Pythagorean Vague b-Open Sets between Pythagorean Vague Topological Spaces

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Abstract: In this paper we devoted to study a new class of Pythagorean vague b-open sets between Pythagorean vague topological spaces. And we also contribute to study several properties of Pythagorean vague b-open sets and also discussed various relationship between Pythagorean vague semi closure and Pythagorean vague pre closure are presented.

Keywords: Pythagorean Vague b-Open set(PVbOs), Pythagorean Vague b-Closed set (PVbCs), Pythagorean Vague b-closure, Pythagorean Vague b-interior, Pythagorean vague semi-interior, Pythagorean vague semi-closure, Pythagorean vague pre-interior, Pythagorean vague pre- closure.

I. Introduction:

In 1996 Andrijevic [2] introduced b-open sets in topological spaces. After that Caldas and jafari [4] proposed the application of b-open set in topological spaces in 2007. In 2008 Chandrashekhara and Vaithillinghum [5] presented b-open set in Topological spaces. Characterization of b-open sets are presented by Vidyottama Kumari et.al[10]. The concept of fuzzy set was introduced by Zadeh [16] in 1965. The membership of an element to a fuzzy set a single value between 0 and 1. Chang [3] was the first to introduce the concept of fuzzy topology on a set X. Atanassov [1] initiated the concept of intuitionistic fuzzy set (IFS), which is a generalization of zadesh fuzzy set in 1986. The theory of vague set was proposed by Gaw and Buchere [6] as an extension of fuzzy set theory. Yager[14,15] introduced Pythagorean fuzzy sets characterized by a membership degree and non-membership degree which satisfies that the condition that the square sum of its membership degree and non membership degree is less than or equal to 1.In 2015 Peng.et.al [7] proposed the concept of Pythagorean fuzzy soft sets. Peng and Yang [8] studied some results of Pythagorean fuzzy sets in 2015. In 2016 Gou, Xu and Ren

[13] presented the properties of continuous Pythagorean fuzzy information. Vinnarasi and Nirmala Irudayam[11] introduce the Pythagorean vague topological space in 2018. Taha Yasin and Adem [9] studied some structures on Pythagorean fuzzy topological spaces in 2020.

II. PreliminariesDefinition 2.1[1]:

A fuzzy set $A = \{< u, \mu A(u) > | u \in U\}$ in a universe of discourse U is characterized by a membership function, μA, as follows: $\mu A : U \to [0, 1]$.

Definition 2.2[3]:

Let A and B be fuzzy sets in a space X = {x}, with the grades of membership of x in A and B denoted by $\mu A(x)$ and $\mu B(x)$, respectively. Then

A = B iff $\mu A(x) = \mu B(x)$ A ⊆ B iff $\mu A(x) \leq \mu B(x)$.

A ∪ B = Max[$\mu A(x)$, $\mu B(x)$] for all x ∈X. A ∩ B = Min[$\mu A(x)$, $\mu B(x)$] for all x ∈X. $A^c = 1 - \mu A(x)$ for all x ∈X.

Definition 2.3[1]:

Let X be a non-empty set. Then A is called an Intuitionistic Fuzzy set (in short, IFS) of X , if it is an object having the form A = { < x, μA ,γA > | x ∈ X } where the function μA : X→[0,1] and γA : X → [0,1] denote the degree of membership μA (x) and degree of non membership $\gamma A(x)$ of each element x∈ X to the set A and satisfies the condition that, $0 \leq \mu A(x) + \gamma A(x) \leq 1$.

Definition 2.4[1]:

If A and B are two IFSs of the set X, then A^c = {< x, $vA(x)$, $\mu A(x)$ > |x ∈ X},

A ⊆ B iff ∀ x ∈ X, $\mu A(x) \leq \mu B(x)$ and $vA(x) \geq vB(x)$, A = B iff ∀ x ∈ X, $\mu A(x) = \mu B(x)$ and $vA(x) = vB(x)$,

A ∪ B = {< x, max($\mu A(x)$, $\mu B(x)$), min($vA(x)$, $vB(x)$) > |x ∈ X}, and

A ∩ B = {< x, min($\mu A(x)$, $\mu B(x)$), max($vA(x)$, $vB(x)$) > | x ∈ X}.

Definition 2.5[6]:

A vague set V in a universe of discourse X is characterized by a true membership function t_V , and a false membership function f_V, as follows: t_V : U → [0, 1], f_V: U → [0, 1], and $t_V+f_V \leq 1$, where t_V (x) is a lower bound on the grade of membership of x derived from the evidence for x, and f_V (x) is a lower bound on the grade of membership of the negation of x derived from the evidence against x. The vague set A is written as,

A={ < x, $t_A(x)$, 1-$f_A(x)$> |x ∈ X}.

Definition 2.6[6]:

Let A and B be vague sets of the form A={ < x, $t_A(x)$, 1-$f_A(x)$> |x ∈ XB={ < x, $t_B(x)$, 1-$f_B(x)$> |x ∈ X}.Then

$A \square B$ if and only if $t_A(x) \leq t_B(x)$ And $1-f_A(x) \leq 1-f_B(x)$.

A=B if and only if $A \subseteq B$ and $B \subseteq A$.

A^c ={ < x, 1-f_A(x),t_A(x) > |x ∈ X}.

A ∪ B = { < x, max (t_A(x), t_B(x)), max (1-f_A(x), 1-f_B(x)) > |x ∈ X}.

A ∩ B = { < x, min (t_A(x), t_B(x)), min (1-f_A(x), 1-f_B(x)) > |x ∈ X}.

Definition 2.7 [6]:

A vague topology (VT in short) on X is a family A of vague sets(VS in short) in X satisfying the following axioms:

(1) 0,1 ∈ A;

(2) G_1 ∩ G_2 ∈ A for any G_1, G_2 ∈ A.

(3) ∪ G_i ∈ A for any family {G_i : i ∈ N} ⊆ A.

In this case the pair (X, A) is called a vague topological space (VTS in short) and any vague set in A is known as a vague open set (VOS) in X. The complement A^c of a VOS A in a VTS (X,A) is called a vague closed set (VCS in short) in X.

Definition 2.8[14]

Let X be a universe of discourse. An Pythagorean Fuzzy Set (PFS) P in X is given by P = {< x,μ_P (x), ν_P (x) >| x ∈ X}, where μ_P : X → [0,1] denotes the degree of membership and ν_P : X →

[0,1] denotes the degree of nonmembership of the element x ∈ X to the set P, respectively, withthe condition that $0 \leq (\mu_P(x))^2 + (\nu_P(x))^2 \leq 1$.

DEFINITION 2.9 [11]

Let X be a universe of discourse. A Pythagorean Vague Set (PVS) A in X is denoted as A = {< x, t_A (x), 1-f_A (x) >| x ∈ X}, where t_A(x): X → [0,1] denotes the truth value and 1

-f_A(x)

: X → [0,1] denotes the false value of the element x ∈ X to the set A, respectively, with the $\sqrt{1-[(t_A(x))^2+(f_A(x))^2]}$

condition that $0 \leq t_A{}^2(x) + f_A{}^2(x) \leq 1$. A degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) =$

Definition 2.10[12] *and* $\pi_A(x) \in$[0,1].Let A and B be Pythagorean vague sets of the form A={ < x, t_A(x), 1-f_A(x)> |x ∈ X} andB={ < x, t_B(x), 1-f_B(x)> |x ∈X}.Then

$A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ And 1-f_A(x) ≤1-f_B(x).

A=B if and only if $A \subseteq B$ and $B \subseteq A$.

A^c ={ < x, $f_A(x)$,1-$t_A(x)$ > |x ∈ X}.

A ∪ B = { < x, max ($t_A(x)$, $t_B(x)$), max (1-$f_A(x)$, 1-$f_B(x)$) > |x ∈ X}.

A ∩ B = { < x, min ($t_A(x)$, $t_B(x)$), min (1-$f_A(x)$, 1-$f_B(x)$) > |x ∈ X}.

Definition 2.11[12]

Let τ be the collection of Pythagorean vague sets over X, then τ is said to bePythagorean vague topology on X if

[1] 0,1 belongs to τ .

[2] the union of any two Pythagorean vague sets in τ belongs to τ .

[3] the intersection of any two Pythagorean vague sets in τ belongs to τ .

The (X, τ) is called the a Pythagorean vague topological space(PVTS) and any Pythagoreanvague set in τ is known as a Pythagorean vague open (PVOS) set X.

The complement of an Pythagorean vague open set in (X, τ) is called Pythagoreanvague closed set (PVCs) in X.

Definition 2.12[12]

Let (X, τ) be an Pythagorean vague topological space(PVTS) and Pythagorean vague

set A={x,< $t_A(x)$, $1 - f_A(x)$ >} of X is called,

PVcl(A)= ∩{K / K is an (PVCS) in X and K is containing in A ($A \subseteq K$)} PVint (A)= ∪ {G / G is an (PVOS) in X and G is contained in A ($G \subseteq A$)}

III Pythagorean vague b-open & Pythagorean vague closed sets

Definition 3.1:

A Pythagorean vague set A in a Pythagorean vague topological space

Be (X, τ) is said to

(i) Pythagorean vague semi open set (PVsOs in short) if

(ii) Pythagorean vague pre open set (PVpOs in short) if

(iii) Pythagorean vague α open set (PVα Os in short) if$A \subseteq PVcl(PV\text{int}(A))$. $A \subseteq PV\text{int}(PVcl(A))$.

$A \subseteq PV\text{int}(PVcl(PV\text{int}(A)))$.

(iv) Pythagorean vague β open set (PV β Os in short) if $A \subseteq PVcl(PV\,\text{int}(PVcl(A)))$.

(v) Pythagorean vague regular open set (PVr Os in short) if $A = PV\,\text{int}(PVcl(A))$.

The Complement of the above Pythagorean vague open sets are called the Pythagorean vagueclosed sets.

Definition 3.2:

A Pythagorean vague set A in a PVTS X is called Pythagorean vague b-open set (PVbOs) if Pythagorean vague b-closed set (PVbCs) if

Definition 3.3: $\boldsymbol{A \subseteq PV\,\text{int}(PVcl(A)) \cup PVcl(PV\,\text{int}(A))}$. $\boldsymbol{A \supseteq PV\,\text{int}(PVcl(A)) \cap PVcl(PV\,\text{int}(A))}$.

Let(X, τ) be a PVTS and A be a PVS in X. ThenPythagorean vague b-interior of A (PVbint(A)) is the union of all Pythagorean vague b-open sets of X contained in A.

(i.e) $PVb\text{int}(A) = \cup\{S : S \text{ is a } PVbOS \text{ in } X \text{ and } S \subseteq A\}$

(a) Pythagorean vague b-closure of A (PV b cl(A)) is the intersection of all Pythagoreanvague b-closed sets of X containing in A.

(i.e) $PVbcl(A) = \cup\{T : T \text{ is a } PVbCS \text{ in } X \text{ and } T \supseteq A\}$.

Definition 3.4

Let(X, τ) be a PVTS and A be a PVS in X. Then

(a) Pythagorean vague semi-interior of A (PVsint(A)) is the union of all Pythagorean vagues-open sets of X contained in A.

(i.e) $PVs\,\text{int}(A) = \cup\{U : U \text{ is a } PVsOS \text{ in } X \text{ and } U \subseteq A\}$

(b) Pythagorean vague semi-closure of A (PVscl(A)) is the intersection of all Pythagoreanvague semi-closed sets of X containing in A.

(i.e) $PVscl(A) = \cup\{V : V \text{ is a } PVsCS \text{ in } X \text{ and } V \supseteq A\}$.

Definition 3.5 Let (X, τ) be a PVTS and A be a PVS in X. ThenPythagorean vague pre-interior of A (PVpint(A)) is the union of all Pythagorean vague pre-open sets of X contained in A.

(i.e) $PVp\,\text{int}(A) = \cup\{S : S \text{ is a } PVpOS \text{ in } X \text{ and } S \subseteq A\}$

(a) Pythagorean vague pre-closure of A (PV pcl(A)) is the intersection of all Pythagoreanvague pre-closed sets of X containing in A.

(i.e) $PVpcl(A) = \cup\{T : T \text{ is a } PVpCS \text{ in } X \text{ and } T \supseteq A\}$.

Proposition 3.6 Let (X, τ) be a PVTS and A and B be a PVS in X. Then the Pythagorean vague b

operator satisfies the following properties.

(i) PVbcl(0)=0 PVbcl(1)=1

(ii) $A \subseteq PVbcl(A)$

(iii) $PVb\text{int}(A) \subseteq A$

(iv) If A is Pythagorean vague b closed set then A= PVbcl(PVbcl(A))

(v) $A \subseteq B \Rightarrow PVbcl(A) \subseteq PVbcl(B)$

(vi) $A \subseteq B \Rightarrow PVb\text{int}(A) \subseteq PVb\text{int}(B)$

Lemma 3.7:

Let A be a Pythagorean vague set in a Pythagorean vague topological space X. Then

(i) $[\text{PVbint}(A)]^c = \text{PVbcl}(A^c)$

(ii) $[\text{PVbcl}(A)]^c = \text{PVbint}(A^c)$.

Proof:

(i) Let A be Pythagorean vague set in X.

$PVb\text{int}(A) = \cup\{S : S \text{ is a } PVbOS \text{ in } X \text{ and } S \subseteq A\}$

Then $[PVb\text{int}(A)]^C = [\cup\{S : S \text{ is a } PVbOS \text{ in } X \text{ and } S \subseteq A\}]^C$

$= \cap\{S^c : S^c \text{ is a } PVbCS \text{ in } X \text{ and } A^c \subseteq S^c\}$

Replacing S^c by B, we get

$[PVb\text{int}(A)]^C = \cap\{B : B \text{ is a } PVbCS \text{ in } X \text{ and } B \supseteq A^c\}$.

$[PVb\text{int}(A)]^C = PVbcl(A^c)$.

(ii) $PVbcl(A) = \cap\{E : E \text{ is a } PVbCS \text{ in } X \text{ and } E \supseteq A\}$

Then $[PVbcl(A)]^C = [\cap\{E : E \text{ is a } PVbCS \text{ in } X \text{ and } E \supseteq A\}]^C$

$= \cup\{E^c : E^c \text{ is a } PVbOS \text{ in } X \text{ and}$

Replacing E^c by F, we get $A^c \supseteq E^c\}$

$[PVbcl(A)]^C = \cup\{F : F \text{ is a } PVbOS \text{ in } X \text{ and } F \subseteq A^c\}$.

$[PVbcl(A)]^C = PVb\text{int}(A^c)$.

Theorem 3.8

In Pythagorean vague Topological space (X, τ)

(i) Every Pythagorean vague open set is Pythagorean vague b-open set.

(ii) Every Pythagorean vague pre-open set is Pythagorean vague b-open set.

(iii) Every Pythagorean vague semi-open set is Pythagorean vague b-open set.

Proof:

(i) Let A be a Pythagorean vague open set in X. Then

$A \subseteq PVcl(A)$ *and* $A \subseteq PV\text{int}(A)$, $PV\text{int}(A) \subseteq PV\text{int}(PVcl(A))$ *then* $PV\text{int}(A) \subseteq PVcl(PV\text{int}(A))$ *which implies* $PV\text{int}(A) \subseteq PV\text{int}(PVcl(A)) \cup PVcl(PV\text{int}(A))$.

Hence $A \subseteq PV\text{int}(A) \subseteq PV\text{int}(PVcl(A)) \cup PVcl(PV\text{int}(A))$.

and A is Pythagorean vague b- open set.

(ii) Let A be a Pythagorean vague pre open set in X. Then

$A \subseteq PV\text{int}(PVcl(A)) \Rightarrow A \subseteq PV\text{int}(PVcl(A)) \cup PV\text{int}(A) \subseteq PV\text{int}(PVcl(A)) \cup PVcl(PV\text{int}(A))$

Hence A is Pythagorean vague b- open set.

(iii) Let A be a Pythagorean vague semi open set in X. Then

$A \subseteq PVcl(PV\text{int}(A)) \Rightarrow A \subseteq PVcl(PV\text{int}(A)) \cup PV\text{int}(A) \subseteq PVcl(PV\text{int}(A)) \cup PV\text{int}(PVcl(A))$

Thus A is Pythagorean vague b- open set.

Remark 3.9

Converse of the above theorem need not be true as seen in the following example.

Example 3.10

Let Let (X, τ) be a Pythagorean vague Topological space and X={a,b}.

$G_1 = \{x, (0.1,0.2)/a, (0.2,0.2)/b\}$

$G_2 = \{x, (0.2,0.3)/a, (0.3,0.3)/b\}$

Then $\tau = \{0,1,G_1,G_2\}$ is Pythagorean vague Topological on X.

Let $A = \{x,(0.3,0.3)/a, (0.3,0.4)/b\}$. The set A is Pythagorean vague b-open set but not Pythagorean vague open set.

Example 3.11

Let (X,τ) be a Pythagorean vague Topological space and X={a,b}.

Let $G_1 = \{x,(0.2,0.2)/a,(0.3,0.4)/b\}$

$G_2 = \{x,(0.3,0.3)/a,(0.4,0.4)/b\}$

Then $\tau = \{0,1,G_1,G_2\}$ is Pythagorean vague Topological on X.

Let $A = \{x,(0.3,0.4)/a, (0.4,0.4)/b\}$. The set A is Pythagorean vague b-open set but not Pythagorean vague pre-open set.

Example 3.12

Let (X,τ) be a Pythagorean vague Topological space and X={a,b}.

Let $G_1 = \{x,(0.3,0.3)/a,(0.2,0.2)/b\}$

$G_2 = \{x,(0.2,0.2)/a,(0.2,0.2)/b\}$

Then $\tau = \{0,1,G_1,G_2\}$ is Pythagorean vague Topological on X.

Let $A = \{x,(0.2,0.3)/a, (0.2,0.8)/b\}$. The set A is Pythagorean vague b-open set but not Pythagorean vague semi-open set.

Theorem 3.13

Let A be Pythagorean vague set of a space (X,τ). Then

(i)

(ii)

Proof $\boldsymbol{PVscl(A) = A \cup PV\,\mathrm{int}(PVcl(A))\quad PV\,\sin t(A) = A \cap PVcl(PV\,\mathrm{int}(A))}$

(i) Since $PVscl(A) \subseteq PV\,\mathrm{int}(PVcl(PVscl(A))) \subseteq PV\,\mathrm{int}(PVcl(A))$.

$A \cup PVscl(A) = PVscl(A) \subseteq A \cup PV\,\mathrm{int}(PVcl(A))$

$A \cup PV\,\mathrm{int}(PVcl(A)) \supseteq PVscl(A) \qquad (a)$

$And\ A \square\ PVscl(A), PV\operatorname{int}(PVcl(A)) \square\ PV\operatorname{int}(PVcl(PVsc(A))) \square\ PVscl(A)$

$A \cup PV\operatorname{int}(PVcl(A)) \square\ PVscl(A) \cup A \square\ PVscl(A) \square \square \square (b)$

$From(a) \,\&\, (b)\ PVscl(A) \square\ A \cup PV\operatorname{int}(PVcl(A)).$

(ii) can be proved by taking compliment of (i).

Theorem 3.14

Let A be Pythagorean vague set of a space $(X, \square)$. Then

(i)

(ii)

Proof $\boldsymbol{PVpcl(A) \square\ A \cup PVcl(PV\operatorname{int}(A))\ PVp\operatorname{int}(A) \square\ A \cap PV\operatorname{int}(PVcl(A))}$

(i) Since $PVpcl(A) \square\ PVcl(PV\operatorname{int}(PVp\operatorname{int}(A))) \square\ PVcl(PV\operatorname{int}(A)).$

$A \cup PVpcl(A) \square\ PVpcl(A) \square\ A \cup PVcl(PV\operatorname{int}(A))$

$A \cup PVcl(PV\operatorname{int}(A)) \square\ PVpcl(A) \square \square \square \square \square \square \square \square \square \square \square (a)$

$nd\ A \square\ PVpcl(A), PV\operatorname{int}(PVcl(A)) \square\ PV\operatorname{int}(PVcl(PVpcl(A))) \square\ PVpcl(A)$

$A \cup PVcl(PV\operatorname{int}(A)) \square\ PVpcl(A) \cup A \square\ PVpcl(A) \square \square \square (bFrom(a) \,\&\, (b)\ PVpcl(A) \square\ A \cup PVcl(PV\operatorname{int}(A)).$

(ii) can be proved by taking compliment of (i).

Theorem 3.15

Let A be a Pythagorean vague set in $(X, \square)$. Then

(i)

(ii)

Proof $\boldsymbol{PVbcl(A) \square\ PVscl(A) \cap PVpcl(A)\ PVb\operatorname{int}(A) \square\ PVs\operatorname{int}(A) \cup PVp\operatorname{int}(A)}$

(i) Since PVbcl(A) is a Pythagorean vague b-closed set.

Then $PV\operatorname{int}(PVcl(PVbcl(A))) \cap PVcl(PV\operatorname{int}(PVbcl(A))) \square\ PVbcl(A)$

Which implies that $PV\operatorname{int}(PVcl(A)) \cap PVcl(PV\operatorname{int}(A)) \square\ PVbcl(A)$

(i.e) $A \cup [PV\operatorname{int}(PVcl(A)) \cap PVcl(PV\operatorname{int}(A))] \square\ A \cup PVbcl(A)$

$[A \cup PV\operatorname{int}(PVcl(A))] \cap [A \cup PVcl(PV\operatorname{int}(A))] \square\ PVbcl(A)\ PVscl(A) \cap PVpcl(A) \square\ PVbcl(A) \square \square \square \square \square \square \square \square \square \square (1)$

Since $PVbcl(A) \square PVscl(A) and PVbcl(A) \square PVpcl(A)$

then $PVbcl(A) \square PVscl(A) \cap PVpcl(A) \square \square \square \square \square \square \square \square\square \square$ (2)

form (1) &(2) we get $PVbcl(A) \square PVscl(A) \cap PVpcl(A)$.

(ii) Since PV b int (A) is a Pythagorean vague b-open set.

Then $PVcl(PV\text{int}(PVb\text{int}(A))) \cup PV\text{int}(PVcl(PVb\text{int}(A))) \square PVb\text{int}(A)$

Which implies that $PVcl(PV\text{int}(A)) \cup PV\text{int}(PVcl(A)) \square PVb\text{int}(A)$(i.e) $A \cap [PVcl(PV\text{int}(A)) \cup PV\text{int}(PVcl(A))] \square A \cap PVb\text{int}(A)$

$[A \cap PVcl(PV\text{int}(A))] \cup [A \cap PV\text{int}(PVcl(A))] \square PVb\text{int}(A)$ $PV\sin t(A) \cup PVp\text{int}(A) \square PVb\text{int}(A) \square \square \square \square \square \square \square \square \square$ (1)

Since $PVb\text{int}(A) \square PV\sin t(A) and PVb\text{int}(A) \square PVp\text{int}(A)$

then $PVb\text{int}(A) \square PV\sin t(A) \cup PVp\text{int}(A) \square \square \square \square \square \square \square \square\square \square$ (2)

form (1) &(2) we get

Theorem 3.16 $\boldsymbol{PVb\text{int}(A) \square PV\sin t(A) \cup PVp\text{int}(A)}$**. .**

(i)

(ii)

Proof Let A be Pythagorean vague set of a space $(X, \square)$. Then

$PVscl(PV\sin t(A)) \square PV\sin t(A) \cup PV\text{int}(PVcl(PV\text{int}(A)))$ $PV\sin t(PVscl(A)) \square PVscl(A) \cup PVcl(PV\text{int}(PVcl(A)))$Let $PVscl(PV\sin t(A)) \square PV\sin t(A) \cup PV\text{int}[PVcl(PV\sin t(A))]$

$\square PV\sin t(A) \cup PV\text{int}[PVcl(A \cap PVcl(PV\text{int}(A)))]$

$\square PV\sin t(A) \cup PV\text{int}[PVcl(A) \cap PVcl(PV\text{int}(A))]$

$\square PV\sin t(A) \cup PV\text{int}[PVcl(PV\text{int}(A))]$Now we consider

$PVscl(Pv\sin t(A)) \square PV\sin t(A) \cup PV\text{int}(PVcl(PV\sin t(A))) \square PV\sin t(A) \cup PV\text{int}(PVcl(PV\text{int}(A)))$

$\square$ $PVscl(PV\sin t(A)) \square PV\sin t(A) \cup PV\text{int}(PVcl(PV\text{int}(A)))$.

(i) Similarly we can prove (ii)

Theorem 3.17

Let A be Pythagorean vague set of a space $(X, \square)$. Then

(i)

(ii)

Proof $PVpcl(PVp\,\mathrm{int}(A))$ □ $PVp\,\mathrm{int}(A) \cup PVcl(PV\,\mathrm{int}(A))$ $PVp\,\mathrm{int}(PVpcl(A))$ □ $PVpcl(A) \cup PV\,\mathrm{int}(PVcl(A))$Let $PVpcl(PVp\,\mathrm{int}(A))$ □ $PVp\,\mathrm{int}(A) \cup PVcl[PV\,\mathrm{int}(PVp\,\mathrm{int}(A))]$

□ $PVp\,\mathrm{int}(A) \cup PVcl[PV\,\mathrm{int}(A \cap PV\,\mathrm{int}(PVcl(A)))]$

□ $PVp\,\mathrm{int}(A) \cup PVcl[PV\,\mathrm{int}(A) \cap PV\,\mathrm{int}(PVcl(A))]$

□ $PVp\,\mathrm{int}(A) \cup PVcl[PV\,\mathrm{int}(A)]$Now we consider

$PVpcl(Pvp\,\mathrm{int}(A))$ □ $PVp\,\mathrm{int}(A) \cup PVcl(PVp\,\mathrm{int}(PVp\,\mathrm{int}(A)))$ □ $PVp\,\mathrm{int}(A) \cup PVcl(PV\,\mathrm{int}(A))$

□ $PVpcl(PVp\,\mathrm{int}(A))$ □ $PVp\,\mathrm{int}(A) \cup PVcl(PV\,\mathrm{int}(A))$.

(i) Similarly we can prove (ii)

Theorem 3.18

Let A be a Pythagorean vague set of a space $(X, □)$. Then the following are equivalent:

(i) A is Pythagorean vague b-open set.

(ii)

(iii)

Proof A □ $PVp\,\mathrm{int}(A) \cup PV\sin t(A)$ A □ $PVpcl(PVp\,\mathrm{int}(A))$□ (ii) Let A be a Pythagorean vague b-open set.

$(i.e) A$ □ $PV\,\mathrm{int}(PVcl(A)) \cup PVcl(PV\,\mathrm{int}(A))$ *by* $u\sin g theorem$ 3.15 & 3.16, *we get*,

$PVp\,\mathrm{int}(A) \cup PV\sin t(A)$ □ $(A \cap PV\,\mathrm{int}(PVcl(A))) \cup (A \cap PVcl(PV\,\mathrm{int}(A)))$

□ $A \cap (PV\,\mathrm{int}(PVcl(A))) \cup (PVcl(PV\,\mathrm{int}(A)))$ □ A.

Hence A □ $PVp\,\mathrm{int}(A) \cup PV\sin t(A)$.

(i) □ $(iii) u\sin g theorem$ 3.15 & 3.16,

A □ $PVp\,\mathrm{int}(A) \cup PV\sin t(A)$ □ $PVp\,\mathrm{int}(A) \cup (A \cap PVcl(PV\,\mathrm{int}(A)))$ □ $PVp\,\mathrm{int}(A) \cup PVcl(PV\,\mathrm{int}(A))$

□ $PVpcl(PVp\,\mathrm{int}(A))$.

(ii)) □ $(i)\sin ce, A$ □ $PVp\,\mathrm{int}(A) \cup PVcl(PV\,\mathrm{int}(A))$ □ $PV\,\mathrm{int}(PVcl(A)) \cup PVcl(PV\,\mathrm{int}(A))$.

Therefore A is Pythagorean vague b-open set.

Theorem 3.19 Let A be a Pythagorean vague set of a space $(X, □)$. Then

$PVb\text{int}(PVbcl(A)) \square PVbcl(PVb\text{int}(A))$.

Proof

Let A be a subset of a space X.

$PVb\text{int}(PVbcl(A)) \square PV\sin t(PVbcl(A)) \cup PVb\text{int}(PVbcl(A))$

$\square PVbcl(PV\sin t(A)) \cup PVp\,\text{int}(PVbcl(A))$

$\square PVscl(PV\sin t(A)) \cup PVp\,\text{int}(PVpcl(A)) \square \square \square \square \square \square \square \square \square (a)$

Now, $PVbcl(PVb\text{int}(A)) \square PVbcl(PV\sin t(A)) \cup PVp\,\text{int}(A)$

$\square PVbcl(PV\sin t(A)) \cup PVbcl(PVp\,\text{int}(A))$

$\square PVscl(PV\sin t(A)) \cup PVp\,\text{int}(PVpcl(A)) \square \square \square \square \square \square \square (b)$

From (a)&(b),

Conclusion:

$\boldsymbol{PVb\text{int}(PVbcl(A)) \square PVbcl(PVb\text{int}(A))}$**.**

In this paper, we introduce the concept of Pythagorean vague b-open sets and Pythagorean vague b-closed in Pythagorean vague topological spaces and investigate some of their properties. And also we introduce Pythagorean vague b-interior and Pythagorean vague b-closure and established several properties.

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