $V_{A\times B}^-$(x, y) $\rangle$ / for all x in G and y in H } where $V_{A\times B}^+$(x, y) = rmin { V_A^+(x), V_B^+(y) } and $V_{A\times B}^-$(x, y) = rmax { V_A^-(x), V_B^-(y) } for all x in G and y in H.

Definition 1.13. [5] Let A = $\langle V_A^+, V_A^- \rangle$ be a bipolar valued vague subset in a set S, the strongest bipolar valued vague relation on S, that is a bipolar valued vague relation on A is V = {$\langle$ (x, y), V_V^+(x, y), V_V^-(x, y) $\rangle$ / x and y in S} given by V_V^+(x, y) = rmin{ V_A^+(x), V_A^+(y) } and V_V^-(x, y) = rmax { V_A^-(x), V_A^-(y) } for all x and y in S.

Definition 1.14. [5] Let A = $\langle V_A^+, V_A^- \rangle$ be a bipolar valued vague subset of X. Then the height H(A) = $\langle$ H(V_A^+), H(V_A^-) $\rangle$ is defined as H(V_A^+) = rsupV_A^+(x) for all x in X and H(V_A^-) = rinfV_A^-(x) for all x in X.

Definition 1.15. [5] Let A = $\langle V_A^+, V_A^- \rangle$ be a bipolar valued vague subset of X. Then $^{\oplus}$A = $\langle {}^{\oplus}V_A^+, {}^{\oplus}V_A^- \rangle$ is defined as $^{\oplus}V_A^+$(x) = V_A^+(x) + [1] – H(V_A^+) and $^{\oplus}V_A^-$(x) = V_A^-(x) – [1] – H(V_A^-) for all x in X.

Definition 1.16. [6] Let R be a ring. A bipolar valued vague subring A = $\langle V_A^+, V_A^- \rangle$ of R is said to be a bipolar valued vague normal subring of R if V_A^+(xy) = V_A^+(yx) and V_A^-(xy) = V_A^-(yx) for all x and y in R.

2. Theorems.

Theorem 2.1. [5] Let A = $\langle V_A^+, V_A^- \rangle$ be a BVVSR of a ring R. Then V_A^+(–x) = V_A^+(x), V_A^-(–x) = V_A^-(x), V_A^+(x) ≤ V_A^+(e), V_A^-(x) ≥ V_A^-(e), for all x in R, where e is the identity element in R.

Theorem 2.2. Let A = $\langle V_A^+, V_A^- \rangle$ be a BVVNSR of a ring R. Then V_A^+(–x) = V_A^+(x), V_A^-(–x) = V_A^-(x), V_A^+(x) ≤ V_A^+(e), V_A^-(x) ≥ V_A^-(e), for all x in R, where e is the identity element in R.

Proof. The proof follows from the theorem 2.2.

Theorem 2.3. [5] Let A = $\langle V_A^+, V_A^- \rangle$ be a BVVSR of a ring R. (i) If V_A^+(x–y) = [0] then either V_A^+(x) = [0] or V_A^+(y) = [0] for x, y in R. (ii) If V_A^+(xy) = [0] then either V_A^+(x) = [0] or V_A^+(y) = [0] for x, y in R. (iii) If V_A^-(x–y) = [0] then either V_A^-(x) = [0] or V_A^-(y) = [0] for x, y in R. (iv) If V_A^-(xy) = [0] then either V_A^-(x) = [0] or V_A^- (y) = [0] for x, y in R.

Theorem 2.4. Let A = $\langle V_A^+, V_A^- \rangle$ be a BVVNSR of a ring R. (i) If V_A^+(x–y) = [0] then either V_A^+(x) = [0] or V_A^+(y) = [0] for x, y in R. (ii) If V_A^+(xy) = [0] then either V_A^+(x) = [0] or V_A^+(y) = [0] for x, y in R. (iii) If V_A^-(x–y) = [0] then either V_A^-(x) = [0] or V_A^-(y) = [0] for x, y in R. (iv) If V_A^-(xy) = [0] then either V_A^-(x) = [0] or V_A^- (y) = [0] for x, y in R.

Proof. The proof follows from the theorem 2.3.

Theorem 2.5. [5] If A = $\langle V_A^+, V_A^- \rangle$ is a BVVSR of a ring R, then H = {x∈R / V_A^+(x) = [1], V_A^-(x) = [–1] } is either empty or a subring of R.

Theorem 2.6. If A = $\langle V_A^+, V_A^- \rangle$ is a BVVNSR of a ring R, then H = {x∈R / V_A^+(x) = [1], V_A^-(x) = [–1] } is either empty or a subring of R.

Proof. The proof follows from the theorem 2.5.

Theorem 2.7. [5] If A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ are two BVVSRs of a ring R, then their intersection A$\cap$B is a BVVSR of R.

Theorem 2.8. If A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ are two BVVNSRs of a ring R, then their intersection A$\cap$B is a BVVNSR of R.

Proof. Let C = A$\cap$B and let x, y in R. By the theorem 2.7, A$\cap$B is a BVVSR of R. Now V_C^+(xy) = rmin { V_A^+(xy), V_B^+(xy) } = rmin { V_A^+(yx), V_B^+(yx) }=V_C^+(yx), for all x, y in R. And V_C^-(xy) = rmax { V_A^-(xy), V_B^-(xy) } = rmax { V_A^-(yx), V_B^-(yx) } = V_C^-(yx), for all x, y in R. Hence A$\cap$B is a BVVNSR of R.

Theorem 2.9. [5] The intersection of a family of BVVSRs of a ring R is a BVVSR of R.

Theorem 2.10. The intersection of a family of BVVNSRs of a ring R is a BVVNSR of R.

Proof. The proof follows from the Theorem 2.8 and 2.9.

Theorem 2.11. [5] If A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ are any two BVVSRs of the rings R_1 and R_2 respectively, then A×B = $\langle V_{A\times B}^+, V_{A\times B}^- \rangle$ is a BVVSR of $R_1\times R_2$.

Theorem 2.12. If A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ are any two BVVNSRs of the rings R_1 and R_2 respectively, then A×B = $\langle V_{A\times B}^+, V_{A\times B}^- \rangle$ is a BVVNSR of $R_1\times R_2$.

Proof. Let x_1, x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1\times R_2$. By the theorem 2.11, A×B = $\langle V_{A\times B}^+, V_{A\times B}^- \rangle$ is a BVVSR of $R_1\times R_2$. Now, $V_{A\times B}^+[(x_1, y_1)(x_2, y_2)]$ = $V_{A\times B}^+(x_1x_2, y_1y_2)$ = rmin{$V_A^+(x_1x_2)$, $V_B^+(y_1y_2)$}= rmin{$V_A^+(x_2x_1)$, $V_B^+(y_2y_1)$}= $V_{A\times B}^+(x_2x_1, y_2y_1)$ = $V_{A\times B}^+[(x_2, y_2)(x_1, y_1)]$. And $V_{A\times B}^-[(x_1, y_1)(x_2, y_2)]$ = $V_{A\times B}^-(x_1x_2, y_1y_2)$ = rmax{$V_A^-(x_1x_2)$, $V_B^-(y_1y_2)$ } = rmax{$V_A^-(x_2x_1)$, $V_B^-(y_2y_1)$} = $V_{A\times B}^-(x_2x_1, y_2y_1)$ = $V_{A\times B}^-[(x_2, y_2)(x_1, y_1)]$. Hence A×B is a BVVNSR of $R_1\times R_2$.

Theorem 2.13. If A = $\langle V_A^+, V_A^- \rangle$, B = $\langle V_B^+, V_B^- \rangle$, … K = $\langle V_K^+, V_K^- \rangle$ are BVVSRs of the rings R_A, R_B, …, R_K respectively, then A×B×…×K = $\langle V_{A\times B\times..\times K}^+, V_{A\times B\times..\times K}^- \rangle$ is a BVVSR of $R_A\times R_B\times\ldots\times R_K$.

Proof. The proof follows from the Theorem 2.11.

Theorem 2.14. If A = $\langle V_A^+, V_A^- \rangle$, B = $\langle V_B^+, V_B^- \rangle$, … K = $\langle V_K^+, V_K^- \rangle$ are BVVNSRs of the rings R_A, R_B, …, R_K respectively, then A×B×…×K = $\langle V_{A\times B\times..\times K}^+, V_{A\times B\times..\times K}^- \rangle$ is a BVVNSR of $R_A\times R_B\times\ldots\times R_K$.

Proof. Let (a_1, b_1, …, k_1) and (a_2, b_2, …, k_2) are in $R_A\times R_B\times\ldots\times R_K$. By the theorem 2.13, A×B×…×K = $\langle V_{A\times B\times..\times K}^+, V_{A\times B\times..\times K}^- \rangle$ is a BVVSR of $R_A\times R_B\times\ldots\times R_K$. Now, $V_{A\times B\times..\times K}^+[(a_1, b_1, \ldots, k_1)(a_2, b_2, \ldots, k_2)]$ = $V_{A\times B\times..\times K}^+(a_1a_2, b_1b_2, \ldots, k_1k_2)$ = rmin{ $V_A^+(a_1a_2)$, $V_B^+(b_1b_2)$,

$\ldots, V_K^+(k_1k_2)\}$ = rmin$\{$ $V_A^+(a_2a_1)$, $V_B^+(b_2b_1)$, $\ldots, V_K^+(k_2k_1)\}$ = $V_{A\times B\times..\times K}^+(a_2a_1, b_2b_1, \ldots, k_2k_1)$ = $V_{A\times B\times..\times K}^+[(a_2, b_2, \ldots, k_2)(a_1, b_1, \ldots, k_1)]$. And $V_{A\times B\times..\times K}^-[(a_1, b_1, \ldots, k_1)(a_2, b_2, \ldots, k_2)]$ = $V_{A\times B\times..\times K}^-(a_1a_2, b_1b_2, \ldots, k_1k_2)$ = rmax $\{$ $V_A^-(a_1a_2)$, $V_B^-(b_1b_2)$, $\ldots, V_K^-(k_1k_2)$ $\}$= rmax $\{V_A^-(a_2a_1)$, $V_B^-(b_2b_1)$, $\ldots, V_K^-(k_2k_1)$ $\}$= $V_{A\times B\times..\times K}^-(a_2a_1, b_2b_1, \ldots, k_2k_1)$ = $V_{A\times B\times..\times K}^-[(a_2, b_2, \ldots, k_2)(a_1, b_1, \ldots, k_1)]$. Hence A×B×…×K is a BVVNSR of $R_A\times R_B\times\ldots\times R_K$.

Theorem 2.15. [5] The product of a family of BVVSRs of rings R_i is a BVVSR of $R_1\times R_2\times\ldots$.

Theorem 2.16. The product of a family of BVVNSRs of rings R_i is a BVVNSR of $R_1\times R_2\times\ldots$.

Proof. The proof follows from the Theorem 2.14 and 2.15.

Theorem 2.17. [5] Let A = $\langle V_A^+, V_A^- \rangle$ be a BVVSS of a ring R and V = $\langle V_V^+, V_V^- \rangle$ be the strongest bipolar valued vague relation of R. Then A is a BVVSR of R if and only if V is a BVVSR of R×R.

Theorem 2.18. Let A = $\langle V_A^+, V_A^- \rangle$ be a BVVSS of a ring R and V = $\langle V_V^+, V_V^- \rangle$ be the strongest bipolar valued vague relation of R. Then A is a BVVNSR of R if and only if V is a BVVNSR of R×R.

Proof. Suppose that A is a BVVNSR of R. Then for any x = (x_1, x_2), y = (y_1, y_2) are in R×R. By the theorem 2.17, A is a BVVSR of R if and only if V is a BVVSR of R×R. Now $V_V^+(xy)$ = $V_V^+[(x_1, x_2)(y_1, y_2)]$ = $V_V^+(x_1y_1, x_2y_2)$ = rmin $\{V_A^+(x_1y_1)$, $V_A^+(x_2y_2)$ $\}$= rmin $\{V_A^+(y_1x_1)$, $V_A^+(y_2x_2)$ $\}$= $V_V^+(y_1x_1, y_2x_2)$ = $V_V^+[(y_1, y_2)(x_1, x_2)]$ = $V_V^+(yx)$, for all x and y in R×R. And $V_V^-(xy)$ = $V_V^-[(x_1, x_2)(y_1, y_2)]$ = $V_V^-(x_1y_1, x_2y_2)$ = rmax$\{$ $V_A^-(x_1y_1)$, $V_A^-(x_2y_2)$ $\}$ = rmax $\{V_A^-(y_1x_1)$, $V_A^-(y_2x_2)$ $\}$= $V_V^-(y_1x_1, y_2x_2)$ = $V_V^-[(y_1, y_2)(x_1, x_2)]$ = $V_V^-(yx)$ for all x, y in R×R. This proves that V is a BVVNSR of R×R. Conversely assume that V is a BVVNSR of R×R, then for any x = (x_1, x_2) and y = (y_1, y_2) are in R×R, we have rmin $\{$ $V_A^+(x_1y_1)$, $V_A^+(x_2y_2)$ $\}$ = $V_V^+(x_1y_1, x_2y_2)$ = $V_V^+[(x_1, x_2)(y_1, y_2)]$ = $V_V^+(xy)$ = $V_V^+(yx)$ = $V_V^+[(y_1, y_2)(x_1, x_2)]$ = $V_V^+(y_1x_1, y_2x_2)$ = rmin $\{$ $V_A^+(y_1x_1)$, $V_A^+(y_2x_2)$ $\}$, if $x_2 = y_2 = e$, we get $V_A^+(x_1y_1) = V_A^+(y_1x_1)$, for all x_1 and y_1 in R. Also we have rmax $\{V_A^-(x_1y_1)$, $V_A^-(x_2y_2)$ $\}$ = $V_V^-(x_1y_1, x_2y_2)$ = $V_V^-[(x_1, x_2)(y_1, y_2)]$ = $V_V^-(xy)$ = $V_V^-(yx)$ = $V_V^-[(y_1, y_2)(x_1, x_2)]$ = $V_V^-(y_1x_1, y_2x_2)$ = rmax $\{V_A^-(y_1x_1)$, $V_A^-(y_2x_2)$ $\}$, if $x_2 = y_2 = e$, we get $V_A^-(x_1y_1) = V_A^-(y_1x_1)$, for all x_1 and y_1 in R. Hence A is a BVVNSR of R.

Theorem 2.19. [5] If A = $\langle V_A^+, V_A^- \rangle$ is a BVVSR of a ring R, then $^{\oplus}$A = $\langle {}^{\oplus}V_A^+, {}^{\oplus}V_A^- \rangle$ is a BVVSR of the ring R.

Theorem 2.20. If A = $\langle V_A^+, V_A^- \rangle$ is a BVVNSR of a ring R, then $^{\oplus}$A = $\langle {}^{\oplus}V_A^+, {}^{\oplus}V_A^- \rangle$ is a BVVNSR of the ring R.

Proof. Let x and y in R. By the theorem 2.19, $^{\oplus}$A = ⟨ $^{\oplus}V_A^+$, $^{\oplus}V_A^-$ ⟩ is a BVVSR of the ring R. Now $^{\oplus}V_A^+$ (xy) = V_A^+ (xy) + [1] – H(V_A^+) = V_A^+ (yx) + [1] – H(V_A^+) = $^{\oplus}V_A^+$ (yx), for all x, y in R. And $^{\oplus}V_A^-$ (xy) = V_A^- (xy) – [1] – H(V_A^-) = V_A^- (yx) – [1] – H(V_A^-) = $^{\oplus}V_A^-$ (yx) for all x, y in R. Hence $^{\oplus}$A is a BVVNSR of R.

Theorem 2.21. [5] Let A = ⟨ V_A^+ , V_A^- ⟩ be a BVVSR of a ring R. Then (i) H(V_A^+) = [1] if and only if $^{\oplus}V_A^+$ (x) = V_A^+ (x) for all x in R

(ii) H(V_A^-) = [–1] if and only if $^{\oplus}V_A^-$ (x) = V_A^- (x) for all x in R.

(iii) $^{\oplus}V_A^+$ (x) = [1] if and only if H(V_A^+) = V_A^+ (x) for all x in R

(iv) $^{\oplus}V_A^-$ (x) = [–1] if and only if H(V_A^-) = V_A^- (x) for all x in R.

(v) $^{\oplus}$ ($^{\oplus}$A) = $^{\oplus}$A.

Theorem 2.22. Let A = ⟨ V_A^+ , V_A^- ⟩ be a BVVNSR of a ring R. Then (i) H(V_A^+) = [1] if and only if $^{\oplus}V_A^+$ (x) = V_A^+ (x) for all x in R

(ii) H(V_A^-) = [–1] if and only if $^{\oplus}V_A^-$ (x) = V_A^- (x) for all x in R.

(iii) $^{\oplus}V_A^+$ (x) = [1] if and only if H(V_A^+) = V_A^+ (x) for all x in R

(iv) $^{\oplus}V_A^-$ (x) = [–1] if and only if H(V_A^-) = V_A^- (x) for all x in R.

(v) $^{\oplus}$ ($^{\oplus}$A) = $^{\oplus}$A.

Proof. The proof follows from the theorem 2.21.

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