# Composition of Pathway Integral operator on Extended Unified Mittag - Leffler Function

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**Abstract:** In this article, The compositions of the Extended Unified Mittag-Leffer Function and Pathway Fractional Integral Operator are obtained. When the fractional integral operator is applied to the power multiple of Extended Unified Mittag-Leffer Functions', the result is a multiplication of both the beta function and the Extended Unified Mittag-Leffer Function and also with multiplication of beta function and ${}_1\psi_1$ and convert into k – wright function and hypergeometric form ${}_1F_1$.

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## 1 Introductions

In this paper, C and R denote the sets of complex and real numbers respectively. Also $N_0 = \{0,1,\ldots..\}$, $R^+ = (0,\infty)$ and $Z^- = \{-1,-2,\ldots\ldots\}$.

**Extended Unified Mittag - Leffler function**

Umar Muhammad Abubkar et al. [12] introduced the following extended unified Mittag-Leffler function

$$ {}^{\tau_1,\tau_2}_{\psi_1,\psi_2}M^{\theta_1,\theta_2,\lambda_1,\lambda_2,r}_{\alpha_1,\alpha_2,\gamma_2,\gamma_1,\rho_2,\rho_1}(z;\underline{x},\underline{y},\underline{w};\varphi)=\sum_{r=0}^{\infty}\frac{\prod_{n=1}^{m}B^{\tau_1,\tau_2}_{\psi_1,\psi_2}(w_n,x_n;\varphi)}{\prod_{n=1}^{m}B(w_n,y_n)}\frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}}\frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}}\frac{z^r}{\Gamma(\alpha_1 r+\alpha_2)} \quad (1.1)$$

If $\underline{y}=(y_1,y_2,\ldots\ldots,y_n),\underline{w}=(w_1,w_2,\ldots\ldots,w_n),\underline{x}=(x_1,x_2,\ldots\ldots,x_n),z,\gamma_1,\gamma_2,\alpha_1,\alpha_2$ $\rho_1,\rho_2,\lambda_1,\theta_1,\theta_2,y_n,w_n,x_n,\in C,$ min $\{Re(y_n),Re(w_n),Re(x_n)\}>0,\forall n$; min$\{\mathrm{Re}(\alpha_1),Re(\alpha_2),Re(\gamma_1),Re(\gamma_2),Re(\theta_1)\}>0,\lambda_2\in(0,1)\cup N$ with $\lambda_2+Re(\theta_2)<Re(\gamma_2+\rho_2+\alpha_1),Im(\theta_2)=Im(\gamma_2+\rho_2+\alpha_1)$, and $B^{\tau_1,\tau_2}_{\varphi_1,\varphi_2}(y,x;\varphi)$ is the extended beta function [13] defined by

$$B^{\tau_1,\tau_2}_{\varphi_1,\varphi_2}(y,x;\varphi)=\int_0^1 t^{x-1}(1-t)^{y-1}\varphi^{\left(-\frac{\varphi_1}{t^{\tau_1}}-\frac{\varphi_2}{(1-t)^{\tau_2}}\right)}dt, \quad (1.2)$$

for Re $(\varphi_1)>0$, Re (s, Re $(\tau_1)>0$, $\mathrm{Re}(\tau_2)>0$, $\mathrm{Re}(x)>0$, Re $(\mathrm{y})>0$, $\varphi\in(0,\infty)\setminus\{1\}$.

**Generalized Wright Function and k-Wright Function**

Let $\alpha_j$, $\beta_i \in R\setminus\{0\}$ and $a_j$, $b_i \in$ C, i = 1, 2, , p ; j = 1 , 2, , q, then the generalized Wright function is defined by Wright [[16] - [15] ]in the following manner:

$$ {}_p\psi_q(z) = {}_p\psi_q\left[\begin{matrix}(b_i,\beta_i)_{1,p}\\(a_j,\alpha_j)_{1,q}\end{matrix}\middle| z\right] = \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(b_i+n\beta_i)}{\prod_{j=1}^{q}\Gamma(a_j+n\alpha_j)}\frac{z^n}{n!}, \qquad z\in C \tag{1.3}$$

where the coefficients $\beta_1,\beta_2,.........\beta_p \in R^+$ and $\alpha_1,\alpha_2,.........\alpha_q \in R^+$ such that

$$1+\sum_{j=1}^{q}\alpha_j-\sum_{i=1}^{p}\beta_i \geq 0$$

The gamma function $\Gamma(z)$ introduced by Wright [14] is defined as follows

$$\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt, \qquad R(z)>0$$

Kilbas et al. [6] gave the conditions of existence along with its representation in the form of Mellin - Barnes type integral. Gehlot and Prajapati [4] introduced the generalization of the above function (1.3) named as the generalized K-Wright function, and is defined as

$$ {}_p\psi_q^k(z) = {}_p\psi_q^k\left[\begin{matrix}(b_i,\beta_i)_{1,p}\\(a_j,\alpha_j)_{1,q}\end{matrix}\middle| z\right] = \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(b_i+n\beta_i)}{\prod_{j=1}^{q}\Gamma_k(a_j+n\alpha_j)}\frac{z^n}{n!}, \qquad z\in C \tag{1.4}$$

where, $k\in R^+$ and $(b_i+n\beta_i)$, $(a_j+n\alpha_j)\in C\setminus kZ^-$ for all $n\in N_0$. In the above definition, $\Gamma_k(z)$ denotes the generalized k - gamma function [2] defined as:

$$\Gamma_k(z) = \int_0^\infty e^{-\frac{t^k}{k}}t^{z-1}dt, \qquad R(z)>0;\quad k\in R^+ \tag{1.5}$$

and

$$\Gamma_k(z) = \lim_{n\to\infty}\frac{n!k^n(nk)^{\frac{z}{k-1}}}{(z)_{n,k}}, \qquad k\in R^+,\quad z\in C\setminus kZ^- \tag{1.6}$$

which has the following relationship [2] :

$$\Gamma_k(z+k) = z\Gamma_k(z) \tag{1.7}$$

and

$$\Gamma_k(z) = k^{\frac{z}{k}-1}\Gamma\left(\frac{z}{k}\right) \tag{1.8}$$

Diaz and Pariguan [2] introduced the k-Pochhammer symbol $(z)_{n,k}$ for all complex $z\in C$ and $k\in R$, and is defined as

$$(z)_{n,k} = \begin{cases}1 & if\ \ n=0\\ z(z+k)(z+2k).......(z+(n-1)k) & if\ \ n\in N\end{cases} \tag{1.9}$$

for k=1, the generalized K-Wright function (1.4) reduces to the generalized Wright function

(1.3).

**Pathway Fractional Integral Operator:**

Using the pathway idea of Mathai [7], Nair [[11], p. 239] introduced a new pathway fractional integration operator.

Let $f(x) \in L(\alpha, \beta), \eta \in C, Re(\eta) > 0, \alpha > 0$ and 'a' is taken as a pathway parameter such that $0 < a < 1$, the pathway fractional integration operator is defined as:

$$(P_{0+}^{(\eta,a,\alpha)})(x) = x^{\eta} \int_0^{\frac{x}{\alpha(1-a)}} \left[\frac{\alpha(1-a)}{x}\right]^{\frac{\eta}{(1-a)}} f(t)dt \tag{1.10}$$

where L($\alpha$ , $\beta$ ) is the set of Lebesgue measurable function defined on ($\alpha$ , $\beta$ ).

Mathai [11] introduced the pathway model and further studied by Mathai and Haubold [[8]-[9]].

**Result Required :**

The following result is required [[8], p. 239; see also [5], [10]

$$(P_{0+}^{(\eta,a,\alpha)})\{t^{\beta-1}\} = \frac{t^{\eta+\beta}}{[\alpha(1-a)]^{\beta}} \frac{\Gamma(\beta)\Gamma\left(1+\frac{\eta}{1-a}\right)}{\Gamma\left(\frac{\eta}{1-a}+\beta+1\right)} \tag{1.11}$$

where $0 < a < 1;\ Re(\eta) > 0;\ Re(\beta) > 0$.

The function $H_{p,q}^{m,n}$ was given by C. Fox [3] in 1961 and Braaksma [[1], p.278] gave the convergence condition

$${}_p\psi_q^*\left[\begin{matrix}((\alpha_p, A_p));\\ ((\beta_q, B_q));\end{matrix} z\right] = {}_p\psi_q^*\left[\begin{matrix}(\alpha_1, A_1),\ldots\ldots\ldots(\alpha_p, A_p);\\ (\beta_1, B_1),\ldots\ldots\ldots(\beta_q, B_q);\end{matrix} z\right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_{nA_1}\ldots\ldots\ldots(\alpha_p)_{nA_p}}{(\beta_1)_{nB_1}\ldots\ldots\ldots(\beta_q)_{nB_q}} \frac{z^n}{n!} \tag{1.12}$$

The Fox's H-function on the RHS of makes sense when either

$$\delta = (1 + B_1 + B_2 + \ldots + B_q) - (A_1 + A_2 + \ldots + A_p) > 0$$

and

$$0 < |z| < \infty; \qquad z \neq 0$$

The equality holds only for suitably constrained values of | z | or appropriate | z| i.e. $\delta = 0$ and $0 < |z| < R = A_1^{-A_1} A_2^{-A_2} \ldots\ldots A_p^{-A_p} B_1^{B_1} B_2^{B_2} \ldots\ldots\ldots B_q^{B_q}$

$${}_p\psi_q\left[\begin{matrix}(\alpha_1,1),\ldots\ldots\ldots(\alpha_p,1);\\ (\beta_1,1),\ldots\ldots\ldots(\beta_q,1);\end{matrix} z\right] = \frac{\prod_{j=1}^{p}\Gamma(\alpha_j)}{\prod_{j=1}^{q}\Gamma(\beta_j)} {}_pF_q\left[\begin{matrix}\alpha_1,\ldots\ldots,\alpha_p;\\ \beta_1,\ldots\ldots,\beta_q;\end{matrix} z\right] \tag{1.13}$$

**3. Formula for the Pathway Fractional Integral associated with the Extended unified Mittag - Leffler Function.**

**Theorem 1**. If $\underline{w} = (w_1, w_2, \ldots\ldots w_n)$, $\underline{x} = (x_1, x_2, \ldots\ldots x_n)$, $\underline{y} = (y_1, y_2, \ldots, y_n)$, z , $\alpha_1, \alpha_2, \gamma_1, \gamma_2, \rho_1, \rho_2, \theta_1, \theta_2, \lambda_1, w_n, x_n, y_n \in C$, $\min\{Re(w_n), Re(y_n), Re(x_n)\} > 0$ $, \forall n$, min{Re($\alpha_1$), Re($\alpha_2$), $Re(\gamma_1), Re(\gamma_2), Re(\theta_1)\}$ > 0, $\lambda_2 \in (0,1) \bigcup$ N with $\lambda_2$ + Re ($\theta_2$) < Re

$(\gamma_2 + \rho_2 + \alpha_1)$, $\text{Im}(\theta_2) = \text{Im}(\gamma_2 + \rho_2 + \alpha_1)$, then

$$= t^{\frac{\rho}{k}+\eta} \sum_{r=0}^{\infty} \frac{1}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} B\left(r+\frac{\rho}{k}, 1+\frac{\eta}{1-\alpha}\right) \times_{\varphi_1,\varphi_2}^{\tau_1,\tau_2} M_{\alpha_1,\alpha_2,\gamma_2,\gamma_1,\rho_2,\rho_1}^{\theta_1,\theta_2,\lambda_1,\lambda_2,r}\left(t;\underline{w},\underline{x},\underline{y},\varphi\right) \tag{1.14}$$

**Proof**: Let I be the left hand side of (1.14). Applying (1.1) and interchanging the order of integration and summation, we have

$$I = P_{0+}^{(\eta,\alpha,a)}\left(t^{\frac{\rho}{\kappa}-1}\,{}^{\tau_1,\tau_2}_{\psi_1,\psi_2}M_{\alpha_1,\alpha_2,\gamma_2,\gamma_1,\rho_2,\rho_1}^{\theta_1,\theta_2,\lambda_1,\lambda_2,r}(t;\underline{w},\underline{x},\underline{y},\varphi)\right)$$

$$= \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n,w_n;\varphi)}{\prod_{n=1}^{m} B(y_n,w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{\Gamma(\alpha_1 r+\alpha_2)} P_{0+}^{(\eta,\alpha,a)} t^{r+\frac{\rho}{k}-1}$$

Using (1.11)

$$I = \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n,w_n;\varphi)}{\prod_{n=1}^{m} B(y_n,w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{\Gamma(\alpha_1 r+\alpha_2)} \frac{t^{r+\frac{\rho}{k}+\eta}}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} \frac{\Gamma\left(r+\frac{\rho}{k}\right)\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{\Gamma\left(\frac{\eta}{1-\alpha}+r+\frac{\rho}{k}+1\right)}$$

$$= t^{\frac{\rho}{k}+\eta} \sum_{r=0}^{\infty} \frac{1}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} B\left(r+\frac{\rho}{k}, 1+\frac{\eta}{1-\alpha}\right) \times_{\varphi_1,\varphi_2}^{\tau_1,\tau_2} M_{\alpha_1,\alpha_2,\gamma_2,\gamma_1,,\rho_2,\rho_1}^{\theta_1,\theta_2,\lambda_1,\lambda_2,r}\left(t;\underline{w},\underline{x},\underline{y},\varphi\right)$$

which is the right hand side of (1.14), which completes the proof.

**Theorem 2.** If $\underline{w} = (w_1, w_2, \ldots\ldots w_n)$, $\underline{x} = (x_1, x_2, \ldots\ldots x_n)$, $\underline{y} = (y_1, y_2, \ldots\ldots y_n)$, z , $\alpha_1,\alpha_2,\gamma_1,\gamma_2,\rho_1,\rho_2,\ \theta_1,\theta_2,\lambda_1,w_n,x_n,y_n \in C,$ $\min\{Re(w_n),Re(y_n),Re(x_n)\} > 0$ $,\forall n$, $\min\{\text{Re}(\alpha_1), \text{Re}(\alpha_2), Re(\gamma_1), Re(\gamma_2), Re(\theta_1)\} > 0$, $\lambda_2 \in (0,1) \bigcup$ N with $\lambda_2 + \text{Re}(\theta_2) < \text{Re}$ $(\gamma_2 + \rho_2 + \alpha_1)$, $\text{Im}(\theta_2) = \text{Im}(\gamma_2 + \rho_2 + \alpha_1)$, then

$$P_{0+}^{(\eta,\alpha,a)}(t^{\frac{\rho}{\kappa}-1}\,{}^{\tau_1,\tau_2}_{\psi_1,\psi_2}M_{\alpha_1,\alpha_2,\gamma_2,\gamma_1,\rho_2,\rho_1}^{\theta_1,,\theta_2,\lambda_1,\lambda_2,r}(t;\underline{w},\underline{x},\underline{y},\varphi))$$

$$= t^{\frac{\rho}{k}+\eta} \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n,w_n;\varphi)}{\prod_{n=1}^{m} B(y_n,w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} B\left(r+\frac{\rho}{k}, 1+\frac{\eta}{1-\alpha}\right) \times$$

$$\times {}_1\psi_1\left[\begin{matrix}(1,1)_{1,1} \\ (\alpha_2,\alpha_1)_{1,1}\end{matrix}\middle| t\right] \tag{1.15}$$

**Proof**: Let I be the left hand side of (1.15). Applying (1.1) and interchanging the order of integration and summation, we have

$$I = P_{0+}^{(\eta,\alpha,a)}\left(t^{\frac{\rho}{\kappa}-1}\,{}^{\tau_1,\tau_2}_{\psi_1,\psi_2}M_{\alpha_1,\alpha_2,\gamma_2,\gamma_1,\rho_2,\rho_1}^{\theta_1,\theta_2,\lambda_1,\lambda_2,r}(t;\underline{w},\underline{x},\underline{y},\varphi)\right)$$

$$= \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n, w_n; \varphi)}{\prod_{n=1}^{m} B(y_n, w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{\Gamma(\alpha_1 r + \alpha_2)} P_{0+}^{(\eta,\alpha,a)} t^{r+\frac{\rho}{k}-1}$$

Using (1.11)

$$I = \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n, w_n; \varphi)}{\prod_{n=1}^{m} B(y_n, w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{\Gamma(\alpha_1 r + \alpha_2)} \frac{t^{r+\frac{\rho}{k}+\eta}}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} \frac{\Gamma\left(r+\frac{\rho}{k}\right)\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{\Gamma\left(\frac{\eta}{1-\alpha}+r+\frac{\rho}{k}+1\right)}$$

$$= t^{\frac{\rho}{k}+\eta} \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n, w_n; \varphi)}{\prod_{n=1}^{m} B(y_n, w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} \times$$

$$\times B\left(r+\frac{\rho}{k}, 1+\frac{\eta}{1-\alpha}\right) \frac{\Gamma(r+1)}{\Gamma(\alpha_1 r + \alpha_2)} \frac{t^r}{r!}$$

By using (1.4)

$$I = t^{\frac{\rho}{k}+\eta} \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n, w_n; \varphi)}{\prod_{n=1}^{m} B(y_n, w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} B\left(r+\frac{\rho}{k}, 1+\frac{\eta}{1-\alpha}\right) \times$$

$$\times {}_1\psi_1\left[\begin{matrix}(1,1)_{1,1} \\ (\alpha_2, \alpha_1)_{1,1}\end{matrix}\middle| t\right]$$

which is the right hand side of (1.15), which completes the proof.

**Corollary 1.** The result in (1.15) reduces to the following exponential function by taking $\alpha_1 = \alpha_2 = 0$

$$P_{0+}^{(\eta,\alpha,a)}\left(t^{\frac{\rho}{\kappa}-1} {}^{\tau_1,\tau_2}_{\psi_1,\psi_2}M^{\theta_1,\theta_2,\lambda_1,\lambda_2,r}_{\alpha_1,\alpha_2,\gamma_2,\gamma_1,\rho_2,\rho_1}(t; \underline{w}, \underline{x}, \underline{y}, \varphi)\right)$$

$$= t^{\frac{\rho}{k}+\eta} \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n, w_n; \varphi)}{\prod_{n=1}^{m} B(y_n, w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} B\left(r+\frac{\rho}{k}, 1+\frac{\eta}{1-\alpha}\right) e^t$$

**Corollary 2.** The result in (1.15) reduces to the following k-wright function by using (1.7) and (1.8)

$$P_{0+}^{(\eta,\alpha,a)}\left(t^{\frac{\rho}{\kappa}-1} {}^{\tau_1,\tau_2}_{\psi_1,\psi_2}M^{\theta_1,\theta_2,\lambda_1,\lambda_2,r}_{\alpha_1,\alpha_2,\gamma_2,\gamma_1,\rho_2,\rho_1}(t; \underline{w}, \underline{x}, \underline{y}, \varphi)\right)$$

$$= t^{\frac{\rho}{k}+\eta} \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n, w_n; \varphi)}{\prod_{n=1}^{m} B(y_n, w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} \times$$

$$\times B\left(r+\frac{\rho}{k}, 1+\frac{\eta}{1-\alpha}\right) k^{-r(1-\alpha_1)-(1-\alpha_2)} {}_1\psi_1^k \left[\left.\begin{matrix} (k,k)_{1,1} \\ (\alpha_2 k, \alpha_1 k)_{1,1} \end{matrix}\right| t\right]$$

**Corollary 3**. The result in (1.15) reduces to the following hypergeometric function by using (1.13)

$$P_{0+}^{(\eta,\alpha,a)}\left( t^{\frac{\rho}{\kappa}-1} {}^{\tau_1,\tau_2}_{\psi_1,\psi_2} M^{\theta_1,\theta_2,\lambda_1,\lambda_2,r}_{\alpha_1,\alpha_2,\gamma_2,\gamma_1,\rho_2,\rho_1}(t; \underline{w}, \underline{x}, \underline{y}, \varphi) \right)$$

$$= t^{\frac{\rho}{k}+\eta} \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{m} B_{\psi_1,\psi_2}^{\tau_1,\tau_2}(x_n, w_n; \varphi)}{\prod_{n=1}^{m} B(y_n, w_n)} \frac{(\theta_1)_{\theta_2 r}}{(\gamma_1)_{\gamma_2 r}} \frac{(\lambda_1)_{\lambda_2 r}}{(\rho_1)_{\rho_2 r}} \frac{1}{[a(1-\alpha)]^{r+\frac{\rho}{k}}} \times$$

$$\times B\left(r+\frac{\rho}{k}, 1+\frac{\eta}{1-\alpha}\right) \frac{1}{\Gamma \alpha_2} {}_1F_1\left[\begin{matrix} 1; \\ \alpha_2; \end{matrix} t\right]$$

## Conclusion

In this research, we derive some compositions of Extended Unified Mittag-Leffler function and pathway fractional integral operator. Due to the universal characteristics of hypergeometric functions, the obtained results may be reduced to many new results including several generalised Mittag-Leffler functions. The discoveries are novel and distinct and will be used to expand on several studies.

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