# More on IF-$(\alpha,\beta)$-n-Normal operator on IFH-space

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***Abstract***

In this paper we introduce IF-$(\alpha,\beta)$-n-Normal operator on Hilbert space $\mathcal{H}$.We give some basic properties of these operator .An operator $\delta$ is an intuitionistic fuzzy-$(\alpha,\beta)$-n-normal operator if $\alpha^2\delta^*\delta^n \le \delta^n\delta^* \le \beta^2\delta^*\delta^n$ i.e. $\delta$ commutes with its intuitionistic fuzzy ad joint with $0\le \alpha \le 1 \le \beta$

**Keywords:** Self ad joint , intuitionistic fuzzy$(\boldsymbol{\alpha},\boldsymbol{\beta})$- n-normal operator , intuitionistic fuzzy n-normal operator , partial isometry

## 1. Introduction

Let IF B(H) be the set of all IF-Bounded Linear operators on IF H-space .On Intuitionistic Fuzzy Metric and Norm have been defined by saadati [6] . Then Goudarzi et al . [4] in 2009 ,introduced Intuitionistic Fuzzy Inner product space(IFIP-Space).

Majumdhar and Samanta [7] defined IFIP-Space in 2011 .In 2018

Radharamani et al .[1], [2] have given the definition and properties of Intuitionistic Fuzzy Hilbert space ( IFH-Space ) $\mathcal{H}$ as a triplet $(\mathcal{H},\mathcal{F}_{\mu,v},\mathcal{T})$ and also the cancept of intuitionistic fuzzy adjonint and self-adjont operators (IFA and IFSA , operators) in IF H-space .IF H-space .IF $\delta \in$ IFB(H), $\ni <\delta x,y> = <x\delta^* y>, \forall x,y \in \mathcal{H}$ .AlSo $\delta$ is an IF SA-operator if $\delta = \delta^*$. An operator T is said to be normal if $T^*T = TT^*$, (it is well known that normal operators have translation – invariant properties r ,i.e. ,if T is normal operator, then $(T-\lambda)$is normal operator for evry( $\lambda \in \mathbb{C}$);

Self ad joint if $T^* = T$; projection if $T^2 = T = T^*$. For an operator $T \in H$ , if $\|Tx\|= \|x\|$ for all $x \in H$ (or equivalently $T^*T = I$),

Then T is called an isometric .An on to isometric is called unitary .An operator T $\in$ IF B(H) is called partial isometry if $T^*T$ is projection, An operator S $\in$ IF B(H) is called intuitionistic fuzzy n-normal operator if $S^nS^* = S^*S^n$

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Now we introduced intuitionistic fuzzy$(\alpha,\beta)$-n-normal operator on $\mathcal{H}$ ,if for real number $\alpha,\beta$ with $0\le \alpha \le 1 \le \beta$ $then \alpha^2\, T^*T^n \le T^nT^* \le \beta^2 T^*T^n$

Have we estadlish some theorems and an example for intuitionistic fuzzy$(\alpha,\beta)$- n-normal operator like addition and multiplication of intuitionistic fuzzy$(\alpha,\beta)$- n-normal operator.

## 2. PRELIMINARIES

Definition 2.1: [4]A continuous t – norm T is called continuous t- representable iff $\exists$ a continuous t- norm * and a continuous t- conform $\lozenge$ on the interval [0,1] such that for all
$x = (x_1,x_2)$ , $y = (y_1,y_2) \in L^*$ , $\mathcal{T}(x,y) = (x_1 * y_1 \ , x_2 \lozenge y_2)$.

Definition 2.2: [4] Let $\mu : V^2 \times (0,+\infty) \to [0,1]$ and $\vartheta: V^2 \times (0,\infty) \to [0,1]$ be Fuzzy sets , such that $\mu(x,y,t) + \vartheta(x,y,t) \le 1, \forall x,y \in V \,\&\, t > 0$ . An Intuitionistic Fuzzy Inner Product Space

(IFIP-Space) is a triplet $\left(v, \mathcal{F}_{\mu,v}, \mathcal{T}\right)$ , whrerV is real Vector Space ,$\mathcal{T}$ is a continuous t –representable and $\mathcal{F}_{\mu,v}$ is an Intuitionistic Fuzzy set on $V^2 \times \mathbb{R}$ satisfying the following conditions for all x , y , z $\in$ V and s , r , t $\in \mathbb{R}$ :
(IFI-1) $\mathcal{F}_{\mu,v}(x, y, 0) = 0$ and $\mathcal{F}_{\mu,v}(x, y, t) > 0$ , for every $t > 0$ .
(IFI-2) $\mathcal{F}_{\mu,v}(x, y, t) = \mathcal{F}_{\mu,v}(y, x, t)$ .(IFI-3) $\mathcal{F}_{\mu,v}(x, x, t) \neq H(t)$
for some $t \in \mathbb{R}$ iff $x \neq 0$

$$\text{Where } H(t) = \begin{cases} 1, & \text{if } t > 0 \\ 0, & \text{if } t \leq 0 \end{cases}$$

(IFI-4) For any $\propto \in \mathbb{R}$ ,

$(\mathrm{IFI}-5) \sup\left\{\mathcal{T}\left(\mathcal{F}_{\mu,v}(x,z,s), \mathcal{F}_{\mu,v}(y,z,r)\right)\right\} = \mathcal{F}_{\mu,v}(x+y,y,t)$.
$(\mathrm{IFI}-6) \mathcal{F}_{\mu,v}(x,y,.) : \mathbb{R} \to [0,1]$ is Continuous on $\mathbb{R} \backslash \{0\}$.
$(\mathrm{IFI}-7)\ \lim_{t\to 0}. \mathcal{F}_{\mu,v}(x,y,t) = 1$

Note 2.3: [4]

(i) Here the standard negator $\mathcal{N}_s(x) = 1 - x$ , $\forall x \in [0,1]$
(ii)By putting $\langle x, y \rangle = \mathcal{F}_{\mu,v}(x, y, .)$, it is very simple to show that the Intuitionistic Fuzzy Inner Product acts quite similarly as theOrdinary Inner Product.

Definition 2.4: (IFSA – operator) [2] Let $(V, \mathcal{F}_{\mu,v}, \mathcal{T})$ be an IFH-Space with IP: $\langle x, y \rangle = \sup \left\{ t \in \mathbb{R}: \mathcal{F}_{\mu,v}(x, y, t) < 1 \right\}, \forall x, y \in v$ and let $\mathcal{S} \in$ IFB (V). Then $\mathcal{S}$ is Intuitionistic Fuzzy Self- Ad joint operator , if $\mathcal{S} = \mathcal{S}^*$ ,where of $\mathcal{S}^*$is Intuitionistic Fuzzy Self- ad joint of $\mathcal{S}$.

Theorem 2.5: [2]Let$\left(v, \mathcal{F}_{\mu,v}, \mathcal{T}\right)$be an IFIP – Space , where $\mathcal{T}$is a continuoust– representable for every $x, y \in V$,sup { $t \in \mathbb{R}$ $\mathcal{F}_{\mu,v}$ x, y, t $< 1 < \infty$ . $Define.,.$: $V \times V \to \mathbb{R}$ by x, y =sup $t \in \mathbb{R}$ : $\mathcal{F}_{\mu,vx,y,t<1.}$Then(V,.,.)is an IFIP-space, so that$\left(V, \mathcal{P}_{\mu,v}\right)$is normed space , where $\mathcal{P}_{\mu,v}(x,t) = \langle x, x \rangle^{1/2} \forall x \in V$

## 3. MAIN RESULTS

In this section , we introduced the definition of intuitionistic fuzzy$(\alpha, \beta)$- n-Normal operator in IFH - space and also explain some elementary properties of IF$(\alpha, \beta)$- n-Normal operator in IFH, space in detail.

Definition 3.1: let $(v_1, F_{\mu,v}, T)$ be IFH-space and let $\tau \in IFB(v)$ Then T is called IF-$(\alpha, \beta)$n-normal if for real number $\alpha, \beta$ with $0 \leq \alpha \leq 1 \leq \beta$ ,
$\alpha^2 T^* T^n \leq T^n T^* \leq \beta^2 T^* T^n$
An immediate consequence of a bove definition
$\alpha^2 \langle T^* T^n x, x \rangle \leq \langle T^n T^* x, x \rangle \leq \beta^2 \langle T^* T^n x, x \rangle$For all $x \in r$

Theorem 3.2:if $S_1 S_2$ are commuting IF-$(\alpha, \beta)$ n-normal operator,then $S_1 S_2$ is an IF-$(\alpha, \beta)$ n-normal operator

Proposition 3.3: let T,S is be a commuting IF-$(\alpha, \beta)$ n-normal operator such that $(S + T)^*$commutes with $\sum_{k=1}^{n-1} \binom{n}{k} S^{n-k} T^k$ Then $(S + T)$ is an IF-$(\alpha, \beta)$ n-normal operator

Theorem 3.4: Let $T \in IFB(H)$Then T is IF-$(\alpha, \beta)$-n-normal operator if and only if $T^n$ is IF-$(\alpha, \beta)$-normal operator where $n \in N$

Theorem 3.5: Let $T_1 \ldots \ldots . . T_m$ be commute IF-$(\alpha, \beta)$-n -normal operator in IFB(H) then $(T_1 \oplus \ldots \ldots . . \oplus T_m)$ are IF-$(\alpha, \beta)$-n -normal operator

Theorem 3.6: Let $T_1 \ldots \ldots . . T_m$ be commute IF-$(\alpha, \beta)$-n -normal operator in IFB(H)then$(T_1 \otimes \ldots \ldots . . \otimes T_m)$ are IF-$(\alpha, \beta)$-n-normal operator

Theorem 3.7 :Suppose T is both IF-$(\alpha,\beta)$-k-normal operator and IF-$(\alpha,\beta)$-(k+1)-normal operator for some positive integer K
Then T is IF-$(\alpha,\beta)$-(k+2)-normal operator and hence T is IF-$(\alpha,\beta)$-n-normal operator for all $n \geq K$

Theorem 3-8: Let $T \in B(V)$be an IF-$(\alpha,\beta)$-n-normal operator, if $0 \leq p \leq 1$ or $p \geq 2$,then we have

$$((\mathrm{P}_{\mu,v}(T^*T^n + T^nT^*),t)^2 + (\mathrm{P}_{\mu,v}(T^*T^n - T^nT^*),t)^2)^p \geq \left(\mathrm{P}_{\mu,v}(T^*T^n,t)\right)^{2p} \varphi(\alpha,p)$$

Where $\varphi(\alpha,p) = 2^p(1+\alpha^{2p})^2 + (2^p - 2^2)\alpha^{2p})$ (3.1)

Proof:

We use the following inequality [8, Theorem 8, page 551]:

$\left((\mathrm{P}_{\mu,v}(a+b,t))^2 + (\mathrm{P}_{\mu,v}(a-b,t))^2\right)^p \geq$

$2^p(((\mathrm{P}_{\mu,v}(a,t))^p + (\mathrm{P}_{\mu,v}(b,t))^p)^2 + (2^p - 2^2)(\mathrm{P}_{\mu,v}(a,t))^p(\mathrm{P}_{\mu,v}(b,t))^p)$ (3.2)

Where a and b are two vectors in a Hilbert space and $0 \leq p \leq 1$ or $p \geq 2$

Now, if we put $a = T^*T^nx$ and $b = T^nT^*x$ in (3.3)

Then we obtain

$\left((\mathrm{P}_{\mu,v}(T^*T^nx + T^nT^*x,t)\right)^2 + \left(\mathrm{P}_{\mu,v}(T^*T^nx - T^nT^*x,t)\right)^2)^p \geq$

$\geq 2^p(((\mathrm{P}_{\mu,v}(T^*T^nx,t))^p + (\mathrm{P}_{\mu,v}(T^nT^*x,t))^p)^2 + (2^p - 2^2)$

$(\mathrm{P}_{\mu,v}(T^*T^nx,t))^p\left(\mathrm{P}_{\mu,v}(T^nT^*x,t))^p\right)$

$\geq 2^p((\mathrm{P}_{\mu,v}(T^*T^nx,t))^{2p}(1+\alpha^{2p})^2 + (2^p - 2^2)\alpha^{2p}(\mathrm{P}_{\mu,v}(T^*T^nx,t))^{2p})$(3.4)

$= 2^p\left((\mathrm{P}_{\mu,v}(T^*T^nx,t))^{2p}(1+\alpha^{2p})^2 + (2^p - 2^2)\alpha^{2p}\right)$

$= (\mathrm{P}_{\mu,v}(T^*T^nx,t))^{2p}\varphi(\alpha,p)$

Now , Taking the supremum over $\mathrm{P}_{\mu,v}(x,t) = 1$ in (3.5)

We get

$((\mathrm{P}_{\mu,v}(T^*T^n + T^nT^*),t)^2 + (\mathrm{P}_{\mu,v}(T^*T^n - T^nT^*),t)^2)^p \geq \left(\mathrm{P}_{\mu,v}(T^*T^n,t)\right)^{2p} \varphi(\alpha,p)$

Where $\varphi(\alpha,p) = 2^p(1+\alpha^{2p})^2 + (2^p - 2^2)\alpha^{2p})$□

Theorem (3.9): Let $T \in B(V)$be an IF-$(\alpha,\beta)$-n-normal operator, if $\mathcal{N}(T^n) = 0$ and for any $x \in V$ with $\mathrm{P}_{\mu,v}(x,t) = 1$

We have $\mathrm{P}_{\mu,v}\left(\frac{T^nx}{\mathrm{P}_{\mu,v}(T^*x,t)} - \frac{T^*x}{\mathrm{P}_{\mu,v}(T^nx,t)},t\right) \leq \rho$

Then we have $\alpha^2\left(\mathrm{P}_{\mu,v}(T^*T^n,t)\right)^2 \leq \omega((T^nT^*)^*(T^nT^*)) + \frac{1}{2}\rho^2\alpha^2\left(\mathrm{P}_{\mu,v}(T^*T^nx,t)\right)^2$

Proof :

We use the following reverse of Schwarz's inequality :

$(0 \leq)(\mathrm{P}_{\mu,v}(a,t)(\mathrm{P}_{\mu,v}(b,t)) - |\langle a,b\rangle|$

$\leq \left(\mathrm{P}_{\mu,v}(a,t)\right)\mathrm{P}_{\mu,v}(b,t) - Re\,\langle a,b\rangle \leq \frac{1}{2}\rho^2(\mathrm{P}_{\mu,v}(a,t)\mathrm{P}_{\mu,v}(b,t)$ (3.6)

Which is valid for $a,b \in V/0$ $and$ $\rho > 0$ $,with$ $\mathrm{P}_{\mu,v}\left(\frac{a}{\mathrm{P}_{\mu,v}(b,t)} - \frac{b}{\mathrm{P}_{\mu,v}(a,t)},t\right) \leq \rho$ (see[9]) .

We take $a = T^*T^nx$ and $b = T^nT^*x$ in (3.8) to get

$(\mathrm{P}_{\mu,v}(T^*T^nx,t)(\mathrm{P}_{\mu,v}(T^nT^*x,t)) \leq |\langle T^*T^nx, T^nT^*x\rangle| + \frac{1}{2}\rho^2(\mathrm{P}_{\mu,v}(a,t)\mathrm{P}_{\mu,v}(b,t)$ (3.7)

Thus , we obtain $\alpha^2\left(\mathrm{P}_{\mu,v}(T^*T^nx,t)\right)^2 \leq |\langle T^*T^nx, T^nT^*x\rangle| + \frac{1}{2}\rho^2\beta^2\left(\mathrm{P}_{\mu,v}(T^*T^nx,t)\right)^2$ (3.8)

$\alpha^2\left(\mathrm{P}_{\mu,v}(T^*T^nx,t)\right)^2 \leq |\langle (T^nT^*)^*(T^nT^*)x,x\rangle| + \frac{1}{2}\rho^2\beta^2\left(\mathrm{P}_{\mu,v}(T^*T^nx,t)\right)^2$

Now , Taking the supremum over $\mathrm{P}_{\mu,v}(x,t) = 1$ in (3.10) we get

$$\alpha^2\left(\mathrm{P}_{\mu,v}(T^*T^n,t)\right)^2 \leq \omega((T^nT^*)^*(T^nT^*)) + \frac{1}{2}\rho^2\beta^2\left(\mathrm{P}_{\mu,v}(T^*T^nx,t)\right)^2 \square$$

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