Comparison of Mathematical Modelling Skills of Secondary and Tertiary Students

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Abstract

One of the pillars of scientific endeavor is the use of mathematics to model actual physical or natural processes. Inconsistencies and considerable variations in the analyses and representations or pictures of mathematical modeling are found in previous studies, according to a review of such works. New definitions of a system are created, such as "real-world problem," "mathematical problem," and "mathematical solution," as well as "modeling one system as another" and "connecting a real-world problem to a mathematical problem." This study's main aim is to find out if there are any variations in the mathematical modeling abilities of secondary and tertiary students attending a highly intellectual university, as well as what those differences are. To do this, we evaluated students' modeling abilities using a multiple-choice question instrument that has been widely utilized in experimental investigations at several universities.

Keywords: Modelling, Students, Mathematics, Problem, University.

I. INTRODUCTION

The Ministry of Education and Culture's Regulation No. 22 for the year 2016 outlines the learning principles in the curriculum for 2013, and it focuses on the traditional processes like shifting from learning that emphasizes single answers to learning with answers that are multidimensional in truth and from verbal learning toward practical skills. Additionally, students must complete real-world mathematical challenges. These requirements are also included in the PISA tests, which are part of the OECD's PISA framework. The capacity to transform real-world issues into mathematical ones, which might entail organizing, conceptualizing, establishing assumptions, and formulating a mathematical model corresponding to the real situation, is a key component of mathematical literacy. Making the kids aware of the real-world applications of math issues is one of the most crucial abilities in today's world.

The problem with math today is that students find it difficult to link what they learn in class to real-world situations. In addition to misconceptions, a mistake in changing the information provided to the mathematical sentence, an inability to determine the formula to be used to solve problems, a lack of strong prerequisite material understanding needed for solving problems, a mistake in interpreting the solution, and a lack of accuracy on the part of the students in the calculation are other causes of errors students make when solving problems related to real life. Mathematical modeling, according to Lawson and Marion, is the process of converting issues from the actual world into mathematical terms. In order to represent, analyze, and make a prediction in an effort to solve a problem in the real world, mathematical modeling is the process of converting a problem in the real world to a type of mathematics that cannot be separated from math skills such as reading, communicating, designing, putting problem-solving techniques into practice, or working methodically.

In order to understand the mechanisms at work in nature and to forecast how these real-world systems will behave in the future under various circumstances, it has been helpful to relate a real-world system to mathematics. Many times, the term "mathematical modeling" refers to these procedures as well as others. The mathematical modeling procedures and ideas have been represented in a variety of ways by previous writers, as seen in Figures 1-3 below. The process of dissecting mathematical modeling and creating a representation results in the creation of a mathematical modeling model. Thus, there is a chance that the initial mathematical model and the method of modeling it with an image may be confused.

We distinguish several elements of mathematical modeling in order to clarify our discussion and the comparison of earlier models. Finding a real-world issue and applying it to define a mathematical issue is one of the numerous stages involved in mathematical modeling. The real-world issue may be viewed as a physical system made up of things like forces and objects. A second mathematical system of mathematical objects and processes can be conceived of as the related mathematical issue. Thus, the transformation of a physical system into a mathematical system may be seen as a process. This specific procedure is known as "mathematising," or turning a real-world situation into a mathematical one. Of course, solving the mathematical system and comparing the result to the real world are just two of the crucial steps in mathematical modeling. Analyzing these older graphics of mathematical modeling is helpful in identifying the various systems and processes. Below, these words will be formalized. The theory will then be applied to develop a comprehensive theory that will allow for the reconciliation of all previous pictures.

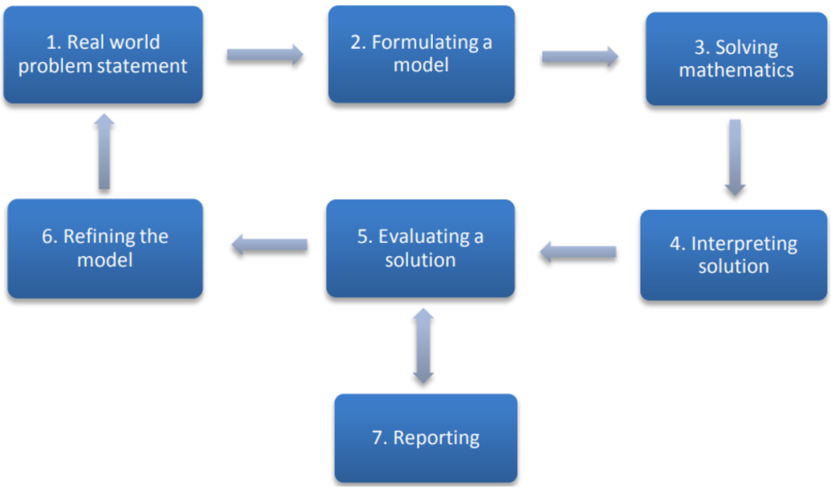

Figure 1: Early image of modeling

The second through seventh phases are procedures in mathematical modeling. Stage 1 is an illustration of the real-world issue statement's system.

In the process of mathematical modeling, new systems are created in order to improve the perception of modeling for teaching and learning. New procedures between all the systems are also looked at. To identify and recognize significant processes in the classroom, the new systems and processes are featured in a new graphic. The previous modeling pictures are then represented and brought into harmony using the new theory and image. The new modeling systems and procedures provide a better representation of modeling in the classroom and an enhanced theory of pedagogical modeling. Using the same design, the new graphic may also be used to illustrate modeling in learning, teaching, and research. Thus, the new picture offers a way to describe the anatomy of modeling and a tool to identify each student's unique challenges with particular mathematical modeling methods.

II. EXPERIMENTAL DETAILS

The examinations were given to 100 first-year students from two universities during the fall semester of 2021. All of the students were enrolled in Introduction to Mathematical Modeling, a one-semester course designed to expose first-year students to some fundamental modeling principles and applications. There are fifteen 90-minute lectures in the course, one every week. Each lesson begins with a modeling example that has a direct connection to the students' everyday lives. These examples may include some fundamental models drawn from engineering, government administration, operations management, marketing, and economics.

In a lecture, the instructor initially exposes the students to the modeling context for a particular modeling challenge before asking them to consider and attempt to solve the problem on their own for roughly 15 minutes. Following some students reporting their views in front of their peers, the teacher then guides student debate. Finally, the instructor offers three tried-and-true mathematical models for the problem, highlighting their fundamental presuppositions and discussing their benefits and drawbacks. After each lecture, students are asked to form groups (each group comprises of three or four students) and work independently to conceive, discuss, and construct mathematical models to address real-world issues of interest. This is done without assigning any homework outside of class. Each group is required to submit a course report to the instructor at the end of the course outlining their accomplishments outside of class. In the students' reports, a wide range of issues and themes from diverse contexts are often discussed.

As first-year students arrived at the university and had just finished the course's first lecture, Test 1 was given. At the conclusion of the first semester, Test 2 was administered. Within 20 minutes, all pupils finished both assessments. We measured secondary students' modeling abilities using the first test's findings, and we measured students' modeling abilities using the second test's results. It was demonstrated in Haines and Crouch that questions with the same number in the two exams were relatively similar and had equal difficulty (2001). Consider Question 6 as an example.

Question 6 in Test 1

Which one of the following options most closely models the height of a sunflower while it is growing (in terms of time t)?

A. $1-e^{-t}$ B. $(1-t)^2$ C. t D. $t-t^2$ E. $1/(1+e^{-t})$

Question 6 in Test 2

Which one of the following options most closely models the speed of a car starting from rest (in terms of time t)?

A. $1-e^{-t}$ B. $(1-t)^2$ C. t D. $t-t^2$ E. $1/(1+e^{-t})$

In order to answer either question, the modeler must consider the mathematical model that will be applied in that case, evaluate a solution, or compare the model to the actual problem. So it seems sense to compare the results of the two exams to see if students' modeling abilities have improved.

In addition to the second exam, we also required the students to submit their midterm results in Calculus and Linear Algebra, the two main first-year students' basic mathematics courses. This data was utilized to examine the relationship between modeling skills and knowledge from introductory mathematics courses.

III. RESULTS

Comparison of the Two Tests

The full score for each test is 12, with each question receiving one correct answer for two points and one or more partial correct responses earning one point. Table 1 summarizes the students' average test results on the two exams.

Table 1: Summary of Test 1 and Test 2

Question number	**1**	**2**	**3**	**4**	**5**	**6**	**Overall average**	**Standard deviation**
Test 1	1.17	1.35	1.68	1.42	1.91	1.78	9.32	1.86
Test 2	1.69	1.26	1.62	1.47	1.73	1.75	9.53	1.79
Sum	2.87	2.61	3.30	2.89	3.64	3.53	–	–

Table 1 shows that the two exams' total average results differ by only 0.21 points on average, with nearly identical standard deviations. Statistics reveal that there is no significant difference in the students' scores on the two tests (in the statistical studies in this chapter, we employ t-tests with the significance level of 0.1). We also counted the number of students who did better on Test 2, and we discovered that 90 of them did better on Test 2 than on Test 1, while 71 of them did worse. Statistics showed that the difference was not significant. Therefore, we draw the conclusion that there is no statistically significant difference between (upper) secondary and (first year) students' proficiency in mathematical modeling. To be more exact, first-year students at Delhi University did not improve their capacity to use mathematical models after just one semester of instruction. We were rather dissatisfied because, in a research similar to ours conducted by Kaiser (2007), students improved significantly from their pre-test score of 8.4 to their post-test score of 9.6 (both tests included eight testing items rather than the study's six). However, Haines et al. (2000) used data from JNU students and came to a similar result to our study. We believe the following are some of the potential causes:

Teachers should promote numerous answers and support students' distinct modeling approaches in order to provide high-quality instruction, but neither we nor our students performed much work along these lines in class or as homework. In reality, we focus more on imposing the tried-and-true patterns on the learners in our class. This encourages us to enhance our instructional strategies going forward to increase the effectiveness of our instruction.

The students that get admitted to Delhi University are among the finest. It is challenging to further improve because they have received very good marks (with an overall average score of 9.32 out of 12) on the pre-test (Test 1). A semester is a very little period of time. It is challenging to develop the pupils' modeling abilities in such a short period of time. We may determine the advantages and disadvantages of our pupils in the appropriate sub-competency by adding the scores obtained for two questions with the same number in the two examinations since questions with the same number assess the same sort of sub-competencies in the modeling process. Despite the fact that the two examinations were completed at different periods, it is appropriate to sum the results together since we have discovered that our kids did not make much improvement throughout one semester.

Table 2: Relative scores of Delhi and JNU students

	Test 1		**Test 2**		**Tests 1 & 2**	
Question	JNU (%)	Delhi (%)	JNU (%)	Delhi (%)	JNU (%)	Delhi (%)
1	15.8	12.6	18.0	17.9	17.0	15.2
2	12.0	14.5	7.2	13.2	9.5	13.9
3	14.7	18.1	22.9	17.0	19.1	17.5
4	21.4	15.2	19.7	15.5	20.5	15.3
5	22.6	20.5	20.5	18.1	21.4	19.3
6	13.5	19.1	11.7	18.3	12.5	18.7
Sum	100.0	100.0	100.0	100.0	100.0	100.0

Haines et al. (2000) checked students' comprehension at several points during the modeling process using a similar strategy. This concept was also used by Haines and Crouch (2001) to create a measure of modeling stage accomplishment. A look at Table 1 reveals that the students appeared to find it more difficult to answer Questions 1, 2, and 4 (only 71.6%, 65.3%, and 72.3% of the total marks were achieved), while they were most successful in answering Questions 3, 5, and 6 (82.6%, 91.1%, and 88.2% of the total marks were achieved). According to this finding, we should focus more on teaching students how to make fair assumptions for situations in the real world, comprehend the objectives of the modeling activities, and identify the necessary parameters, variables, and constants in order to enhance students' modeling skills.

Comparison with students at JNU

Now, we contrast the academic differences between Delhi students (in our experiment) and JNU students (hereinafter referred to as JNU students) with those reported by Haines and Crouch (2001). In our trial, Delhi students generally outperformed JNU students on each question. Noting that it is unfair and meaningless to directly compare Delhi students' test results to those of JNU students. As a result, we made the decision to examine the relative scores represented in Table 2. Here, a student's relative score is determined by dividing their overall test-question average score by their average score on the question in issue. For instance, the average score for Delhi students on the third question of Test 1 was 1.63 points, while their total average score was 9.32. The third question of Test 1's relative score equals 18.1% (=1.63/9.32) after that. It is plausible to assume Delhi students performed better in accordance with the sub-competency relating to the question in the process of modeling if the relative score earned by Delhi students in a question is much greater than that of JNU students.

According to Table 2, students at JNU scored considerably better on Question 4 than students at Delhi did on Question 6. This shows that JNU students have a comparatively better grasp of assigning parameters, variables, and constants than Delhi students, whereas Delhi students have a relatively better understanding of evaluating and selecting an appropriate model than JNU students. The variations in the other questions are very slight. Additionally, Question 2 appears to be the most challenging for all students, while Question 5 is the most straightforward for all.

Relationship with Basic Mathematics Courses

A significant amount of "pure" mathematical expertise and knowledge are required for serious mathematical modeling. It is only reasonable to examine the association between modeling abilities and the students' proficiency levels in fundamental mathematics since questions in our examinations also involve some mathematical reasoning. As in Dan and Xie (2011), we began by calculating the correlation coefficients between students' midterm test scores for two foundational mathematics courses—Calculus and Linear Algebra—and their proficiency in mathematical modeling. Since the correlation coefficients were both less than 0.2, the data cannot be used to make any inferences. However, by reexamining our data from a different angle, as shown below, we were able to arrive at a relevant conclusion.

We consider test 1 or test 2 participants who received 12 points to be strong modelers, and those who received no more than 6 points to be less talented. Similar to this, we see 22–24 points as a sign of excellent modeling abilities and no more than 15 points as an indicative of weak modeling skills when analyzing the combined scores for the two exams. Table 3 provides the comparative outcomes. Table 3 analysis reveals that there is no evidence of a link between students' calculus results and their modeling prowess in the experiment. Contrarily, all of the p-values for the linear algebra course are less than 0.1, which supports the finding that students who excel at modeling also excel at linear algebra, and vice versa, and that students who struggle with modeling tend to excel at linear algebra to a lesser extent. It is unclear why this occurs, and this merits more research and discussion. One explanation for this might be that while not all of the questions in our tests for modeling skills required calculus knowledge, knowledge of linear algebra was important to the test questions we utilized.

IV. DISCUSSION

This chapter presents some preliminary findings from an experiment carried out at Delhi University that compared the differences in modeling abilities between upper secondary and first-year students. We are aware of the challenges that students had when modeling, which will assist us develop our instructional strategy. The comparison with JNU students makes the gap between Delhi and JNU students' modeling abilities clear, which partly reflects the different ways that students in the two nations think.

Table 3 Relationship between modeling skills and mean scores on basic mathematics courses

		Number of students	Mean scores of calculus	Mean scores of linear algebra
Test 1	12 points	20	89.65	88.13
	No more than 6 points	15	87.60	82.00
	p-value	–	0.26	0.07
Test 2	12 points	18	87.50	85.61
	No more than 6 points	13	83.38	77.88
	p-value	–	0.23	0.08
Tests 1 & 2	22~24 points	22	88.23	87.20
	No more than 15 points	21	88.76	80.48
	p-value	–	0.43	0.07

Additionally, it should be emphasized that key competences that are essential for a comprehensive approach to conducting modeling processes were not covered by the tests employed in our study. It is anticipated that a research tool will be created that covers all modeling competencies. It could make sense to assess students' modeling abilities, for instance, using a realistic modeling challenge.

The results of the study are not very exciting because of the restrictions of our experiment (e.g., the students were all from a highly intellectual university, and there was only a very short interval between the pre-test and the post-test). However, the authors believe it is very interesting to look into whether there are differences in modeling abilities between students at various educational levels and what those differences are. The answers to these questions can help with the design of curricula and provide effective instruction for students taking mathematical modeling courses at various educational levels. Future research in this area is anticipated to continue.

V. CONCLUSION

This chapter presents some preliminary findings from an experiment carried out at Delhi University comparing the differences in modeling abilities between upper secondary and first-year tertiary students. We are aware of the challenges that students had when modeling, which will assist us develop our instructional strategy. Additionally, it should be emphasized that key competences that are essential for a comprehensive approach to conducting modeling processes were not covered by the tests employed in our study. Therefore, the development of a research instrument that tackles the entire spectrum of modelling skills is anticipated. For instance, it could be appropriate to assess students' modeling abilities using a genuine modeling activity.

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