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Some Properties of W$\tilde{\alpha}$-Interior and W$\tilde{\alpha}$-Closure in Topological Spaces

V. **Mahadevan**

Assistant Professor, Department of Mathematics,

Government College of Engineering (Autonomous)

(Affiliated to Anna University), Salem-11, Tamilnadu, India.

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Abstract: In this paper, the notion of w$\tilde{\alpha}$-interior is defined and some of its basic properties are studied. I introduce the concept of w$\tilde{\alpha}$-closure in topological spaces using the notions of w$\tilde{\alpha}$-closed sets, and we obtain some related results.

Keywords and Phrases: w$\tilde{\alpha}$-interior, w$\tilde{\alpha}$-closure, τw$\tilde{\alpha}$, sg-closed, w$\tilde{\alpha}$-closed.

1. Introduction

Sheik John [7] introduced ω-closed sets in topological spaces. After the coming of these ideas, generalized topological spaces presented different types of generalized closed sets and concentrated on their major properties. Levine [4] introduced generalized closed sets in general topology as a generalization of closed sets. This idea was viewed as valuable and many outcomes overall generalized topology were gotten to the next level. Many researchers like Veerakumar [9] introduced ĝ-closed sets in topological spaces. Ravi and Ganesan [6] presented and $\ddot{g}$-closed sets in general topolgy as one more generalization of closed sets and demonstrated that the class of $\ddot{g}$-closed sets appropriately lies between the class of closed sets and the class of g-closed sets. Pious Missier et al. [5] presented the idea of g'''-closed sets and concentrated on their most basic properties in topological spaces. In this paper, the notion of w$\tilde{\alpha}$-interior is defined and some of its basic properties are studied. I introduce the concept of w$\tilde{\alpha}$-closure in topological spaces using the notions of w$\tilde{\alpha}$-closed sets, and we obtain some related results.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent topological spaces (briefly TPS) on which no separation axioms are assumed unless otherwise mentioned. For a subset P of a space X, cl(P), int(P) and P^c or X$\mid$ P or X − P denote the closure of P, the interior of P and the complement of P, respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1

A subset P of a space X is called:

(i) semi-open [3] if $P \subseteq cl(int(P))$;

(ii) semi-preopen [1] if $P \subseteq cl(int(cl(P)))$;

(iii) regular open [8] if $P = int(cl(P))$.

The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.2

A subset P of a space X is called a

(i) semi-generalized closed (briefly sg-closed) [2] if $scl(P) \subseteq B$ whenever $P \subseteq B$ and B is semi-open in X.

(ii) ω-closed [10] if $cl(P) \subseteq B$ whenever $P \subseteq B$ and B is semi-open in X.

The complements of the above mentioned closed sets are called their respective open sets.

Definition 2.3

A subset $\mathcal{N}$ of a TPS is called a weakly $\tilde{\alpha}$-closed (briefly w$\tilde{\alpha}$-closed) [9] if $cl(int(\mathcal{N})) \subseteq B$ whenever $\mathcal{N} \subseteq B$ and B is sg-open in X.

The complements of the above mentioned closed set are called their respective open set.

3. w$\tilde{\alpha}$-INTERIOR

I introduce the following definition.

Definition 3.1

w$\tilde{\alpha}$-int(P) is as the union of all w$\tilde{\alpha}$-open sets contained in P, for all $P \subseteq X$,. i.e., w$\tilde{\alpha}$-int(P) = $\cup$ {G : $G \subseteq P$ and G is w$\tilde{\alpha}$-open}.

Lemma 3.2

If for any $P \subseteq X$, then $int(P) \subseteq w\tilde{\alpha}\text{-}int(P) \subseteq P$.

Proof

It follows from Definition 3.1.

Proposition 3.3

If for any $P \subseteq X$, then

(i) $w\tilde{a}$-int(P) is the largest $w\tilde{a}$-open set contained in P.

(ii) P is $w\tilde{a}$-open iff $w\tilde{a}$-int(P) = P.

Proof:

(i) $w\tilde{a}$-int(P) is an open set,By definition, $w\tilde{a}$-int(P) $\subseteq$P. Let B be another open set $\subseteq$ P. Then B $\subseteq$ G $\subseteq$ $w\tilde{a}$-int(P) .So $w\tilde{a}$-int(P) is the largest open set contained in P

(ii) $w\tilde{a}$-int(P) = $\cup$ {G : G $\subseteq$ P and G is $w\tilde{a}$-open}.Then by definition itself, we can say that $w\tilde{a}$-int(P) = P.

Proposition 3.4

For any subsets A and B of X,

(i) If M $\subseteq$ N, then $w\tilde{a}$-int(M) $\subseteq$ $w\tilde{a}$-int(N).

(ii) $w\tilde{a}$-int (M $\cap$ N) = $w\tilde{a}$-int(M) $\cap$ $w\tilde{a}$-int(N).

(iii) $w\tilde{a}$-int(M $\cup$ N) $\supseteq$ $w\tilde{a}$-int(M) $\cup$ $w\tilde{a}$-int(N).

(vi) $w\tilde{a}$-int(X) = X and $w\tilde{a}$-int(ϕ) = ϕ.

Proof:

(i) Let x $\in$ $w\tilde{a}$-int(M), there exit a real number r>0 such that G $\subseteq$ M $\subseteq$ N, x $\in$ $w\tilde{a}$-int(N). Thus $w\tilde{a}$-int(M) $\subseteq$ $w\tilde{a}$-int(N).

(ii) Since M $\cap$ N $\subseteq$ M, by (i) $w\tilde{a}$-int(M $\cap$ N) $\subseteq$ $w\tilde{a}$-int(M), since M $\cap$ N $\subseteq$ N, By (i) $w\tilde{a}$-int(M $\cap$ N) $\subseteq$ $w\tilde{a}$-int(N). So $w\tilde{a}$-int(M $\cap$ N) $\subseteq$ $w\tilde{a}$-int(M) $\cap$ $w\tilde{a}$-int(N). By definition $w\tilde{a}$-int(M) $\subseteq$ M and $w\tilde{a}$-int(M) $\subseteq$ N. Thus $w\tilde{a}$-int(M) $\cap$ $w\tilde{a}$-int(N) $\subseteq$ M $\cap$ N. But $w\tilde{a}$-int(M $\cap$ N) is the largest open set contained in M $\cap$ N. So $w\tilde{a}$-int(M) $\cap$ $w\tilde{a}$-int(N) $\subseteq$ $w\tilde{a}$-int(M $\cap$ N). Thus $w\tilde{a}$-int(M $\cap$ N) = $w\tilde{a}$-int(M) $\cap$ $w\tilde{a}$-int(N).

(iii) Since M $\subseteq$ M $\cup$ N, B$\subseteq$ M $\cup$ N, by (i) $w\tilde{a}$-int(M) $\subseteq$ $w\tilde{a}$-int(M $\cup$ N) and

$w\tilde{a}$-int(N) $\subseteq$ $w\tilde{a}$-int(M $\cup$ N).Thus $w\tilde{a}$-int(M $\cup$ N) $\supseteq$ $w\tilde{a}$-int(M) $\cup$ $w\tilde{a}$-int(N).

(iv) we can directly get this by definition.

4. $w\tilde{a}$-CLOSURE

In this paper, I define $w\tilde{a}$-closure of a set and we prove that $w\tilde{a}$-closure is a Kuratowski closure operator on X.

Definition 4.1

$w\tilde{a}$-closure of P is defined to be the intersection of all $w\tilde{a}$-closed sets containing P , for all P $\subseteq$ X.

Otherwise, $w\tilde{\alpha}$-cl(P) = $\cap$ {F : P $\subseteq$ F $\in$ $w\tilde{\alpha}$C(X)}.

Lemma 4.2

If for any P $\subseteq$ X, P $\subseteq$ $w\tilde{\alpha}$-cl(P) $\subseteq$ cl(P).

Proof

By definition, $w\tilde{\alpha}$-cl(P) is the smallest $w\tilde{\alpha}$-closed containing P, we have any closed set is $w\tilde{\alpha}$-closed containing P. Thus P $\subseteq$ $w\tilde{\alpha}$-cl(P) $\subseteq$ cl(P).

Remark 4.3

Both containment relations in Lemma 4.2 may be proper as seen from the following example.

Example 4.4

Let X = {w_1, w_2, w_3} and τ = {ϕ, {w_1, w_2}, X}. Let P = {w_2}. Then $w\tilde{\alpha}$-cl(P) = {w_2, w_3} and cl(P) = X. so P $\subset$ $w\tilde{\alpha}$-cl(P) $\subset$ cl(P).

Lemma 4.5

For any P$\subseteq$ X, ω-cl(P) $\subseteq$ $w\tilde{\alpha}$-cl(P), where ω-cl(P) is given by ω-cl(P) = $\cap$ {F :P $\subseteq$ F $\in \omega$ C(X)}.

Proof

Since, any $w\tilde{\alpha}$-cld is ω-cld and by definition, $w\tilde{\alpha}$-cl(P) is the smallest $w\tilde{\alpha}$-cld containing P. Thus for any P $\subseteq$ X, ω-cl(P) $\subseteq$ $w\tilde{\alpha}$-cl(P)

Remark 4.6

The above lemma 4.5 may be as seen from the following example.

Example 4.7

Let X = {w_1, w_2, w_3, w_4} with τ = {ϕ, {w_1}, {w_2, w_3}, {w_1, w_2, w_3}, X}. Then $w\tilde{\alpha}$C(X) = {ϕ, {w_4}, {w_1, w_4}, {w_1, w_2, w_3}, X} and ωC(X) = {ϕ, {w_4}, {w_1, w_4}, {w_2, w_4}, {w3, w_4}, {w_1, w_2, w_4}, {w_2, w_3, w_4}, X}. Let P = {w_1, w_2, w_4}. Then $w\tilde{\alpha}$-cl(P) = X and ω-cl(P) ={w_1, w_2, w_3}. So, ω-cl(P) $\subset$ $w\tilde{\alpha}$-cl(P).

Theorem 4.8

$w\tilde{\alpha}$-closure is a Kuratowski closure operator on X.

Proof

(i) $w\tilde{\alpha}$-cl(ϕ) = ϕ.

(ii) P $\subseteq$ $w\tilde{\alpha}$-cl(P), by Lemma 4.2.

(iii) Let $P_1 \cup P_2 \subseteq F \in w\tilde{\alpha}C(X)$, then $P_i \subseteq F$ and by definition, $w\tilde{\alpha}$-cl(P_i) $\subseteq F$ for i = 1, 2. Therefore $w\tilde{\alpha}$-cl(P_1) $\cup$ $w\tilde{\alpha}$-cl(P_2) $\subseteq \cap \{F : P_1 \cup P_2 \subseteq F \in w\tilde{\alpha}C(X)\}$ = $w\tilde{\alpha}$-cl($P_1 \cup P_2$). To prove the reverse inclusion, let $x \in w\tilde{\alpha}$-cl($P_1 \cup P_2$) and suppose that $x \notin w\tilde{\alpha}$-cl(P_1) $\cup$ $w\tilde{\alpha}$-cl(P_2). Then there exists $w\tilde{\alpha}$-closed sets F_1 and F_2 with $P_1 \subseteq F_1$, $P_2 \subseteq F_2$ and $x \notin F_1 \cup F_2$. We have $P_1 \cup P_2 \subseteq F_1 \cup F_2$ and $F_1 \cup F_2$ is an $w\tilde{\alpha}$-closed set such that $x \notin F_1 \cup F_2$. Thus $x \notin w\tilde{\alpha}$-cl($P_1 \cup P_2$) which is a contradiction to $x \in w\tilde{\alpha}$-cl($P_1 \cup P_2$). Hence $w\tilde{\alpha}$-cl($P_1 \cup P_2$) = $w\tilde{\alpha}$-cl(P_1) $\cup$ $w\tilde{\alpha}$-cl(P_2).

(iv) Let $P \subseteq F \in w\tilde{\alpha}C(X)$. Then by Definition 4.1, $w\tilde{\alpha}$-cl(P) $\subseteq$ F and $w\tilde{\alpha}$-cl(GΩ-cl(P)) $\subseteq$ F. Since $w\tilde{\alpha}$-cl($w\tilde{\alpha}$-cl(P)) $\subseteq$ F, we have $w\tilde{\alpha}$-cl($w\tilde{\alpha}$-cl(P)) $\subseteq \cap \{F : A \subseteq F \in w\tilde{\alpha}C(X)\}$ = $w\tilde{\alpha}$-cl(P). By Lemma 4.2, $w\tilde{\alpha}$-cl(P) $\subseteq$ $w\tilde{\alpha}$-cl($w\tilde{\alpha}$-cl(P)) and therefore, $w\tilde{\alpha}$-cl($w\tilde{\alpha}$-cl(P)) = $w\tilde{\alpha}$-cl(P). Hence, $w\tilde{\alpha}$-closure is a Kuratowski operator on X.

Lemma 4.9

For an $x \in X$, $x \in w\tilde{\alpha}$-cl(P) if and only if $V \cap P \neq \phi$ for any $w\tilde{\alpha}$-open set V containing x.

Proof

Let $x \in w\tilde{\alpha}$-cl(P) for any $x \in X$. To prove $V \cap P \neq \phi$ for any $w\tilde{\alpha}$-open set V containing x. Prove this by contradiction. Suppose there exists a $w\tilde{\alpha}$-open set V contains x such that $V \cap P = \phi$. Then $P \subset V^c$ and V^c is $w\tilde{\alpha}$-cld. We have $w\tilde{\alpha}$-cl(P) $\subset V^c$. This implies that $x \notin w\tilde{\alpha}$-cl(P) which is a contradiction. Hence $V \cap P \neq \phi$ for all $w\tilde{\alpha}$-open set V contains x.

Conversely, let $V \cap P \neq \phi$ for all $w\tilde{\alpha}$-open set V contains x. To prove $x \in w\tilde{\alpha}$-cl(P). We prove this by contradiction. Suppose $x \notin w\tilde{\alpha}$-cl(P). Then there exists a $w\tilde{\alpha}$-closed set F containing P such that $x \notin F$. Then $x \in F^c$ and F^c is $w\tilde{\alpha}$-open. Also $F^c \cap P = \phi$, which is a contradiction. Thus $x \in w\tilde{\alpha}$-cl(P).

Definition 4.10

Let $\tau w\tilde{\alpha}$ be the topology on X generated by $w\tilde{\alpha}$-closure. Otherwise $\tau w\tilde{\alpha} = \{U \subseteq X : w\tilde{\alpha}\text{-cl}(U^c) = U^c\}$.

Theorem 4.11

For any topology τ on X, $\tau \subseteq \tau w\tilde{\alpha} \subseteq \tau_{\omega}$, where $\tau_{\omega} = \{U \subseteq X : \omega\text{-cl}(U^c)\} = U^c$.**Proof**

It follows from Definition 4.1 and Lemma 4.5.

The following two Propositions are easy consequences from definitions.

Proposition 4.12

For any $P \subseteq X$, the following hold:

(i) $w\tilde{\alpha}$-cl(P) is the smallest $w\tilde{\alpha}$-cld contains P.

(ii) P is $w\tilde{\alpha}$-cld if and only if $w\tilde{\alpha}$-cl(P) = P.

Proposition 4.13

For any two subsets M and N of X, the following hold:

(i) If $M \subseteq N$, then $w\tilde{\alpha}\text{-cl}(M) \subseteq w\tilde{\alpha}\text{-cl}(N)$.

(ii) $w\tilde{\alpha}\text{-cl}(M \cap N) \subseteq w\tilde{\alpha}\text{-cl}(M) \cap w\tilde{\alpha}\text{-cl}(N)$.

Proof

From the Definition 4.1.

Theorem 4.14

Let P be any subset of X. Then

(i) $(w\tilde{\alpha}\text{-int}(P))^c = w\tilde{\alpha}\text{-cl}(P^c)$.

(ii) $w\tilde{\alpha}\text{-int}(P) = (w\tilde{\alpha}\text{-cl}(P^c))^c$.

(iii) $w\tilde{\alpha}\text{-cl}(P) = (w\tilde{\alpha}\text{-int}(P^c))^c$.

Proof

(i) Let $x \in (w\tilde{\alpha}\text{-int}(P))^c$. Then $x \notin w\tilde{\alpha}\text{-int}(P)$. Then, all $w\tilde{\alpha}$-open set U contains x is such that $U \not\subseteq P$. Thus, all $w\tilde{\alpha}$-open set U contains x is such that $U \cap P^c \neq \phi$. By Lemma 4.9, $x \in w\tilde{\alpha}\text{-cl}(P^c)$ and therefore $(w\tilde{\alpha}\text{-int}(P))^c \subseteq w\tilde{\alpha}\text{-cl}(P^c)$.

Conversely, If $x \in w\tilde{\alpha}\text{-cl}(P^c)$ then by Lemma 4.9, all $w\tilde{\alpha}$-open set U contains x is such that $U \cap P^c \neq \phi$. That is, all $w\tilde{\alpha}$-open set U contains x is such that $U \not\subseteq P$. This shows by Definition 3.1, $x \notin w\tilde{\alpha}\text{-int}(P)$. That is, $x \in (w\tilde{\alpha}\text{-int}(P))^c$ and so $w\tilde{\alpha}\text{-cl}(P^c) \subseteq (w\tilde{\alpha}\text{-int}(P))^c$. Thus $(w\tilde{\alpha}\text{-int}(P))^c = w\tilde{\alpha}\text{-cl}(P^c)$.

(ii) Taking complement of (i), we get the answer.

(iii) We get this by replacing P by P^c in (i).

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