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Applications to Extremally Disconnected Spaces

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Abstract: In this paper, the mixed and ordinary operators are characterized in the classes of rω -sets and r*ω-sets. Certain topological sets that are inherited from ω-open set, open set and δ-open set are characterized using rω -sets and r*ω-sets. More over the behavior of rω -sets and r*ω-sets in Extremally Disconnected spaces are investigated. The Inclusion chains among the mixed and ordinary operators are refined in the domains of rω -sets and r*ω-sets.

1 INTRODUCTION

In the year 1982, Hdeib introduced the notion of a ω-closed set. A subset B of a topological space is called ω-closed if it contains all its condensation points. Recently general topologists introduced and studied new types of topological sets by mixing interior, closure operators with δ-interior, δ-closure operators. In this paper, our investigations on ω-open sets and ω-closed sets in the sense of Hdeib, lead to the development in the following domains of topology.

- Investigations on the relationships among the interior and closure operators; δ-interior and δ-closure operators; ω-interior, ω-closure operators.
- New topological sets that will be defined by mixing the interior, closure operators with ω-interior, ω-closure operators and also by mixing the interior, closure operators with ω-interior, ω-closure operators and also by mixing interior, closure operators with ω-interior, ω-closure operators.
- Characterizations of recent concepts in the literature of topology by using the newly defined sets.
- Applications of Extremally disconnected spaces by using the newly defined concepts and their derivations.

In this paper, the mixed and ordinary operators are characterized in the classes of rω -sets and r*ω-sets. Certain topological sets that are inherited from ω-open set, open set and δ-open set are characterized using rω -sets and r*ω-sets. More over the behavior of rω -sets and r*ω-sets in Extremally Disconnected spaces are investigated. The Inclusion chains among the mixed and ordinary operators are refined in the domains of rω -sets and r*ω-sets.

2 PRELIMINARIES

Throughout this paper (X, τ) (or X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. The concept of δ-closure was introduced and studied by Velicko in the year 1968. A point x is in the δ-closure of A if every regular open nbd of x intersects A. $Cl_\delta A$ denotes the δ-closure of A.

Definition 2.1 A subset A of X is δ-closed [11] if A = $Cl_\delta A$. The complement of a δ-closed set is δ-open. The collection of all δ-open sets is a topology denoted by τ^δ. This τ^δ is called the semi - regularization of τ.

Let $Int_\delta A$ and $Cl_\delta A$ denote the δ-interior and δ-closure of A respectively. Velicko established that the operators *Cl*(.) and Cl_δ(.) have the same effect on the class of open sets and the operators *Int*(.) and Int_δ(.) coincide on the class of closed sets.

Lemma 2.2 [11]

(i) For any open set A, $Cl_\delta A = ClA$,

(ii) For any closd set B, $Int_\delta B = IntB$.

Definition 2.3 A space (X,τ) is said to be Extremally Disconnected [7] if the closure every open set is open.

Definition 2.4

A subset M of a Space X is called:

(i) semi-open [4] if M □ cl(int(M));

(ii) α-open [6] if M □ int(cl(int(M)));

(iii) regular open [8] if M = int(cl(M));

(vi) pre-open [5] if M □ int(cl(M)).

The complements of the above-mentioned open sets are called their respective closed sets.

Definition 2.5 [**2**] Let H be a subset of a space (X, τ), a point p in X is called a condensation point of H if for each open set U containing p, U ∩ H is uncountable.

Definition 2.6 [**2**] A subset H of a space (X, τ) is called ω-closed if it contains all its condensation points.

The complement of an ω-closed set is called ω-open. The family of all ω-closed sets is denoted by ωC(X, τ). The family of all ω-open sets is denoted by ωO(X). It is well known that a subset W of a space (X, τ) is ω-open [2] if and only if for each x ∈ W, there exists U ∈

τ such that $x \in U$ and $U - W$ is countable. The family of all ω-open sets, denoted by τ_ω, is a topology on X, which is finer than τ. The interior and closure operator in (X, τ_ω) are denoted by int_ω and cl_ω respectively.

Definition 2.7 The set A of a space X is called

(i) α-ω-open [9] if $A = Int_\omega Cl\ Int_\omega A$

(ii) semi-ω-open [9] if $A \subseteq Cl\ Int_\omega A$

(iii) pre-ω-open [9] if $A \subseteq Int_\omega Cl\ A$

The complements of the above-mentioned open sets are called their respective closed sets.

3 rω-SETS AND r*ω-SETS

Proposition 3.1 For any subset A of a space (X,τ), the following always hold.

(i) $IntCl_\omega IntA \subseteq IntCl_\omega Int_\omega A \subseteq IntClInt_\omega A \subseteq Int_\omega ClInt_\omega A$

(ii) $IntCl_\omega IntA \subseteq IntClIntA \subseteq IntClInt_\omega A \subseteq Int_\omega ClInt_\omega A$

(iii) $IntCl_\omega IntA \subseteq Int_\omega Cl_\omega IntA \subseteq Int_\omega ClIntA \subseteq Int_\omega ClInt_\omega A$

(iv) $IntCl_\omega IntA \subseteq Int_\omega Cl_\omega IntA \subseteq Int_\omega Cl_\omega Int_\omega A \subseteq Int_\omega ClInt_\omega A$

Proof.

$IntCl_\omega IntA \subseteq IntCl_\omega Int_\omega A$ by using $Int(.) \subseteq Int_\omega(.)$

$\subseteq IntClInt_\omega A$ by using $Cl_\omega(.) \subseteq Cl(.)$

$\subseteq Int_\omega ClInt_\omega A$ by using $Int(.) \subseteq Int_\omega(.)$. This proves (i)

$IntCl_\omega IntA \subseteq IntClIntA$ by using $Cl_\omega(.) \subseteq Cl(.)$

$\subseteq IntCl\ Int_\omega A$ by using $Int(.) \subseteq Int_\omega(.)$

$\subseteq Int_\omega ClInt_\omega A$ by using $Int(.) \subseteq Int_\omega(.)$. This proves (ii)

$IntCl_\omega IntA \subseteq Int_\omega Cl_\omega IntA$ by using $Int(.) \subseteq Int_\omega(.)$

$\subseteq Int_\omega Cl\ IntA$ by using $Cl_\omega(.) \subseteq Cl(.)$

$\subseteq Int_\omega Cl\ Int_\omega A$ by using $Int(.) \subseteq Int_\omega(.)$. This proves (iii)

$IntCl_\omega IntA \subseteq Int_\omega Cl_\omega IntA$ by using $IntA \subseteq Int_\omega A$

$\subseteq Int_\omega Cl_\omega\ Int_\omega A$ by using $Int(.) \subseteq Int_\omega(.)$

$\subseteq Int_\omega Cl\ Int_\omega A$ by using $Cl_\omega(.) \subseteq Cl(.)$. This proves (iv).

Proposition 3.2

(i) $Cl_{\omega}Int_{\delta}A \subseteq ClInt_{\delta}A = Cl_{\delta}Int_{\delta}A \subseteq Cl_{\delta}IntA = ClIntA \subseteq ClInt_{\omega}A \subseteq Cl_{\delta}Int_{\omega}A$

(ii) $Cl_{\omega}Int_{\delta}A \subseteq Cl_{\omega}IntA \subseteq Cl_{\omega}Int_{\omega}A \subseteq ClInt_{\omega}A \subseteq Cl_{\delta}Int_{\omega}A$

(iii) $Int_{\delta}Cl_{\omega}A \subseteq IntCl_{\omega}A \subseteq IntClA = Int_{\delta}ClA \subseteq Int_{\delta}Cl_{\delta}A = IntCl_{\delta}A \subseteq Int_{\omega}Cl_{\delta}A$

(iv) $Int_{\delta}Cl_{\omega}A \subseteq IntCl_{\omega}A \subseteq Int_{\omega}Cl_{\omega}A \subseteq Int_{\omega}ClA \subseteq Int_{\omega}Cl_{\delta}A$

Proof. $Int_{\delta}A \subseteq IntA \subseteq Int_{\omega}A \subseteq A \subseteq Cl_{\omega}A \subseteq ClA \subseteq Cl_{\delta}A$.

$Cl_{\omega}Int_{\delta}A \subseteq ClInt_{\delta}A$ using $Cl_{\omega}(.) \subseteq Cl(.)$

$= Cl_{\delta}Int_{\delta}A$ using Lemma 2.2(i)

$\subseteq Cl_{\delta}IntA$ using $Int_{\delta}(.) \subseteq Int(.)$

$= ClIntA$ using Lemma 2.2(i)

$\subseteq ClInt_{\omega}A$ using $Int(.) \subseteq Int_{\omega}(.)$

$\subseteq Cl_{\delta}Int_{\omega}A$ using $Cl(.) \subseteq Cl_{\delta}(.)$.

Therefore $Cl_{\omega}Int_{\delta}A \subseteq ClInt_{\delta}A = Cl_{\delta}Int_{\delta}A \subseteq Cl_{\delta}IntA = ClIntA \subseteq ClInt_{\omega}A \subseteq Cl_{\delta}Int_{\omega}A$.

This proves (i).

Now $Cl_{\omega}Int_{\delta}A \subseteq Cl_{\omega}IntA$ using $Int_{\delta}(.) \subseteq Int(.)$

$\subseteq Cl_{\omega}Int_{\omega}A$ using $Int(.) \subseteq Int_{\omega}(.)$

$\subseteq ClInt_{\omega}A$ using $Cl_{\omega}(.) \subseteq Cl(.)$

$\subseteq Cl_{\delta}Int_{\omega}A$ using $Cl(.) \subseteq Cl_{\delta}(.)$.

Therefore $Cl_{\omega}Int_{\delta}A \subseteq Cl_{\omega}IntA \subseteq Cl_{\omega}Int_{\omega}A \subseteq ClInt_{\omega}A \subseteq Cl_{\delta}Int_{\omega}A$.

This proves (ii).

Again now $Int_{\delta}Cl_{\omega}A \subseteq IntCl_{\omega}A$ using $Int_{\delta}(.) \subseteq Int(.)$

$\subseteq IntClA$ using $Cl_{\omega}(.) \subseteq Cl(.)$

$= Int_{\delta}ClA$ using Lemma 2.2(ii)

$\subseteq Int_{\delta}Cl_{\delta}A$ using $Cl(.) \subseteq Cl_{\delta}(.)$

$= IntCl_{\delta}A$ using Lemma 2.2(ii)

$\subseteq Int_{\omega}Cl_{\delta}A$ using $Int(.) \subseteq Int_{\omega}(.)$.

Therefore $Int_{\delta}Cl_{\omega}A \subseteq IntCl_{\omega}A \subseteq IntClA = Int_{\delta}ClA \subseteq Int_{\delta}Cl_{\delta}A = IntCl_{\delta}A \subseteq Int_{\omega}Cl_{\delta}A$.

This proves (iii).

$Int_\delta Cl_\omega A \subseteq IntCl_\omega A$ using $Int_\delta(.) \subseteq Int(.)$

$\subseteq Int_\omega Cl_\omega A$ using $Int(.) \subseteq Int_\omega(.)$

$\subseteq Int_\omega ClA$ using $Cl_\omega(.) \subseteq Cl(.)$

$\subseteq Int_\omega Cl_\delta A$ using $Cl(.) \subseteq Cl_\delta(.)$.

Therefore $Int_\delta Cl_\omega A \subseteq IntCl_\omega A \subseteq Int_\omega Cl_\omega A \subseteq Int_\omega ClA \subseteq Int_\omega Cl_\delta A$.

This proves (iv).

Proposition 3.3 For any subset A of a space (X,τ), the following always hold.

(i) $Cl_\omega IntCl_\omega A \subseteq Cl_\omega IntClA \subseteq Cl_\omega Int_\omega ClA \subseteq ClInt_\omega ClA$

(ii) $Cl_\omega IntCl_\omega A \subseteq Cl_\omega Int_\omega Cl_\omega A \subseteq Cl_\omega Int_\omega ClA \subseteq ClInt_\omega ClA$

(iii) $Cl_\omega IntCl_\omega A \subseteq ClIntCl_\omega A \subseteq ClInt_\omega Cl_\omega A \subseteq ClInt_\omega ClA$

(iv) $Cl_\omega IntCl_\omega A \subseteq ClIntCl_\omega A \subseteq ClIntClA \subseteq ClInt_\omega ClA$

Proof. $Cl_\omega IntCl_\omega A \subseteq Cl_\omega IntClA$ by using $Cl_\omega(.) \subseteq Cl(.)$

$\subseteq Cl_\omega Int_\omega ClA$ by using $Int(.) \subseteq Int_\omega(.)$

$\subseteq ClInt_\omega ClA$ by using $Cl_\omega(.) \subseteq Cl(.)$. This proves (i)

$Cl_\omega IntCl_\omega A \subseteq Cl_\omega Int_\omega Cl_\omega A$ by using $Int(.) \subseteq Int_\omega(.)$

$\subseteq Cl_\omega Int_\omega ClA$ by using $Cl_\omega(.) \subseteq Cl(.)$

$\subseteq ClInt_\omega ClA$ by using $Cl_\omega(.) \subseteq Cl(.)$. This proves (ii).

$Cl_\omega IntCl_\omega A \subseteq ClIntCl_\omega A$ by using $Cl_\omega(.) \subseteq Cl(.)$

$\subseteq ClInt_\omega Cl_\omega A$ by using $Int(.) \subseteq Int_\omega(.)$

$\subseteq ClInt_\omega ClA$ by using $Cl_\omega(.) \subseteq Cl(.)$. This proves (iii).

$Cl_\omega IntCl_\omega A \subseteq ClIntCl_\omega A$ by using $Cl_\omega(.) \subseteq Cl(.)$

$\subseteq ClIntClA$ by using $Cl_\omega(.) \subseteq Cl(.)$

$\subseteq ClInt_\omega ClA$ by using $Int(.) \subseteq Int_\omega(.)$. This proves (iv).

The assertion (i) of Proposition 3.2 will be used to find the inclusion chains among some mixed three level operators as shown in the next proposition.

Proposition 3.4

(i) $Int_\delta Cl_\omega Int_\delta A \subseteq IntCl_\omega Int_\delta A \subseteq IntClInt_\delta A = IntCl_\delta Int_\delta A \subseteq IntCl_\delta IntA$

(ii) $IntCl_\delta IntA=IntClIntA\subseteq IntClInt_\omega A\subseteq IntCl_\delta Int_\omega A\subseteq Int_\omega Cl_\delta Int_\omega A$

(iii) $Int_\delta Cl_\omega Int_\delta A\subseteq Int_\delta ClInt_\delta A=Int_\delta Cl_\delta Int_\delta A\subseteq Int_\delta Cl_\delta IntA=Int_\delta ClIntA$

(iv) $Int_\delta ClIntA\subseteq Int_\delta ClInt_\omega A\subseteq Int_\delta Cl_\delta Int_\omega A\subseteq IntCl_\delta Int_\omega A\subseteq Int_\omega Cl_\delta Int_\omega A$

(v) $Int_\delta Cl_\omega Int_\delta A\subseteq IntCl_\omega Int_\delta A\subseteq Int_\omega Cl_\omega Int_\delta A\subseteq Int_\omega ClInt_\delta A=Int_\omega Cl_\delta Int_\delta A$

(vi) $Int_\omega Cl_\delta Int_\delta A\subseteq Int_\omega Cl_\delta IntA=Int_\omega ClIntA\subseteq Int_\omega ClInt_\omega A\subseteq Int_\omega Cl_\delta Int_\omega A$

Proof.

Taking interior, δ-interior and ω-interior operators on the assertion (i) of Proposition 3.2 we get the following three equations.

$IntCl_\omega Int_\delta A\subseteq IntClInt_\delta A=IntCl_\delta Int_\delta A\subseteq IntCl_\delta IntA=IntClIntA$

$\subseteq IntClInt_\omega A\subseteq IntCl_\delta Int_\omega A.$

$Int_\delta Cl_\omega Int_\delta A\subseteq Int_\delta ClInt_\delta A=Int_\delta Cl_\delta Int_\delta A\subseteq Int_\delta Cl_\delta IntA=Int_\delta ClIntA$

$\subseteq Int_\delta ClInt_\omega A\subseteq Int_\delta Cl_\delta Int_\omega A.$

$Int_\omega Cl_\omega Int_\delta A\subseteq Int_\omega ClInt_\delta A=Int_\omega Cl_\delta Int_\delta A\subseteq Int_\omega Cl_\delta IntA=Int_\omega ClIntA$

$\subseteq Int_\omega ClInt_\omega A\subseteq Int_\omega Cl_\delta Int_\omega A.$

$Int_\delta Cl_\omega Int_\delta A\subseteq IntCl_\omega Int_\delta A\subseteq IntClInt_\delta A=IntCl_\delta Int_\delta A$

$\subseteq IntCl_\delta IntA=IntClIntA\subseteq IntClInt_\omega A$

$\subseteq IntCl_\delta Int_\omega A\subseteq Int_\omega Cl_\delta Int_\omega A.$

This proves (i) and (ii).

$Int_\delta Cl_\omega Int_\delta A\subseteq Int_\delta ClInt_\delta A=Int_\delta Cl_\delta Int_\delta A$

$\subseteq Int_\delta Cl_\delta IntA=Int_\delta ClIntA$

$\subseteq Int_\delta ClInt_\omega A\subseteq Int_\delta Cl_\delta Int_\omega A$

$\subseteq IntCl_\delta Int_\omega A\subseteq Int_\omega Cl_\delta Int_\omega A.$

This proves (iii) and (iv).

$Int_\delta Cl_\omega Int_\delta A\subseteq IntCl_\omega Int_\delta A\subseteq Int_\omega Cl_\omega Int_\delta A$

$\subseteq Int_\omega ClInt_\delta A=Int_\omega Cl_\delta Int_\delta A$

$\subseteq Int_\omega Cl_\delta IntA=Int_\omega ClIntA$

$\subseteq Int_\omega ClInt_\omega A\subseteq Int_\omega Cl_\delta Int_\omega A.$

This proves (v) and (vi).

The assertion (iii) of Proposition 3.2 will be used to find the inclusion chains among some mixed three level operators as shown in the next proposition.

Proposition 3.5

(i) $Cl_\omega Int_\delta Cl_\omega A \subseteq ClInt_\delta Cl_\omega A \subseteq ClIntCl_\omega A \subseteq ClIntClA = ClInt_\delta ClA \subseteq ClInt_\delta Cl_\delta A$

(ii) $ClInt_\delta Cl_\delta A = ClIntCl_\delta A \subseteq ClInt_\omega Cl_\delta A \subseteq Cl_\delta A\ Int_\omega Cl_\delta A$

(iii) $Cl_\omega Int_\delta Cl_\omega A \subseteq ClInt_\delta Cl_\omega A \subseteq Cl_\delta Int_\delta Cl_\omega A \subseteq Cl_\delta IntCl_\omega A \subseteq Cl_\delta IntClA$

(iv) $Cl_\delta IntClA = Cl_\delta Int_\delta ClA \subseteq Cl_\delta Int_\delta Cl_\delta A = Cl_\delta IntCl_\delta A \subseteq Cl_\delta Int_\omega Cl_\delta A$

(v) $Cl_\omega Int_\delta Cl_\omega A \subseteq Cl_\omega IntCl_\omega A \subseteq Cl_\omega IntClA = Cl_\omega Int_\delta ClA \subseteq Cl_\omega Int_\delta Cl_\delta A$

(vi) $Cl_\omega Int_\delta Cl_\delta A = Cl_\omega IntCl_\delta A \subseteq Cl_\omega Int_\omega Cl_\delta A \subseteq ClInt_\omega Cl_\delta A \subseteq Cl_\delta Int_\omega Cl_\delta A$.

Proof. Taking closure, δ-closure and ω-closure operators on the assertion (iii) of Proposition 3.2 we get the following three equations.

$ClInt_\delta Cl_\omega A \subseteq ClIntCl_\omega A \subseteq ClIntClA = ClInt_\delta ClA$

$\subseteq ClInt_\delta Cl_\delta A = ClIntCl_\delta A \subseteq ClInt_\omega Cl_\delta A$.

$Cl_\delta Int_\delta Cl_\omega A \subseteq Cl_\delta IntCl_\omega A \subseteq Cl_\delta IntClA = Cl_\delta Int_\delta ClA$

$\subseteq Cl_\delta Int_\delta Cl_\delta A = Cl_\delta IntCl_\delta A \subseteq Cl_\delta Int_\omega Cl_\delta A$.

$Cl_\omega Int_\delta Cl_\omega A \subseteq Cl_\omega IntCl_\omega A \subseteq Cl_\omega IntClA = Cl_\omega Int_\delta ClA$

$\subseteq Cl_\omega Int_\delta Cl_\delta A = Cl_\omega IntCl_\delta A \subseteq Cl_\omega Int_\omega Cl_\delta A$.

$Cl_\omega Int_\delta Cl_\omega A \subseteq ClInt_\delta Cl_\omega A \subseteq ClIntCl_\omega A \subseteq ClIntClA = ClInt_\delta ClA$

$\subseteq ClInt_\delta Cl_\delta A = ClIntCl_\delta A \subseteq ClInt_\omega Cl_\delta A \subseteq Cl_\delta A\ Int_\omega Cl_\delta A$.

This proves (i) and (ii).

$Cl_\omega Int_\delta Cl_\omega A \subseteq ClInt_\delta Cl_\omega A \subseteq Cl_\delta Int_\delta Cl_\omega A \subseteq Cl_\delta IntCl_\omega A$

$\subseteq Cl_\delta IntClA = Cl_\delta Int_\delta ClA$

$\subseteq Cl_\delta Int_\delta Cl_\delta A = Cl_\delta IntCl_\delta A \subseteq Cl_\delta Int_\omega Cl_\delta A$.

This proves (ii) and (iii).

$Cl_\omega Int_\delta Cl_\omega A \subseteq Cl_\omega IntCl_\omega A \subseteq Cl_\omega IntClA = Cl_\omega Int_\delta ClA$

$\subseteq Cl_\omega Int_\delta Cl_\delta A = Cl_\omega IntCl_\delta A \subseteq Cl_\omega Int_\omega Cl_\delta A \subseteq ClInt_\omega Cl_\delta A \subseteq Cl_\delta Int_\omega Cl_\delta A$.

This proves (v) and (vi).

Proposition 3.6 For any subset A of an Extremally Disconnected space (X,τ), the following results hold.

(i) $ClInt$A is open

(ii) $IntCl$A is closed

(iii) $IntClInt$A = $ClInt$A

(iv) $ClIntCl$A = $IntCl$A

(v) $ClInt_\delta$A is open

(vi) $IntCl_\delta$A is closed

(vii) $IntClInt_\delta$A = $ClInt_\delta$A

(viii) $ClIntCl_\delta$A = $IntCl_\delta$A

Proof. The assertions (i), (ii), (v) and (vi) follow from Definition 2.1. The assertions (iii) and (iv) follow from (i) and (ii). The assertions (vii) and (viii) follow from (v) and (vi).

Proposition 3.7 For a subset A of an Extremally Disconnected space (X,τ), the following results hold.

(i) $IntCl_\omega Int A \subseteq ClInt A \subseteq IntClInt_\omega A \subseteq Int_\omega ClInt_\omega A$

(ii) $Cl_\omega IntCl_\omega A \subseteq ClIntCl_\omega A \subseteq IntCl A \subseteq ClInt_\omega Cl A$

(iii) $Int_\delta Cl_\omega Int_\delta A \subseteq IntCl_\omega Int_\delta A \subseteq ClInt_\delta A = IntCl_\delta Int_\delta A \subseteq IntCl_\delta Int A$

(iv) $IntCl_\delta Int A = ClInt A \subseteq IntClInt_\omega A \subseteq IntCl_\delta Int_\omega A \subseteq Int_\omega Cl_\delta Int_\omega A$

(v) $Cl_\omega Int_\delta Cl_\omega A \subseteq ClInt_\delta Cl_\omega A \subseteq ClIntCl_\omega A \subseteq IntCl A = ClInt_\delta Cl A \subseteq ClInt_\delta Cl_\delta A$

(vi) $ClInt_\delta Cl_\delta A = IntCl_\delta A \subseteq ClInt_\omega Cl_\delta A \subseteq Cl_\delta Int_\omega Cl_\delta A$

Proof. The assertion (i) follows from Proposition 3.1(ii).

The assertion (ii) follows from Proposition 3.3(iv).

The assertions (iii) & (iv) follow from Proposition 3.4(i) & (ii).

The assertions (v) & (vi) follow from Proposition 3.5(i) & (ii).

Corollary 3.8 Let A be a subset of an Extremally Disconnected space (X,τ),

(i) If A is semi-open then it is α-ω-open.

(ii) If A is semi-closed then it is α-ω-closed.

Proof. Obvious.

Example 3.9 Let R denote the set of all real numbers and τ be the standard topology on R. Let Q be the set of all rational numbers and Q^c denote the set of all irrational numbers. It is easy to prove the following.

- Q is ω-closed and Q^c is ω-open.
- Any super set of Q^c is ω-open.
- A subset of Q is ω-closed.
- Any countable subset of R is ω -closed.
- Every open interval is ω-open.
- Any ω-open set U in R is of the form $U=V\cup A$ where V is a subset of Q^c and A is a subset of Q.

Definition 3.10 A subset A of a space (X,τ) is

(i) an rω-set if $IntCl_\omega A = IntClA$,
(ii) an r*ω-set if $ClInt_\omega A = ClIntA$ and
(iii) an rr*ω-set if it is both an rω-set and an r*ω-set.

As seen from Example 3.9, it is evident that

(i) Q is an r*ω-set but not an rω-set; Q^c is an rω-set but not an r*ω-set;
(ii) $Q\cup(a, b)$ is an r*ω-set but not an rω-set and
(iii) $Q^c\cup(a, b)$ is an rω-set but not an r*ω-set where $a<b$.

Proposition 3.11 A subset A of a space (X,τ) is

(i) an rω-set $\Leftrightarrow IntCl_\omega A = Int_\delta ClA$,

(ii) an r*ω-set $\Leftrightarrow ClInt_\omega A = Cl_\delta IntA$

Proof. Using Proposition 3.2(i) & (iii), we have $IntClA= Int_\delta ClA$ and $Cl_\delta IntA= ClIntA$. This along with Definition 3.10, proves the proposition.

Corollary 3.12

(i) The set A is an rω-set $\Leftrightarrow IntCl_\omega A = IntClA= Int_\delta ClA$.

(ii) The set A is an r*ω-set $\Leftrightarrow ClInt_\omega A = ClIntA = Cl_\delta IntA$.

Proof. Follows from Proposition 3.11.

Proposition 3.13 Let A be an rω-set in an Extremally Disconnected space (X,τ). Then

$Cl_\omega IntCl_\omega A= ClIntCl_\omega A = IntClA= IntCl_\omega A = Int_\delta ClA= ClInt_\delta ClA$ is valid.

Proof. Let A be an rω-set in an Extremally Disconnected space (X,τ). Then using Proposition 3.7(ii), we get

$Cl_\omega IntCl_\omega A\subseteq ClIntCl_\omega A \subseteq IntClA\subseteq ClInt_\omega ClA$.

Using Corollary 3.12(i) in the above expression we have

$Cl_\omega IntCl_\omega A \subseteq ClIntCl_\omega A \subseteq IntClA = IntCl_\omega A = Int_\delta ClA \subseteq ClInt_\omega ClA$. (G)

Since $IntCl_\omega A \subseteq Cl_\omega IntCl_\omega A$, (G) becomes

$Cl_\omega IntCl_\omega A \subseteq ClIntCl_\omega A \subseteq IntClA = IntCl_\omega A = Int_\delta ClA \subseteq Cl_\omega IntCl_\omega A$ that implies

$Cl_\omega IntCl_\omega A = ClIntCl_\omega A = IntClA = IntCl_\omega A = Int_\delta ClA$. (H)

Now using Proposition 3.7(v), we get

$Cl_\omega Int_\delta Cl_\omega A \subseteq ClInt_\delta Cl_\omega A \subseteq ClIntCl_\omega A \subseteq IntClA = ClInt_\delta ClA \subseteq ClInt_\delta Cl_\delta A$.

Using Corollary 3.12(i) in the above expression we have

$Cl_\omega Int_\delta Cl_\omega A \subseteq ClInt_\delta Cl_\omega A \subseteq ClIntCl_\omega A \subseteq IntClA = IntCl_\omega A = Int_\delta ClA$

$= ClInt_\delta ClA \subseteq ClInt_\delta Cl_\delta A$ (I)

. Since $IntCl_\omega A \subseteq ClIntCl_\omega A$, (I) becomes

$IntCl_\omega A \subseteq ClIntCl_\omega A \subseteq IntClA = IntCl_\omega A = Int_\delta ClA = ClInt_\delta ClA$ that implies

$ClIntCl_\omega A = IntClA = IntCl_\omega A = Int_\delta ClA = ClInt_\delta ClA$. (J)

Combining (H) and (J) we have

$Cl_\omega IntCl_\omega A = ClIntCl_\omega A = IntClA = IntCl_\omega A = Int_\delta ClA = ClInt_\delta ClA$. This proves the proposition.

Proposition 3.14 Let A be an rω-set in an Extremally Disconnected space (X,τ). The followings are equivalent.

(i) A is α-ω-closed

(ii) $ClIntCl_\omega A \subseteq A$

(iii) A is semi-ω-closed

(iv) A is semi-closed

(v) $Int_\delta ClA \subseteq A$

(vi) $ClInt_\delta ClA \subseteq A$.

Proof. Let A be an rω-set in an Extremally Disconnected space (X,τ). Then using Proposition 3.13, we have

$Cl_\omega IntCl_\omega A = ClIntCl_\omega A = IntClA = IntCl_\omega A = Int_\delta ClA = ClInt_\delta ClA$. (K)

Therefore

A is α-ω-closed $\Leftrightarrow Cl_\omega IntCl_\omega A \subseteq A \Leftrightarrow ClIntCl_\omega A \subseteq A$

$\Leftrightarrow IntCl_\omega A \subseteq A \Leftrightarrow$ A is semi-ω-closed

$\Leftrightarrow IntClA \subseteq A \Leftrightarrow$ A is semi-closed

$\Leftrightarrow Int_\delta ClA \subseteq A$.

$\Leftrightarrow ClInt_\delta ClA \subseteq A$.

This proves the proposition.

Proposition 3.15 Let A be an rω-set in an Extremally Disconnected space (X,τ). The followings are equivalent.

(i) A is regular open

(ii) $A=ClIntCl_\omega A$

(iii) $A= ClIntCl_\omega A$

(iv) $A= IntCl_\omega A$

(v) $A= Int_\delta ClA$

(vi)$A=ClInt_\delta ClA$.

Proof. Let A be an rω-set in an Extremally Disconnected space (X,τ).

Then using (K) we have

A is regular open $\Leftrightarrow A = IntClA$

$\Leftrightarrow A=Cl_\omega IntCl_\omega A$

$\Leftrightarrow A= ClIntCl_\omega A$

$\Leftrightarrow A= IntCl_\omega A$

$\Leftrightarrow A= Int_\delta ClA$

$\Leftrightarrow A=ClInt_\delta ClA$

This proves the proposition.

Proposition 3.16 Let A be an rω-set in an Extremally Disconnected space (X,τ). The followings are equivalent.

(i) A is pre-open

(ii) $A\subseteq ClIntCl_\omega A$

(iii) $A\subseteq ClIntCl_\omega A$

(iv) A$\subseteq$*IntCl*$_\omega$A

(v) A$\subseteq$*Int*$_\delta$*Cl*A

(vi)A$\subseteq$*ClInt*$_\delta$*Cl*A .

Proof. Let A be an rω-set in an Extremally Disconnected space (X,τ).

Then using (K) we have

A is pre-open $\Leftrightarrow$ A $\subseteq$*IntCl*A

$\Leftrightarrow$ A$\subseteq$*Cl*$_\omega$*IntCl*$_\omega$A

$\Leftrightarrow$ A$\subseteq$*ClIntCl*$_\omega$A

$\Leftrightarrow$ A$\subseteq$*IntCl*$_\omega$A

$\Leftrightarrow$ A$\subseteq$*Int*$_\delta$*Cl*A

$\Leftrightarrow$ A$\subseteq$*ClInt*$_\delta$*Cl*A

This proves the proposition.

Proposition 3.17 Let A be an r*ω-set in an Extremally Disconnected space (X,τ). Then

*ClInt*A= *ClInt*$_\omega$A= *Cl*$_\delta$*Int*A $\subseteq$*IntClInt*$_\omega$A = *Int*$_\omega$*ClInt*$_\omega$A = *IntCl*$_\delta$*Int*A is valid.

Proof . Let A be an r*ω-set in an Extremally Disconnected space (X,τ). Then using Proposition 3.7(i) we have *IntCl*$_\omega$*Int*A$\subseteq$*ClInt*A $\subseteq$*IntClInt*$_\omega$A $\subseteq$*Int*$_\omega$*ClInt*$_\omega$A.

Using Corollary 3.12(ii) in the above expression we get

IntCl$_\omega$*Int*A$\subseteq$*ClInt*A= *ClInt*$_\omega$A= *Cl*$_\delta$*Int*A $\subseteq$*IntClInt*$_\omega$A $\subseteq$*Int*$_\omega$*ClInt*$_\omega$A that implies

*ClInt*A= *ClInt*$_\omega$A= *Cl*$_\delta$*Int*A $\subseteq$*IntClInt*$_\omega$A $\subseteq$*Int*$_\omega$*ClInt*$_\omega$A $\subseteq$ *ClInt*$_\omega$A so that

*ClInt*A= *ClInt*$_\omega$A= *Cl*$_\delta$*Int*A $\subseteq$*IntClInt*$_\omega$A = *Int*$_\omega$*ClInt*$_\omega$A (L)

Now using Proposition 3.7(iv) we have

IntCl$_\delta$*Int*A= *ClInt*A$\subseteq$*IntClInt*$_\omega$A $\subseteq$*IntCl*$_\delta$*Int*$_\omega$A $\subseteq$*Int*$_\omega$*Cl*$_\delta$*Int*$_\omega$A.

Using Corollary 3.12(ii) in the above expression we get

IntCl$_\delta$*Int*A= *ClInt*$_\omega$A= *Cl*$_\delta$*Int*A =*ClInt*A$\subseteq$*IntClInt*$_\omega$A $\subseteq$*IntCl*$_\delta$*Int*$_\omega$A $\subseteq$*Int*$_\omega$*Cl*$_\delta$*Int*$_\omega$A that implies

IntCl$_\delta$*Int*A= *ClInt*$_\omega$A= *Cl*$_\delta$*Int*A =*ClInt*A$\subseteq$*IntClInt*$_\omega$A $\subseteq$*ClInt*$_\omega$A so that

IntCl$_\delta$*Int*A= *ClInt*$_\omega$A= *Cl*$_\delta$*Int*A =*ClInt*A=*IntClInt*$_\omega$A (M)

Combining (L) and (M) we get

$ClInt$A= $ClInt_{\omega}$A= $Cl_{\delta}Int$A $\subseteq IntClInt_{\omega}$A = $Int_{\omega}ClInt_{\omega}$A = $IntCl_{\delta}Int$A. This proves the proposition.

Proposition 3.18 Let A be an r*ω-set in an Extremally Disconnected space (X,τ). The followings are equivalent.

(i) A is α-ω-open

(ii) A $\subseteq IntClInt_{\omega}$A

(iii) A is semi-ω-open

(iv) A is semi-open

(v)A $\subseteq Cl_{\delta}Int$A.

(vi)A $\subseteq IntCl_{\delta}Int$A.

Proof. Let A be an r*ω-set in an Extremally Disconnected space (X,τ). Then using Proposition 3.17, we have

$ClInt$A= $ClInt_{\omega}$A= $Cl_{\delta}Int$A $\subseteq IntClInt_{\omega}$A = $Int_{\omega}ClInt_{\omega}$A = $IntCl_{\delta}Int$A. (N)

Therefore

A is α-ω-open $\Leftrightarrow$ A$\subseteq Int_{\omega}ClInt_{\omega}$A $\Leftrightarrow$ A $\subseteq IntClInt_{\omega}$A

$\Leftrightarrow$ A $\subseteq ClInt_{\omega}$A $\Leftrightarrow$ A is semi-ω-open

$\Leftrightarrow$ A $\subseteq ClInt$A $\Leftrightarrow$ A is semi-open

$\Leftrightarrow$ A $\subseteq Cl_{\delta}Int$A$\Leftrightarrow$ A $\subseteq IntCl_{\delta}Int$A.

This proves the proposition.

Proposition 3.19 Let A be an r*ω-set in an Extremally Disconnected space (X,τ). The followings are equivalent.

(i) A is regular closed

(ii) A= $Int_{\omega}ClInt_{\omega}$A

(iii) A = $IntClInt_{\omega}$A

(iv) A = $ClInt_{\omega}$A

(v) A = $Cl_{\delta}Int$A

(vi) A = $IntCl_{\delta}Int$A

Proof. Let A be an r*ω-set in an Extremally Disconnected space (X,τ). Then using (N) we have

A is regular closed $\Leftrightarrow$ A $\subseteq$ *ClInt*A

$\Leftrightarrow$ A= $Int_{\omega}ClInt_{\omega}$A

$\Leftrightarrow$ A = $IntClInt_{\omega}$A

$\Leftrightarrow$ A = $ClInt_{\omega}$A

$\Leftrightarrow$ A = $Cl_{\delta}Int$A

$\Leftrightarrow$ A = $IntCl_{\delta}Int$A

This proves the proposition.

Proposition 3.20 Let A be an r*ω-set in an Extremally Disconnected space (X,τ). The followings are equivalent.

(i) A is pre-closed

(ii) A$\supseteq$ $Int_{\omega}ClInt_{\omega}$A

(iii) A$\supseteq$ $IntClInt_{\omega}$A

(iv) A $\supseteq ClInt_{\omega}$A

(v) A $\supseteq Cl_{\delta}Int$A

(vi) A $\supseteq IntCl_{\delta}Int$A.

Proof. Let A be an r*ω-set in an Extremally Disconnected space (X,τ). Then using (N) we have

A is pre-closed $\Leftrightarrow$ A $\supseteq$ *ClInt*A

$\Leftrightarrow$ A$\supseteq$ $Int_{\omega}ClInt_{\omega}$A

$\Leftrightarrow$ A$\supseteq$ $IntClInt_{\omega}$A

$\Leftrightarrow$ A $\supseteq ClInt_{\omega}$A

$\Leftrightarrow$ A$\supseteq$= $Cl_{\delta}Int$A

$\Leftrightarrow$ A $\supseteq IntCl_{\delta}Int$A

This proves the proposition.

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