Heat Generation and Absorption Effects on Thin Film Flow of an Unsteady Casson Fluid Across a Penetrable Flat Plate

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Article Info
Page Number: 2006-2023
Publication Issue:
Vol. 72 No. 1 (2023)

Article History
Article Received: 15 November 2022
Revised: 24 December 2022
Accepted: 18 January 2023

ABSTRACT

The present study describes the motion of a thin film Casson liquid through an infinite flat plate over a penetrable medium with the effect of non-uniform, heat source/sink, temperature radiation and uniform magnetic field. By the similarity transformation, the considered PDE's are transferred into suitableODE's . The R-K method of 4^{th} order with shooting method is implemented to solve, the obtained dimensionless ODE's with the given frontier constraints. The obtained Nusselt number and skin friction results are compared with the earlier published researches. Physical behaviour of different parametric quantities represented are listed and compared with the previously published results. The integrated property of thermal radiation parameter with space and temperature dependent parameters(A* and B*) are effective in managing the rate of heat transfer and increase in Hartmann number(M) increases the temperature where as, in the case of velocity field, rate of thermic transfer and wall friction the inverse upshot has been very practical. and also increase in casson parametric quantity decreases velocity profile which enhances temperature profiles within the boundary region.

Key words: Thin fluid film; Heat generation/absorption; Hartmann number; Casson fluid; Penetrable flat plate.

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1. Introduction

A layer formed from the specified material which ranges from nano meter to several micrometers in width is a thin film. Since from, last few century, a lot of work has been done on thin film fluid flows as it has many significance and importance in industry, engineering and technological field. In the period of twentieth century, the technique of depositing thin film has been accredited in large scale of technical developments in the region of electronic transistor, semiconducting material and embedded passive components, LED's optical coat, striating of food stuff, exclusion of alloy and polymer, constant framing, fluidisation of the device, elastic drawing sheets, chemical analysis equipments, energy generation in the form of solar cells and thin film batteries for storage. By seeing such importance, the author given more attention towards working on, evaluation of thin fluid film over an elongating surface. An interpretation of thin film flow is usually preferred to increase, the external properties of the firms. In strengthening nanotechnology also, thin film acts an an important part. crane[1] introduced the motion of a thin Newtonian fluid on a extended plane and a characteristics of a flow in the liquid thin film recovering over the stretched plane was developed by Anderson et al[2]. Chen[3,4] got the numerical solution of thermal energy transfer and velocity of flow owing an unsteady stretching sheet over a thin fluid film in a power law liquid. Abel et al[5] described the influence of Lorentz force and frictional heating to describe the profiles of temperature and velocity of motion over an unsteady stretching surface on a liquid thin film. Aziz et.al[6] have obtained the HAM solution to test the velocity and motion of a heat in a thin liquid film on an unsteady stretching sheet to study the impact of temperature inside by specifying a general temperature on the surface. Vajravelu et al[7,8] described an impact of heat depending on thermal conductivity and viscous heating on an unsteady motion and temperature transformation over, thin film of power law fluid on the permeable surface with stretching. Khader and megahed[9] have obtained the numerical solutions for velocity and temperature using, an implicit finite difference method to test an impact of radiative heat energy transport over the motion of a thin film fluid resting on the elongating sheet through permeable medium.

In thermal oxidization, internal energy sources of temperature transmission in liquid saturated penetrable median is very important. Groşan et.al [10,11] described theoretically that fluid transmission on a vertical plane in inconsistent surface temperature situated in a fluid-saturated permeable median in the non-Newtonian liquid having the constant internal heat source . A.J.Chamkha et.al[12,13] described the inconsistent temperature and mass transformation by mixed transmission of liquid on a perpendicular penetrable cone revolving in a surrounding liquid with the time-dependent angular velocity in the existence of a magnetic field and heat generation/absorption impacts. Mahdy[14,15] has studied the combined effect of temperature and mass transformation over double-diffusive transmission nearby the perpendicular frustum, in the liquid saturated penetrable median, in existence of the first order chemical reaction and temperature generation/absorption with irregular viscidity.

G. Sarojamma et al[16] have studied the degeneration of the narrow viscid liquid film over an elongating sheet inserted in a penetrable median subjected to the magnetic field and thermocapillarity effect by considering temperature radiation and internal heat generation/absorption. Several researches are made for introducing an effect which are proportionally had a remarkable results regarding the study of soap films. Recently, it has been investigated widely on an another symmetric liquid films like saturated and false amalgamation films on solids . Reliability of narrow liquid layer is studied in the form of, dis joining strain isograms, whose shape replicates the character of layer. Thin film liquid flow on the extended plane of inconsistent liquid characteristics with the impact of thermophoresis have studied by Liaqat ali et al[17], in thermal radiation, transversal electromagnetic field is also applicable to the liquid motion. Researches of Refs.[18-29] should be cognizant of revising the liquid characteristics, of the thin film flow of an inconsistent flat plate. And the resultant liquid motion are, linear and nonlinear diversity of liquid transmission with the boundary conditions.

The Researchers [30-36] are continued, study of power law liquids and Casson liquids of earlier published researches. By referring the above researches it is confirmed that, the characteristics of radiation, heat generation and absorption and magneto hydrodynamics of inconsistent Casson thin film flow over permeable flat plate has not yet been discussed by any of the researchers. And the current study describes, the difficulty of two dimensional motion neighbouring to frontier stratum and thermic transfer over an inconsistent motion of the thin film, Casson liquid on a permeable flat plate with different constraints.

2. Mathematical Formulation

Following figure reflects the Casson fluid thin film flow over an inconsistent permeable flat plate. Movement of the flat plate creates unstable movement of the liquid inside the film.

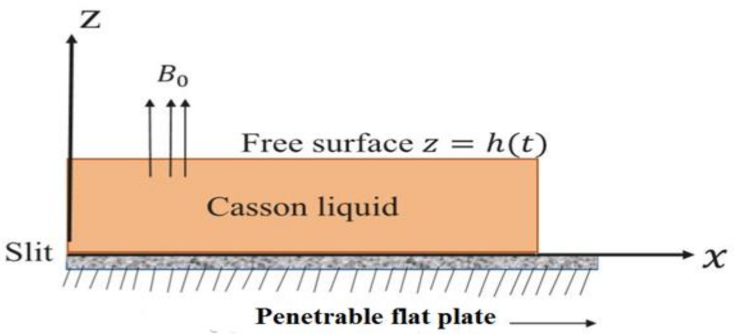

Fig. 1. Geometrical sketch of the flow problem.

In the beginning, considering the prescribed point the expanded velocity is given by

$$\mathrm{U(x,t)} = \frac{\mathrm{bx}}{1-\alpha\mathrm{t}} \quad (1)$$

α and β are the variables represented for measurement and time. Considering the heat transfer in the flat plate from the gas h at the difference of length x as below

$$T_0 - T_S(x,t) - = T_{ref}\left[\frac{bx^2}{2v}\right](1-\alpha t)^{-\frac{3}{2}} \quad (2)$$

The current flow problem is represented by the physicochemical equation as below(See Ref.[27]).

$$\tau_{pq} = \begin{cases} 2\left(\xi_B + \frac{R_y}{\sqrt{2\pi}}\right) e_{pq}, & \phi > \phi_c \\ 2\left(\xi_B + \frac{R_y}{\sqrt{2\pi_c}}\right) e_{pq}, & \phi < \phi_c \end{cases}$$
(3)

At this point, value of ϕ_c dependent on $\phi = e_{pq}e_{pq}$ and that will be the element of rate of distortion e_{pq} which merely the representation of Non- Newtonian liquid, The effective density is represented by ξ_B , yield stress and stress tensor are represented by R_y *and* τ_{pq} respectively.

Approximate boundary value governing to the unstable stretching sheet of Casson liquid film is given by the equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
(4)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1 + \frac{1}{\lambda}\right)\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B^2 u}{\rho} - \frac{\nu}{K}u$$
(5)

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(1 + \frac{1}{\lambda}\right)\left(\frac{\partial u}{\partial y}\right)^2 + \frac{16\sigma^* T_\infty^{\,3}}{3\rho C_p k^*}\frac{\partial^2 T}{\partial y^2} + \frac{q'''}{\rho C_p}$$
(6)

Here, the parameters u and v respectively represents the velocity of the fluid in horizontal and vertical directions, μ represents the dynamic viscosity, $k$denotes thermal conductivity, k^* represents absorption coefficient, σ denotes electrical conductivity, $\lambda = \mu_B\frac{\sqrt{2\pi_c}}{P_y}$ is the casson constraint , C_p denotes specific temperature at constant pressure, σ^* represents Stefen Boltzman constant, T and ρ represents temperature and fluid density .

Equation (7) represents the heat generation/absorption coefficient

$$q''' = \left\{\frac{[A^* f'(T_s - T_0) + B^*(T - T_0)]ku_w(x)}{xv}\right\} \quad (7)$$

Where A^* and $B*$ respectively represents the space and temperature dependents, $A^* < 0$ *and* $B^* < 0$ denotes the case of heat sink where as the heat source is represented by $A^* > 0$ *with* $B^* > 0$.

The suitable boundary conditions for the current flow problem is given below

$$T = T_s,\ \ u = U,\ \ v = 0\ \ at\ \ y = 0 \quad (8)$$

$$\frac{\partial T}{\partial y} = \frac{\partial u}{\partial y} = 0\ \ at y = h \quad (9)$$

$$v = \frac{dh}{dt}\ \ at\ \ y = h$$
(10)

Here it is demonstrated that the problem constructed is completely generated for $x > 0$ only, and ensure to overcome the difficulties on the surface waves, the surface is considered to be smooth and planar. The heat flux and viscous shear stress is represented by $q = -k\left(\frac{\partial T}{\partial y}\right)$ $and\ \tau = \mu\left(\frac{\partial u}{\partial y}\right)_{y=h}$.

Below equations represents the similarity transformation,

$$\eta = \left[\frac{b}{v(1-\alpha t)}\right]^{\frac{1}{2}} y, \tag{11}$$

$$\psi = x\left[\frac{bv}{1-\alpha t}\right]^{\frac{1}{2}} f(\eta), \tag{12}$$

$$T = T_o - T_{ref}\left[\frac{bx^2}{2v(1-\alpha t)^{\frac{3}{2}}}\right]\theta(\eta), \text{where} \theta(\eta) = \frac{T-T_0}{T_s-T_0} \tag{13}$$

u and v the velocity components and $\psi(x, y)$ the stream function is obtained by

$$u = \frac{\partial\psi}{\partial y} = \frac{bx}{1-\alpha t} f'(\eta) \text{ and } v = -\frac{\partial\psi}{\partial x} = -\left(\frac{bv}{1-\alpha t}\right)^{\frac{1}{2}} f(\eta) \tag{14}$$

The problem articulations represented in equations (4-6) are converted to the Non-linear ordinary differential equations are given below.

$$S\left(f' + \frac{\eta}{2} f''\right) + (f')^2 + (M + D)f' - ff'' = \left(1 + \frac{1}{\lambda}\right) f''' \tag{15}$$

$$f\theta' - 2f'\theta - \frac{S}{2}(\eta\theta' + 3\theta) + Ec\left(1 + \frac{1}{\lambda}\right)(f'')^2 + \frac{1}{Pr}[A^* f + B^*\theta] = \frac{1}{Pr}\left(\frac{4}{3} Nr - 1\right)\theta'' \tag{16}$$

Along the boundary conditions as

$$\theta(0) = 1, f(0) = 0 \text{ and } f'(0) = 1 \tag{17}$$

$$\theta'(\beta) = 0 \text{ and } f''(\beta) = 0 \tag{18}$$

$$f(\beta) = S\beta/2 \tag{19}$$

Here, $S \equiv {}^{\alpha}/_{b}$ represents the unsteadiness parameter , $M = \frac{\sigma {B_0}^2}{\rho b}$ denotes Hartmann number , porosity constraint$D = \frac{v}{k\prime b}(1 - \alpha t)$, Prandtl number $Pr = \frac{\rho C_p v}{k}$, thermal radiation constraint $Nr = \frac{4\sigma^* {T_\infty}^3}{k\, k^*}$ and Eckert number $Ec = \frac{U^2}{C_p(T_s - T_0)}$, the value of the similarity variable η is denoted by the unknown parameter β at the free surface and the rate of film thickness is shown as below

$$\beta = h\sqrt{\frac{b}{v}}(1 - \alpha t)^{-\frac{1}{2}} \quad (20)$$

$$\frac{\alpha\beta}{2}\sqrt{\frac{v}{b}}(1 - \alpha t)^{\frac{1}{2}} = -\frac{dh}{dt} \quad (21)$$

The profiles $-f''(0)$ and the temperature transformation of convective to conductive ratio $-\theta'(0)$ are represented by $C_{fx}\ and\ Nu_x$ respectively that approximates the rate of heat transfer and drag force on the surface are given beneath

$$2\left(1 + \frac{1}{\lambda}\right)f''(0) = -C_{fx}{Re_x}^{\frac{1}{2}}, \quad (22)$$

and $$\theta'(0) = Nu\, {Re_x}^{-\frac{1}{2}} \quad (23)$$

Where, the local Reynolds number is, $Re_x = Ux/v$

3. Numerical solution

Integrated Non-linear Ordinary differential equations mentioned in eqns (15-16) along the satisfactory boundary conditions mentioned in eqns(17-19) are derived for heat and velocity profiles $\theta(\eta)\ \ and\ f'(\eta)$ respectively using R-K Fehlberg method with shooting technique. For the above solution, higher order ordinary differential equations are transferred to the first order ordinary differential equations given below.

$$\frac{df_0}{d\eta} = f_1 \quad (24)$$

$$\frac{df_1}{d\eta} = f_2 \quad (25)$$

$$S\left(f_1 + \frac{\eta}{2}f_2\right) + {f_1}^2 - f_0 f_2 + (M + D)f_1 = \left(1 + \frac{1}{\lambda}\right)\frac{df_2}{d\eta} \quad (26)$$

$\frac{d\theta_0}{d\eta} = \theta_1$
(27)

$Pr\left(\frac{S}{2}(3\theta_0 + \eta\theta_1) + 2\theta_0 f_1 - \theta_1 f_0 - Ec\left(1+\frac{1}{\lambda}\right){f_2}^2\right) - A^* f_0 - B^*\theta_0 = \left(1+\frac{4}{3}Nr\right)\frac{d\theta_1}{d\eta}$
(28)

The corresponding Boundary conditions assumes the form as follows

$\theta_0(0) = 1,\ f_0(0) = 0,\ \ f_1(0) = 1$
(29)

$\theta_1(\beta) = 0,\ f_0(\beta) = S\beta/2,\ \ f_2(\beta) = 0$
(30) Here taking $\theta(\eta) = \theta_0(\eta)$ and $f(\eta) = f_0(\eta)$ for performing the solution numerically using the conditions $\theta_1(0)$ and $f_2(0)$ initially. By opting the appropriate presumptions of the interval 0.01, number of approximations were carried out and integrated. The above mentioned boundary conditions eqn 18, satisfies the resulting β value with the lenience of 10^{-7}. and the obtained results have been arranged and compared with properties of flow and thermic transfer in Table 1, table, and table 3.

Table 1: shows the Comparitive consequences of profile$-f''(0)\ for\ M = 0$ and film thickness β as β → ∞ for an inconsistent parameter S for the inaccuracy of lenience10^{-8}.

S	Wang[30]		Megahed [31]		Kalyani et al[32]		Present study	
	β	$-f''(0)$	β	$-f''(0)$	β	$-f''(0)$	β	$-f''(0)$
0.4	4.981450	1.134096	5.122490	1.307785	4.981455	1.134098	4.981448	1.19677904
0.6	3.131710	1.195126	3.131250	1.195155	3.131710	1.195128	3.131710	1.21744369
0.8	2.151994	1.245806	2.151990	1.245795	2.151990	1.245805	2.151990	1.23784971
1.0	1.543616	1.277769	2.543620	1.277762	1.543617	1.277769	1.543616	1.25799643
1.2	1.127781	1.279172	1.127780	1.279177	1.127780	1.279171	1.127780	1.27788890
1.4	0.821032	1.233545	0.821032	1.233549	0.821033	1.233545	0.821033	1.29753250
1.6	0.576173	1.114938	0.576173	1.491137	0.576176	1.114937	0.576176	1.31693527
1.8	0.356389	0.867414	0.356383	0.867414	0.356390	0.867416	0.356390	1.33609566

Table 2: shows the Comparative consequences of $-\left(1+\frac{1}{\lambda}\right)f''(0)\ and\ -\theta'(0)$ for the inaccuracy of lenience10^{-8}.

S	λ	M	Kalyani et al [32]		Present results	
			$-\left(1+\frac{1}{\lambda}\right)f''(0)$	$-\theta'(0)$	$-\left(1+\frac{1}{\lambda}\right)f''(0)$	$-\theta'(0)$

0.8			2.473527	1.355718	2.50341116	0.47073398
1.0	0.5	0.5	2.504972	1.409314	2.59394915	0.56141444
1.2			2.478698	1.450178	2.68156421	0.63299825
1.4			2.365328	1.463985	2.76655711	0.69337347
	1.0		1.577477	1.289227	2.04553902	0.47226884
0.5	2.0	0.5	1.585218	1.276834	1.77172679	0.47292167
	3.0		1.591939	1.269177	1.67043460	0.47300301
	4.0		1.596655	1.264258	1.61740280	0.47299459
		0.0	2.157798	1.365686	2.17840898	0.50714984
0.5	0.5	1.0	2.752908	1.346290	2.78884729	0.44032952
		2.0	3.239791	1.327678	3.28431898	3899588552
		3.0	3.662303	1.309009	3.71354914	0.34795885

Table 3: shows the comparative results of $-\theta'(0)\ for\ S = 0.8$ with various constraint at $D = 0$with an error lenience of 10^{-16}

Pr	Ec	Nr	$A*$	$B*$	$-\theta'(0)$ Present results
0.7					0.9046063997738008
1.0	0.1	0.5	0.5	0.5	1.1186002872168028
2.0					1.6698174025017352
3.0					2.0964865663263215
	0.0				1.1643828770992193
1.0	1.0	0.5	0.5	0.5	0.7065569740949929
	2.0				0.24873104492238055
	3.0				0.20909488177119873
		0.5			1.1186002872168028
1.0	0.1	1.0	0.5	0.5	0.9156571594318765
		1.5			0.7865060510100959

		2.0			0.6954794446052055
1.0	0.1	0.5	−0.5	0.5	1.1047458586867287
			0.0		1.1047458585644556
			0.5		1.1047458585088834
			1.0		1.1047458584927620
1.0	0.1	0.5	0.5	−0.5	1.1047458588576420
				0.0	1.1047458586760088
				0.5	1.1047458585088834
				1.0	1.1047458583587242

4. Results and Discussion:

For the proper values of various parameters of the ordinary differential equations introduced in equations(15-16) on the temperature and velocity profiles $\theta(\eta)$ and $f'(\eta)$ respectively are discussed in the present flow problem numerically and the results obtained are demonstrated by figures (2-11). The present outcomes are verified with the results of Wang et.al.[30], Megahed et.al[31] and Kalyani [32].

Figure 2 describes that the velocity profile $f'(\eta)$ decreases with the increase in the film thickness β in the liquid film with the constant values of $S = 0.8$. In the figure 3, it is observed that increase in the value of M creates resistance in the movement of liquid with an $S = 0.8$, which decreases the amount of film thinning. Figure 4 depicts the accelerated fluid for higher values of S and this higher velocity takes place in the entire region and make film thinner. Figure 5 depicts that increase in the porous parameter D, decreases the movement of liquid , and because of this increase in D, film thinning process decreases with S = 0.8 .

Figure 6 demonstrate the increasing values of the radiation parameter Nr shows noticeable increment in temperature with S=0.8. Figure 7 represents decrease in the thermal boundary layer for the higher values of Prandtl number Pr for $S = 0.8$. Figure 8 describes for higher values of Eckert number *Ec*, temperature profile decreases , where as figure 9 shows the temperature profile, which results for higher values of Casson parameter λ, decreases the amount of film thinning.

Figure 10 and 11 shows the impact of a heat generation/absorption on temperature profile $\theta(\eta)$ for the constant values of an instability constraint $S = 0.8$. The internal heat generator accelerates temperature $\theta(\eta)$ for $A^* > 0$ *and* $B^* > 0$ where as the temperature $\theta(\eta)$ shows decrement for $A^* < 0$ and $B^* < 0$which acts as the temperature absorber that decreases film thickness .

5. Conclusion :

Present study describes, the impact of heat generation/absorption on magneto hydrodynamics and the effect of radiation on, thin film flow of Casson liquid motion over penetrable flat plate. Numerical simulation of the flow problems are achieved and compared with the previously published results. Below conclusions are obtained from the present study,

- Casson parameter and porosity both have the decreasing effect in the process of film thinning.
- In the liquid film, temperature increases for higher values of Casson constraint, Hartmann number and porosity constraint.
- Temperature in the fluid film is decreased by thermal radiation at certain height.

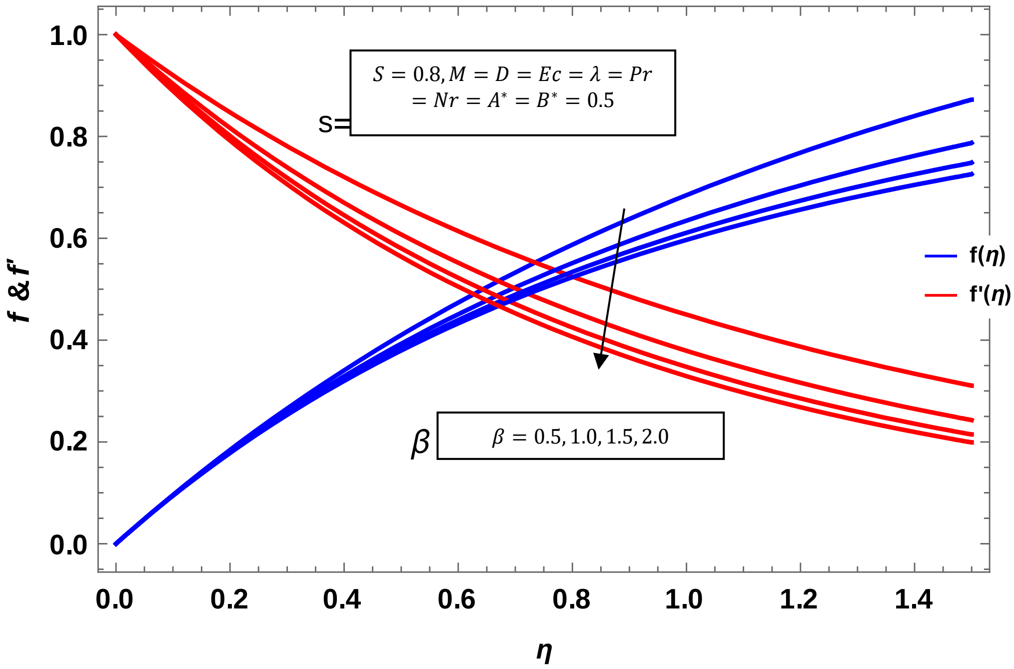

Fig.2.Effect of film thickness βon f and $f'(\eta)$for S=0.8

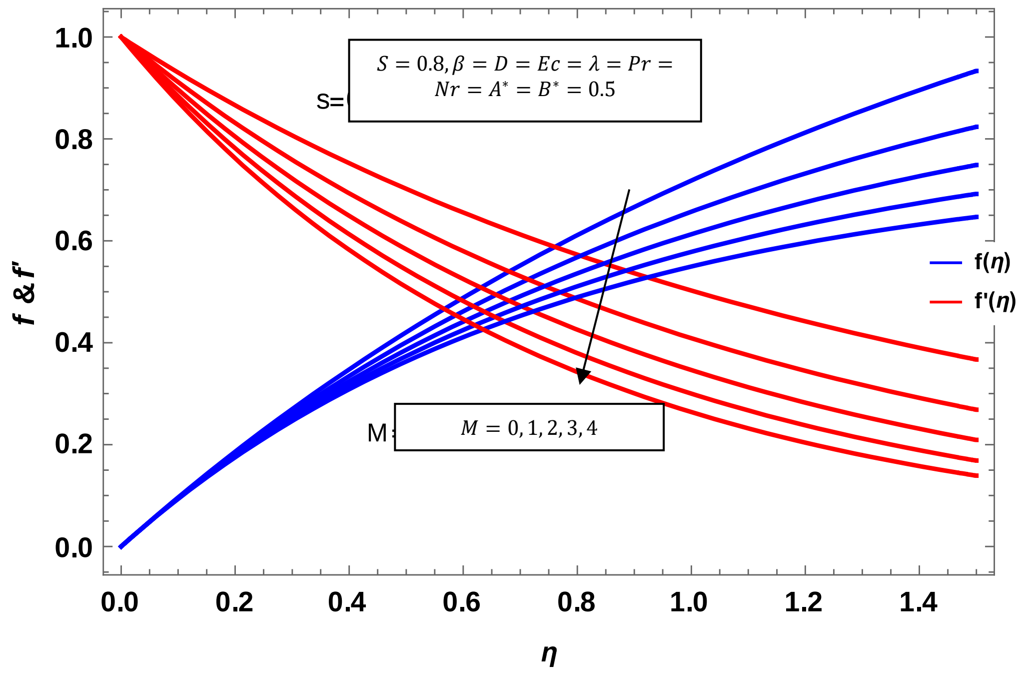

Fig.3.Effect of Hartmann number Mon$f$ and $f'(\eta)$ for S = 0.8

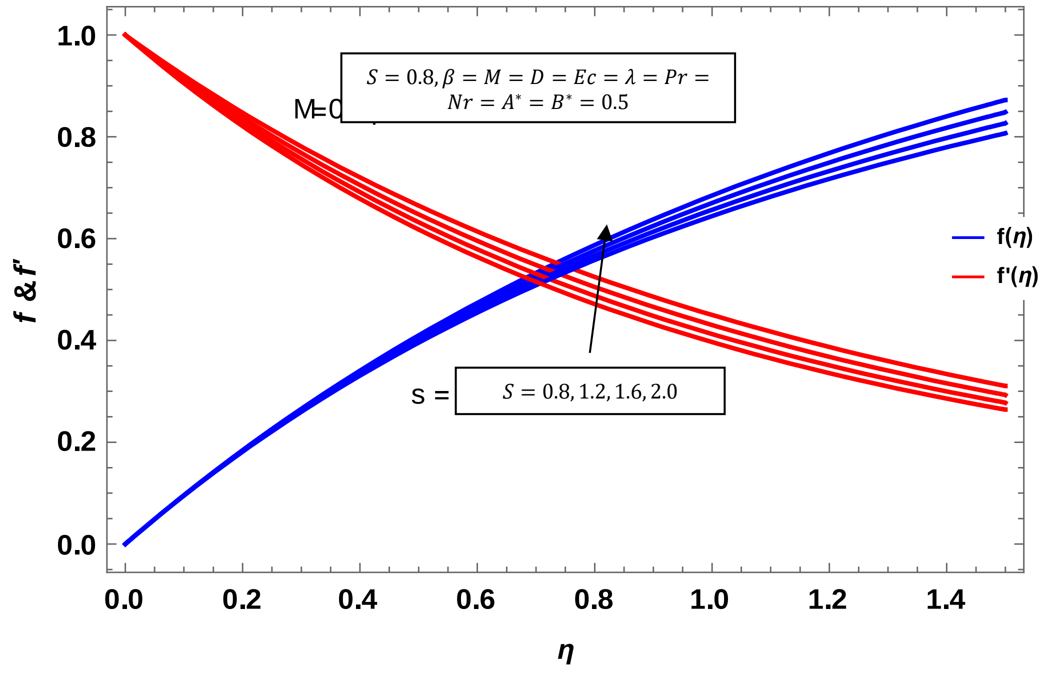

Fig 4 : Effect of unsteadiness parameter Son f and $f'(\eta)$

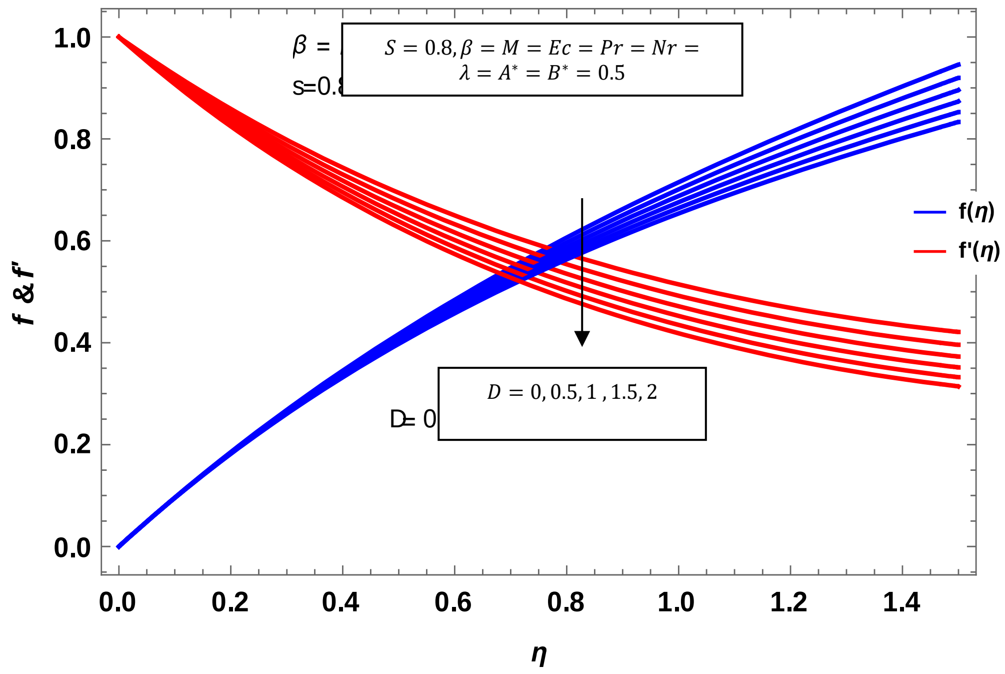

Fig.5. Impact of D, the permeable parameter over f and $f'(\eta)$ for S = 0.8

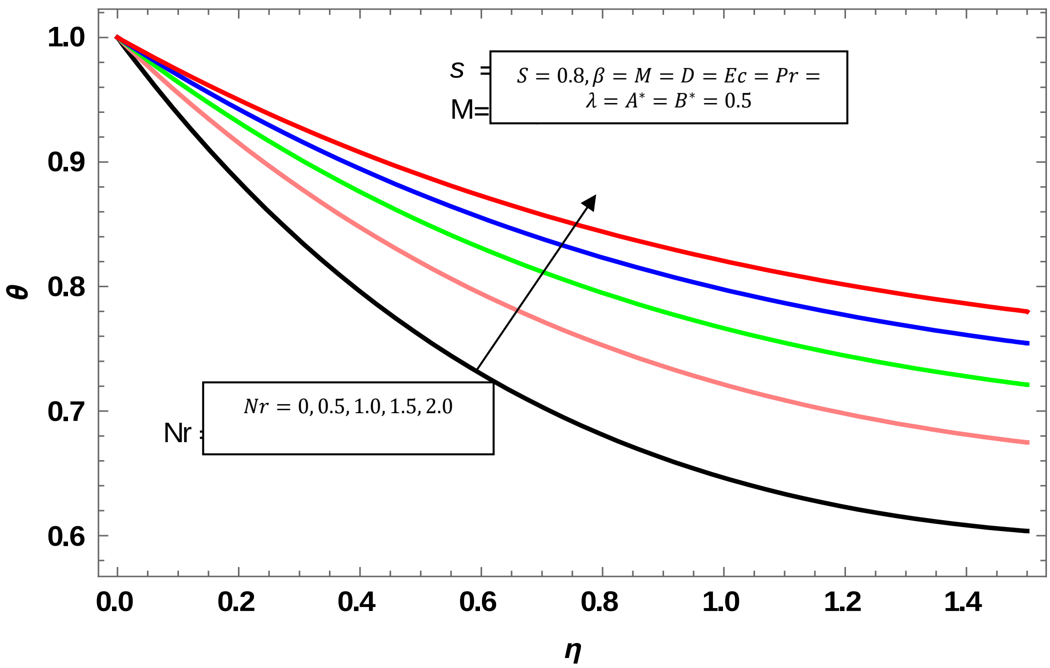

Fig.6. For S = 0.8, impact of Nr , the Radiation parameter on $\theta(\eta)$.

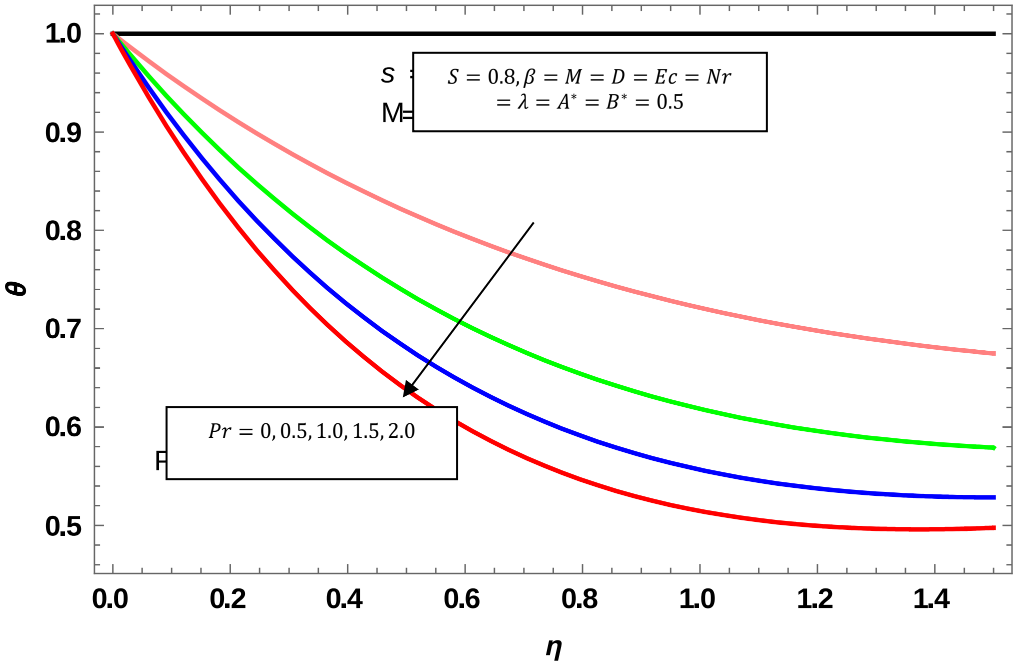

Fig.7. For $\mathrm{S} = 0.8$, impact of Pr , the Prandtl number on $\theta(\eta)$

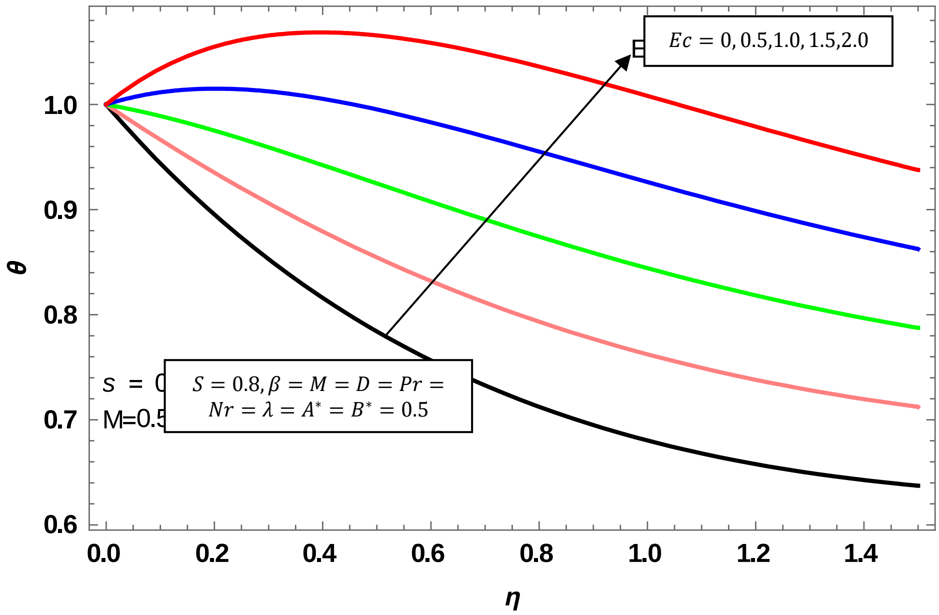

Fig.8. For $\mathrm{S} = 0.8$, impact of Ec , the Eckert number on $\theta(\eta)$

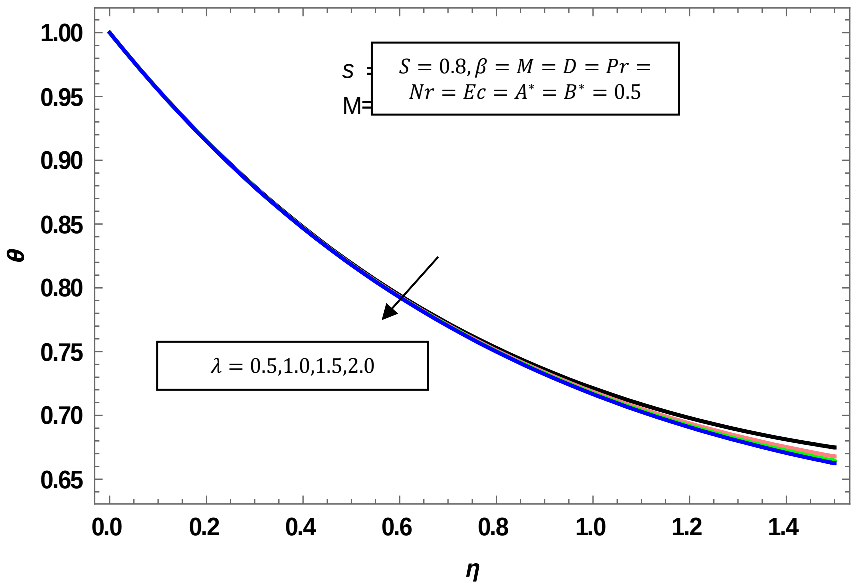

Fig.9. For S $= 0.8$, impact of λ, the Casson parameter on $\theta(\eta)$

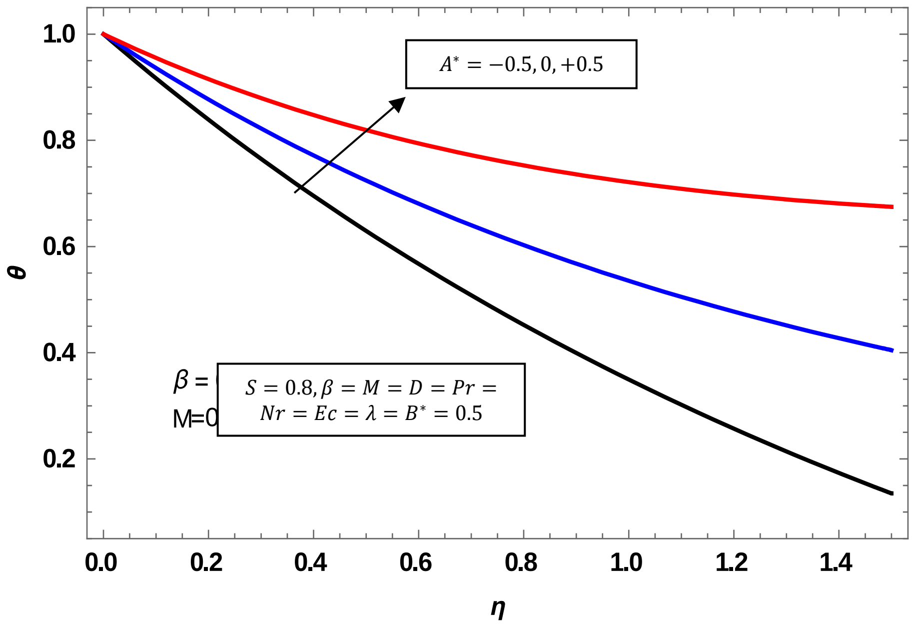

Fig. 10. Effects of A^* on $\theta(\eta)$ for S = 0.8

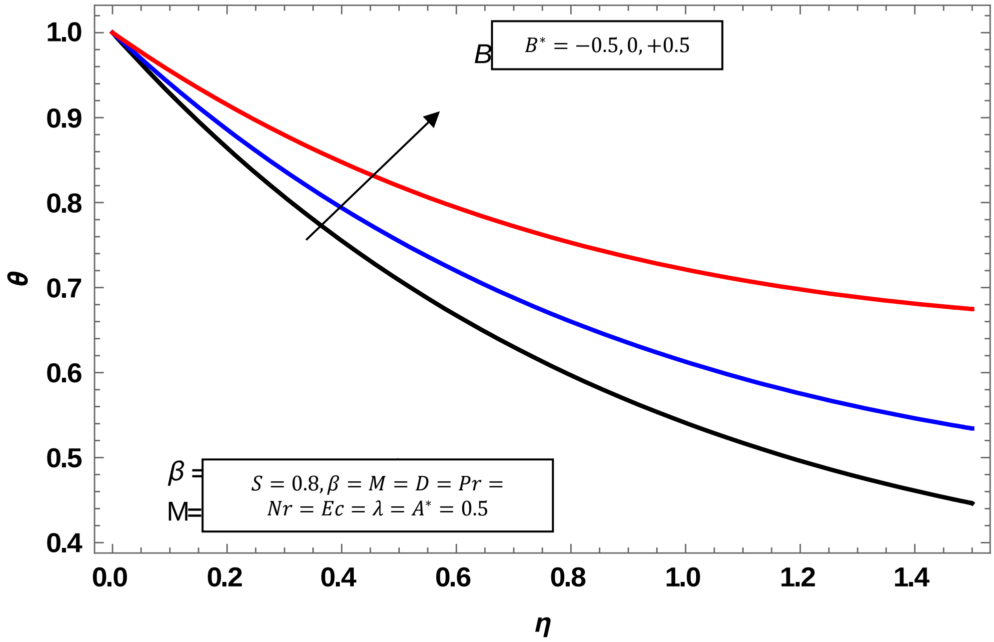

Fig.11: For S = 0.8 , impact of B^* on $\theta(\eta)$.

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