Decision Making Method Related to Pythagorean Neutrosophic Hypersoft Sets

[1]G. Ramya, [2]A. Francina Shalini

[1]Research Scholar, Nirmala College for Women, Coimbatore, Tamil Nadu;

[2]Assistant Professor, Nirmala College for Women, Coimbatore, Tamil Nadu;

Abstract:

PNHSS is a new approach towards decision making under uncertainty. This study presents a new algorithm for group decision-making solutions using PNHSMs and confident weight is given by experts. PNHSS is deriving from the combination of Pythagorean Neutrosophic set and Hypersoft set. PNHSM is also the matrix representation of PNHSS. To demonstrate the validity of the proposed approach, a numerical example has been presented. The obtained results show that our developed approach is most effective.

1. Introduction

Numerous theories are utilized in the literature to address the ambiguity and uncertainty ofNumerous problems that arise in Engineering, Economics, etc., All ideas, however have their own drawbacks. Multiple qualities and uncertainty are present in multi-criteria decision-making situations, Since NSs fully address indeterminacy, whereas NSSs address vagueness and uncertainty, they are utilized to deal with such kinds. The concept of NSS cannot be applied to such problems when qualities are multiple and further divided.

To get around these issues, Smarandache [3] demonstrated a different method of handling uncertainty by extending the SS to the HS and its hybrids, such as the FHS, IFHS, and NHSS, by converting the function into a multi-argument function. The matrix representation and aggregate operators of this idea were presented by Delhi and Broumi in [6]. In this article, a new algorithmic approach for the group D-M method is proposed.

2. Preliminaries

2.1. PNHSS [4].

Let U be the universal set and $\mathcal{P}(U)$ be a power set of U. consider $E_1, E_2, \ldots, E_k$ for k $\geq$ 1 be k well - defined attributes, whose corresponding attributive values are respectively the set $C_1, C_2, \ldots, C_k$ with $C_i \cap C_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, \ldots, k\}$ and their relation $C_1 \times C_2, \ldots \times C_k = \alpha$, then the pair $(\gamma, C_1 \times C_2 \times \ldots \times C_k)$ is said to be Pythagorean Neutrosophic Hypersoft set (PNHSS in short) over U where $\gamma: C_1 \times C_2 \times \ldots \times C_k \to \mathcal{P}(U)$ and $\gamma(C_1 \times C_2 \times \ldots \times C_k) = \{(\alpha, < x, T_{\gamma(\alpha)}(x), I_{\gamma(\alpha)}(x), F_{\gamma(\alpha)}(x) >): x \in U, \alpha \in C_1 \times C_2 \times \ldots \times C_k\}$ where T is the membership value of truthiness, I is the membership value of indeterminacy and F is the membership value of falsity such that $T_{\gamma(\alpha)}(x), I_{\gamma(\alpha)}(x), F_{\gamma(\alpha)}(x) \in [0,1]$ also $0 \leq \left(T_{\gamma(\alpha)}(x)\right)^2 + \left(I_{\gamma(\alpha)}(x)\right)^2 + \left(F_{\gamma(\alpha)}(x)\right)^2 \leq 2$.

2.2. PNHSM [5].

Let U = $\{u^1, u^2, \ldots u^r\}$ be the Universal set and $P(U)$ be the Power set of U. Consider $E_1, E_2, \ldots, E_c, for \ c \geq 1$, where c is well-defined attributes, whose corresponding attributive values are, respectively, the set $E_1{}^a, E_2{}^b, \ldots, E_c{}^z$ and their relation $E_1{}^a \times E_2{}^b \times \ldots \times E_c{}^z$, where a, $b, \ldots, z = 1, 2, \ldots, n$; then, the pair $(\gamma, E_1{}^a \times E_2{}^b \times \ldots \times E_c{}^z)$ is said to be PNHSS over U, where $\gamma: (E_1{}^a \times E_2{}^b \times \ldots \times E_c{}^z) \to P(U)$ and it is defined as $\gamma(E_1{}^a \times E_2{}^b \times \ldots \times E_c{}^z) = \{(\alpha, < x, T_{\gamma(\alpha)}(x), I_{\gamma(\alpha)}(x), F_{\gamma(\alpha)}(x) >): x \in U, \alpha \in E_1{}^a \times E_2{}^b \times \ldots \times E_c{}^z\}$.

If $O_{ij} = X_{\mathcal{R}_{\gamma(\alpha)}}(u^i, E_j{}^k)$, where i = 1, 2, 3... r, j = 1, 2, 3...., c, k = a, $b, \ldots, z$ then a matrix is defined as

$$[O_{ij}]_{r \times c} = \begin{pmatrix} O_{11} & O_{12} \ldots & O_{1c} \\ O_{21} & O_{22} \ldots & O_{2c} \\ & \ldots \ldots \ldots \ldots & \\ & \ldots \ldots \ldots \ldots & \\ O_{r1} & O_{r2} \ldots & O_{rc} \end{pmatrix}, \text{ Where } O_{ij} = \left(T_{E_{\hat{j}}{}^k}(u_i), I_{E_{\hat{j}}{}^k}(u_i), F_{E_{\hat{j}}{}^k}(u_i)\right), (u_i) \in$$

$U, E_{\hat{j}}{}^k \in (E_1{}^a \times E_2{}^b \times \ldots \times E_c{}^z) = (T^o{}_{ijk}, I^o{}_{ijk}, F^o{}_{ijk})$. The collection of all

PNHSMs over U is denoted by PNHSM$(U)_{r \times c}$.

2.3. Basic Operations on PNHSMs[5].

Let O = [O_{ij}], M =[M_{jm}] $\in$ PNHSM$(U)_{r\times c}$,   Where $O_{ij} = \left(T^o_{ijk}, I^o_{ijk}, F^o_{ijk}\right)$, $M_{jm} = \left(T^m_{jkm}, I^m_{jkm}, F^m_{jkm}\right)$. If $O = \left[O_{ij}\right]_{r\times c}$ & $M = \left[M_{jm}\right]_{c\times \hat{\gamma}}$, then $O \otimes M = [\delta_{im}]_{r\times \hat{\gamma}}$, where

$[\delta_{im}] = \left(max_{jk}min\left(T^o_{ijk}, T^m_{jkm}\right), min_{jk}max\left(I^o_{ijk}, I^m_{jkm}\right), min_{jk}max\left(F^o_{ijk}, F^m_{jkm}\right)\right)$ and

$O + M = \left[max\left(T^o_{ijk}, T^m_{jkm}\right), min\left(I^o_{ijk}, I^m_{jkm}\right), min\left(F^o_{ijk}, F^m_{jkm}\right)\right]$.

3. Decision Making method based on PNHSS sets.

Definition 3.1. Choice Matrix (CM) and CombinedChoice Matrix (CHMX).

CM is a square matrix whose rows and columns both indicates parameters.

CM =

$\begin{bmatrix} (1,0,0), & if\ i^{th}\ and\ j^{th}\ both\ parameters\ are\ the\ choice\ parameters\ of\ decision\ maker \\ (0,1,1), & if\ atleast\ one\ of\ the\ i^{th}\ or\ j^{th}\ parameters\ be\ not\ under\ choice\ of\ the\ decision\ maker \end{bmatrix}$

In combined CHMX, rows indicate choice parameters of single decision maker and columns indicate combined choice parameters (obtained by the intersection of parameter sets) of the remaining decision makers. In CM, both row and column indicate the attributes of same decision maker.

Definition 3.2. The cardinal set of the PNHSS $(\gamma, \Im)$ over U is defined as

Car $(\gamma) = \left\{ \dfrac{\left(T_{Car\gamma(\alpha)}(x), I_{Car\gamma(\alpha)}(x), F_{Car\gamma(\alpha)}(x)\right)}{x} : x \in \Im \right\}$, where

$T_{Car\gamma(\alpha)}(x) = \sum_{d\in U} \dfrac{T_{\gamma(\alpha)}(d)}{|U|}$ ……………………….. (1)

$I_{Car\gamma(\alpha)}(x) = \sum_{d\in U} \dfrac{I_{\gamma(\alpha)}(d)}{|U|}$ ……………………….. (2)

$F_{Car\gamma(\alpha)}(x) = \sum_{d\in U} \dfrac{F_{\gamma(\alpha)}(d)}{|U|}$ ……………………….. (3)

(1), (2) and (3) are called scalar cardinalities of PNHSS γ, and $|U|$ represents cardinality of the universe U.

Cardinal Score (CSc) of a cardinal set (CS) is defined by

$$SC\ (Car\ \gamma) = \sum_{x\in\mathfrak{I}}\left[T_{Car\gamma(\alpha)}(x)\right]^2 - \sum_{x\in\mathfrak{I}}\left[I_{Car\gamma(\alpha)}(x)\right]^2 - \sum_{x\in\mathfrak{I}}\left[F_{Car\gamma(\alpha)}(x)\right]^2.$$

A novel DM approach based on PNHSS will be created and its algorithm will be provided in this section. The foundation of this algorithm is made up of the Non-normalized PNHSM and the Normalized PNHSM situations. The flowchart of this algorithm is shown in figure 1.
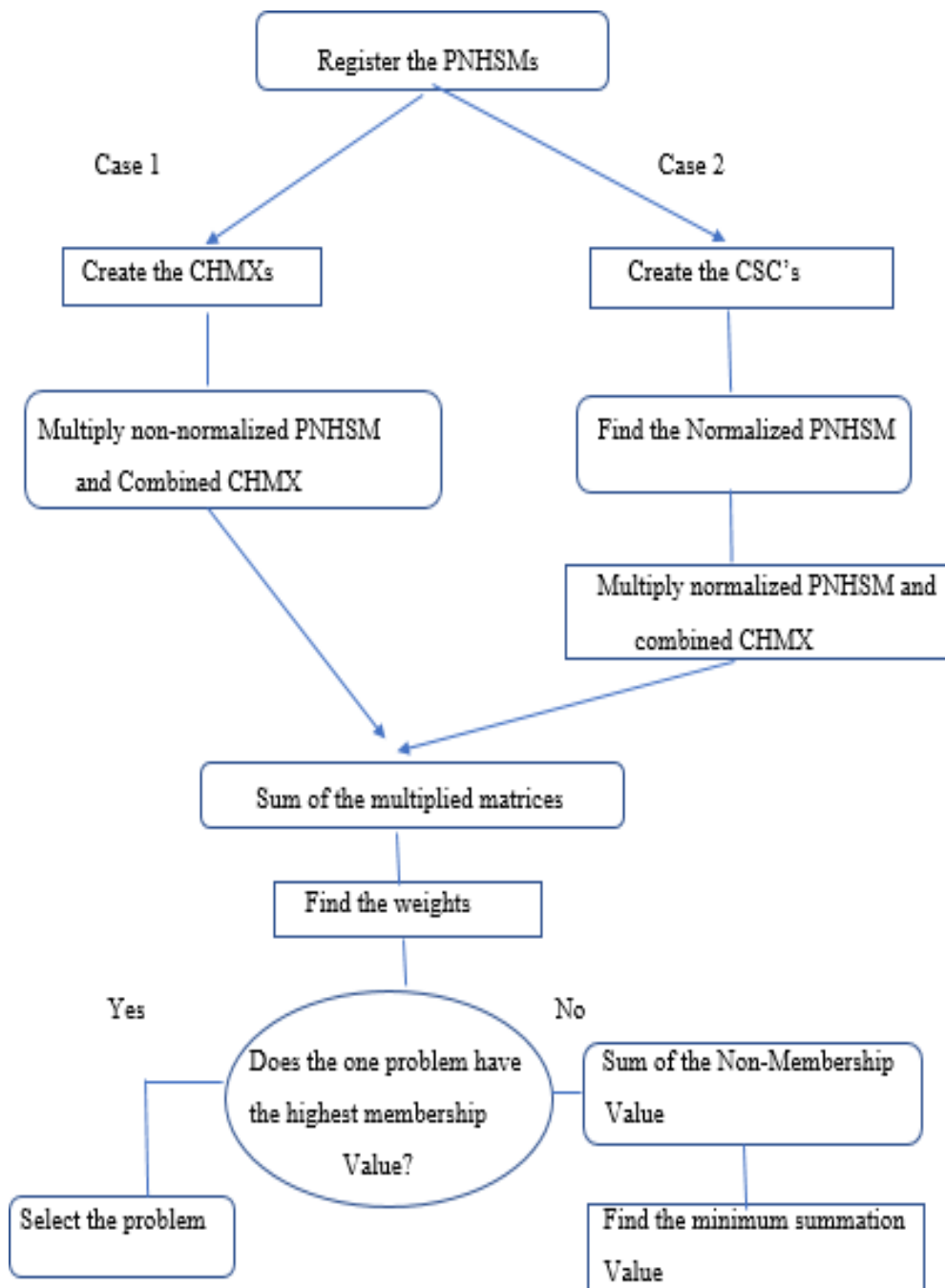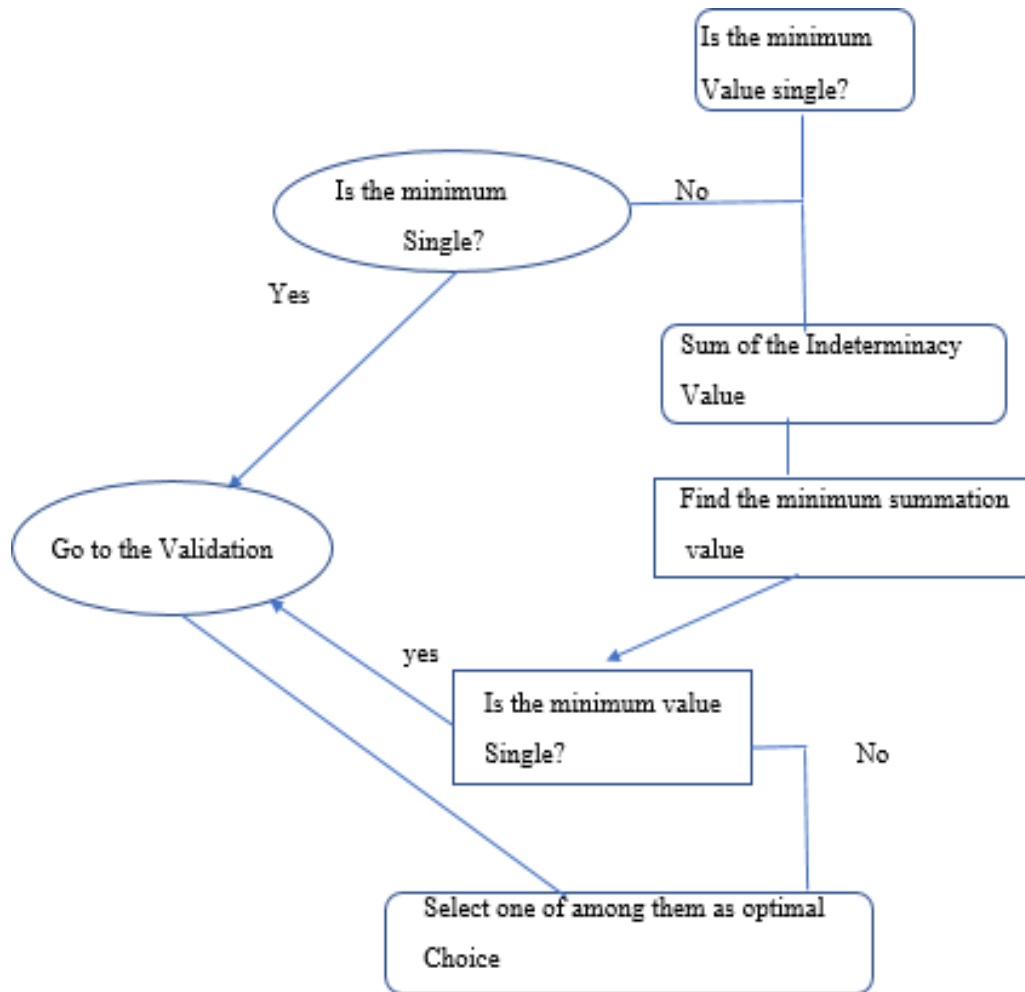
3.3. Algorithms. Case 1 (Non-Normalized PNHSM)

Step 1: Create the Non-Normalized PNHSMs.

Step 2: Construct CHMX'S.

Step 3: Evaluate the Max- Min product of Non-Normalized PNHSM with CHMX.

Step 4: Compute the sum of the product matrices in step 3.

Step 5: Find the weights by summing the Membership values.

Step 6: Compute the best alternative that is having highest Membership summation value.

Step 7: Go to the step 8, if more than one alternative has the same Membership summation

Value.

Step 8: Compute sum of the non-membership values.

Step 9: Find the best alternative that is having minimum non-membership summation value.

Step 10: Go to step 11, if more than one alternative has the same non-membership

summation value.

Step 11: Compute sum of the Indeterminacy values.

Step 12: Find the best alternative that is having minimum indeterminacy summation value.

Step 13: If more than one alternative has the same indeterminacy summation value, then any

Alternative among them is the optimal choice.

Case 2 (Normalized PNHSM)

Step 1: Create the PNHSMs.

Step 2: Compute the cardinal score (CSCs).

Step 3: Find the Normalized PNHSMs which is defined by $[a_{ij}] = [CSc * (a_{ij})]_{m \times n}$.

Step 4: Evaluate the Max- Min product of Normalized PNHSM with CHMX.

Step 5: Compute the sum of the product matrices in step 3.

Step 6: Find the weights by summing the Membership values.

Step 7: Compute the best alternative that is having highest Membership summation value.

Step 8: Go to the step 9, if more than one alternative has the same Membership summation

Value.

Step 9: Compute sum of the non-membership values.

Step 10: Find the best alternative that is having minimum non-membership summation value.

Step 11: Go to step 12, if more than one alternative has the same non-membership

summation value.

Step 12: Compute sum of the Indeterminacy values.

Step 13: Find the best alternative that is having minimum indeterminacy summation value.

Step 14: If more than one alternative has the same indeterminacy summation value, then any

Alternative among them is the optimal choice.

4. Formulation of the Problem.

The top five most common Learning Disability problems such as Dyslexia, ADHD, Dyscalculia, Dysgraphia, Dyspraxia ($LD = \{Ld_1, Ld_2, Ld_3, Ld_4, Ld_5\}$) were chosen based on the number of children affected by Learning Disability problems in recent years.

Let U be the set of common Learning Disabilities affected by the children's:

U = $\{Ld_1, Ld_2, Ld_3, Ld_4, Ld_5\}$.

Also, consider the set of attributes as

$\mathcal{A}_1$ = Age, $\mathcal{A}_2$ = Gender, $\mathcal{A}_3$ = Symptoms, $\mathcal{A}_4$ = Behavioural Mode, $\mathcal{A}_5$ = IQ.

Parameters:

1) $\mathcal{A}_1^a$ = Age $= \{1-2, 3-6, 7-12, Above\ 12\}$
2) $\mathcal{A}_2^b$ = Gender $= \{Male, Female\}$
3) $\mathcal{A}_3^c$ = Symptoms = $\{Difficulty\ Memorising, Absent-Mindedness, Short\ attention\ span, Speech\ impairement\}$
4) $\mathcal{A}_4^d$ = Behavioural Mode = $\{Anger, Boredom, Mood\ swings, Aggression\}$
5) $\mathcal{A}_5^e$ = IQ = $\{High\ IQ, Average\ IQ, Above\ IQ\}$

The function F: $\mathcal{A}_1^a \times \mathcal{A}_2^b \times \mathcal{A}_3^c \times \mathcal{A}_4^d \times \mathcal{A}_5^e \to$ P(U).

Let us assume the relation F$\left(\mathcal{A}_1^a \times \mathcal{A}_2^b \times \mathcal{A}_3^c \times \mathcal{A}_4^d \times \mathcal{A}_5^e\right) =$

F$(3-6, Female, Difficulty\ Memorising, Mood\ swings, Average\ IQ)$.

Choose a group of four Psychologists as P = $\{P_1, P_2, P_3, P_4\}$. Let us consider these Psychologists are examining the parameters of the Learning Disability problems among children's group. According to the observed range of attributes, the PNHSMs $p_{ij}, q_{ij}, r_{ij}, t_{ij}$ are constructed for Psychologists P_i.

.

Case 1: In this case, we can use non-normalized PNHSM.

Step 1: The opinions of the Psychologists related to the set of attributes and problems of a child are registered in PNHSMs.

$$
P = [p_{ij}] = \begin{bmatrix}
(0.3,0.2,0.1) & (0.8,0.4,0.3) & (0.6,0.3,0.1) & (0.7,0.5,0.2) & (0.6,0.3,0.2) \\
(0.8,0.3,0.2) & (0.6,0.2,0.1) & (0.5,0.2,0.1) & (0.6,0.3,0.1) & (0.4,0.2,0.1) \\
(0.7,0.2,0.1) & (0.5,0.3,0.1) & (0.6,0.3,0.2) & (0.3,0.2,0.1) & (0.8,0.3,0.1) \\
(0.6,0.4,0.3) & (0.4,0.2,0.1) & (0.7,0.4,0.2) & (0.4,0.3,0.1) & (0.5,0.2,0.1) \\
(0.5,0.3,0.1) & (0.3,0.2,0.1) & (0.6,0.2,0.1) & (0.5,0.2,0.1) & (0.4,0.2,0.1)
\end{bmatrix}
$$

$$
Q = [q_{ij}] = \begin{bmatrix}
(0.6,0.3,0.1) & (0.5,0.2,0.1) & (0.4,0.2,0.1) & (0.8,0.5,0.3) & (0.7,0.3,0.1) \\
(0.7,0.4,0.3) & (0.4,0.2,0.1) & (0.5,0.3,0.2) & (0.4,0.3,0.2) & (0.8,0.4,0.1) \\
(0.5,0.3,0.1) & (0.6,0.3,0.2) & (0.9,0.5,0.2) & (0.5,0.2,0.1) & (0.6,0.3,0.1) \\
(0.4,0.2,0.1) & 0.8,0.5,0.3 & (0.6,0.3,0.1) & (0.6,0.3,0.1) & (0.5,0.2,0.1) \\
(0.8,0.3,0.2) & (0.9,0.3,0.1) & (0.5,0.2,0.1) & (0.3,0.2,0.1) & (0.7,0.3,0.2)
\end{bmatrix}
$$

$$
R = [r_{ij}] = \begin{bmatrix}
(0.7,0.4,0.2) & (0.8,0.3,0.1) & (0.8,0.5,0.3) & (0.7,0.6,0.5) & (0.7,0.5,0.3) \\
(0.8,0.3,0.1) & (0.6,0.4,0.2) & (0.6,0.3,0.2) & (0.5,0.3,0.2) & (0.6,0.3,0.2) \\
(0.6,0.2,0.1) & (0.5,0.3,0.1) & (0.7,0.2,0.1) & (0.4,0.2,0.1) & (0.5,0.2,0.1) \\
(0.5,0.3,0.2) & 0.3,0.2,0.1 & (0.5,0.2,0.1) & (0.6,0.3,0.1) & (0.3,0.2,0.1) \\
(0.4,0.2,0.1) & (0.4,0.3,0.1) & (0.6,0.3,0.2) & (0.8,0.4,0.3) & (0.7,0.3,0.2)
\end{bmatrix}
$$

$$
T = [t_{ij}] = \begin{bmatrix}
(0.8,0.5,0.2) & (0.7,0.3,0.1) & (0.6,0.3,0.2) & (0.5,0.3,0.1) & (0.3,0.2,0.1) \\
(0.6,0.3,0.1) & (0.5,0.2,0.1) & (0.3,0.2,0.1) & (0.8,0.5,0.1) & (0.6,0.4,0.2) \\
(0.7,0.6,0.2) & (0.8,0.3,0.2) & (0.5,0.3,0.1) & (0.6,0.2,0.1) & (0.7,0.5,0.2) \\
(0.5,0.3,0.1) & 0.6,0.2,0.1 & (0.4,0.2,0.1) & (0.8,0.3,0.2) & (0.7,0.3,0.2) \\
(0.6,0.2,0.1) & (0.7,0.3,0.2) & (0.6,0.5,0.1) & (0.5,0.3,0.1) & (0.6,0.3,0.1)
\end{bmatrix}
$$

Step 2: The CHMX are created according to the Psychologist's CPs. The CHMX are obtained for all DMKRs. Psychologist A = P_1, Psychologist B = P_2, Psychologist C = P_3, Psychologist D = P_4 as follows:

$$
CM_{P_1} = \begin{bmatrix}
(1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) & (0,1,1) \\
(1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) & (0,1,1) \\
(1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) & (0,1,1) \\
(1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) & (0,1,1) \\
(1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) & (0,1,1)
\end{bmatrix}
$$

$$
CM_{P_2} = \begin{bmatrix}
(1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) & (1,0,0) \\
(1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) & (1,0,0) \\
(1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) & (1,0,0) \\
(1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) & (1,0,0) \\
(0,1,1) & (0,1,1) & (0,1,1) & (0,1,1) & (0,1,1)
\end{bmatrix}
$$

$$CM_{P_3} = \begin{bmatrix} (1,0,0) & (1,0,0) & (0,1,1) & (1,0,0) & (0,1,1) \\ (1,0,0) & (1,0,0) & (0,1,1) & (1,0,0) & (0,1,1) \\ (1,0,0) & (1,0,0) & (0,1,1) & (1,0,0) & (0,1,1) \\ (0,1,1) & (0,1,1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (1,0,0) & (1,0,0) & (0,1,1) & (1,0,0) & (0,1,1) \end{bmatrix}$$

$$CM_{P_4} = \begin{bmatrix} (1,0,0) & (1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) \\ (1,0,0) & (1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) \\ (0,1,1) & (0,1,1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (1,0,0) & (1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) \\ (1,0,0) & (1,0,0) & (1,0,0) & (0,1,1) & (0,1,1) \end{bmatrix}$$

Step 3: Multiply PNHSMs with CHMXs. The multiplication process here will be done according to the multiplication rule of PNHSMs.

$$[p_{ij}] \times CM_{P_1} =$$

$$\begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.6,0.2,0.1) & (0.6,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \end{bmatrix}$$

$$[q_{ij}] \times CM_{P_2} =$$

$$\begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0.8,0.2,0.1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) & (0.7,0.2,0.1) \\ (0.5,0.3,0.1) & (0.9,0.2,0.1) & (0,1,1) & (0,1,1) & (0.9,0.2,0.1) \\ (0.4,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0.8,0.2,0.1) \\ (0.8,0.3,0.2) & (0.9,0.2,0.1) & (0,1,1) & (0,1,1) & (0.9,0.2,0.1) \end{bmatrix}$$

$$[r_{ij}] \times CM_{P_3} =$$

$$\begin{bmatrix} (0.8,0.3,0.1) & (0.8,0.3,0.1) & (0,1,1) & (0.8,0.3,0.1) & (0,1,1) \\ (0.8,0.3,0.1) & (0.8,0.3,0.1) & (0,1,1) & (0.8,0.3,0.1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0.7,0.2,0.1) & (0,1,1) \\ (0.5,0.2,0.1) & (0.5,0.2,0.1) & (0,1,1) & (0.5,0.2,0.1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0.7,0.2,0.1) & (0,1,1) \end{bmatrix}$$

$$[t_{ij}] \times CM_{P_4} =$$

$$\begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) \end{bmatrix}$$

Step 4: The sum of the product matrices obtained in step 3.

$$\begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.6,0.2,0.1) & (0.6,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \end{bmatrix} +$$

$$\begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0.8,0.2,0.1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) & (0.7,0.2,0.1) \\ (0.5,0.3,0.1) & (0.9,0.2,0.1) & (0,1,1) & (0,1,1) & (0.9,0.2,0.1) \\ (0.4,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0.8,0.2,0.1) \\ (0.8,0.3,0.2) & (0.9,0.2,0.1) & (0,1,1) & (0,1,1) & (0.9,0.2,0.1) \end{bmatrix} +$$

$$\begin{bmatrix} (0.8,0.3,0.1) & (0.8,0.3,0.1) & (0,1,1) & (0.8,0.3,0.1) & (0,1,1) \\ (0.8,0.3,0.1) & (0.8,0.3,0.1) & (0,1,1) & (0.8,0.3,0.1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0.7,0.2,0.1) & (0,1,1) \\ (0.5,0.2,0.1) & (0.5,0.2,0.1) & (0,1,1) & (0.5,0.2,0.1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0.7,0.2,0.1) & (0,1,1) \end{bmatrix} +$$

$$\begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) \end{bmatrix}$$

$$= \begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.3,0.1) & (0.8,0.2,0.1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.3,0.1) & (0.7,0.2,0.1) \\ (0.9,0.2,0.1) & (0.9,0.2,0.1) & (0.8,0.2,0.1) & (0.7,0.2,0.1) & (0.9,0.2,0.1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.5,0.2,0.1) & (0.8,0.2,0.1) \\ (0.9,0.2,0.1) & (0.9,0.2,0.1) & (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0.9,0.2,0.1) \end{bmatrix}$$

Step 5: Calculate the weights of these problems:

$W(d_1) = 4.0$, $W(d_2) = 3.9$, $W(d_3) = 4.2$, $W(d_4) = 3.7$, $W(d_5) = 4.1$

Step 6: Now as the LD'P d_2 has the highest MV, therefore it is the optimal observation by all of the Psychologists. More precisely, it is stated that all the four Psychologists have reached a

common opinion from their individual observations that the child is affected by Dyslexia problem.

Case 2: (Normalized PNHSMs)

Step 1: Let's use the PNHSMs p_{ij}, q_{ij}, r_{ij}, t_{ij} in case 1.

Step 2: Let's first calculate the cardinals for $[p_{ij}]$, $[q_{ij}]$, $[r_{ij}]$, $[t_{ij}]$. CMat of the PNHSM,

$[p_{ij}]_{1\times5}$ =
$[(0.58,0.28,0.16) \quad (0.52,0.26,0.14) \quad (0.6,0.28,0.14) \quad (0.5,0.3,0.12) \quad (0.54,0.24,0.12)]$
and SC (Car P) = 1.5084-0.372-0.0936 = 1.04 $\cong$ 1.

$[q_{ij}]_{1\times5}$ =
$[(0.6,0.3,0.16) \quad (0.64,0.3,0.16) \quad (0.58,0.3,0.14) \quad (0.52,0.3,0.16) \quad (0.66,0.3,0.12)]$ and SC (Car Q) = 1.812-0.45-0.1108 = 1.2512$\cong$ 1.

$[r_{ij}]_{1\times5}$ =
$[(0.6,0.28,0.14) \quad (0.52,0.3,0.12) \quad (0.64,0.3,0.18) \quad (0.6,0.36,0.24) \quad (0.56,0.3,0.18)]$ and SC (Car R) = 1.7136-0.478-0.1564 = 1.0792 $\cong$ 1.

$[t_{ij}]_{1\times5}$ =
$[(0.64,0.38,0.14) \quad (0.66,0.26,0.14) \quad (0.48,0.3,0.12) \quad (0.64,0.32,0.12) \quad (0.58,0.34,0.16)]$
and SC (Car T) = 1.8216-0.52-0.0936 = 1.208$\cong$ 1.

Step 3: Normalized PNHSMs are NP = $N_{PNHSM}[p_{ij}] = [1.0 * p_{ij}]$, NQ = $N_{PNHSM}[q_{ij}] = [1.0 * q_{ij}]$, NR = $N_{PNHSM}[r_{ij}] = [1.0 * r_{ij}]$, NT = $N_{PNHSM}[t_{ij}] = [1.0 * t_{ij}]$ are as follows:

$$NP = \begin{bmatrix} (0.3,0.2,0.1) & (0.8,0.4,0.3) & (0.6,0.3,0.1) & (0.7,0.5,0.2) & (0.6,0.3,0.2) \\ (0.8,0.3,0.2) & (0.6,0.2,0.1) & (0.5,0.2,0.1) & (0.6,0.3,0.1) & (0.4,0.2,0.1) \\ (0.7,0.2,0.1) & (0.5,0.3,0.1) & (0.6,0.3,0.2) & (0.3,0.2,0.1) & (0.8,0.3,0.1) \\ (0.6,0.4,0.3) & (0.4,0.2,0.1) & (0.7,0.4,0.2) & (0.4,0.3,0.1) & (0.5,0.2,0.1) \\ (0.5,0.3,0.1) & (0.3,0.2,0.1) & (0.6,0.2,0.1) & (0.5,0.2,0.1) & (0.4,0.2,0.1) \end{bmatrix}$$

$$NQ = \begin{bmatrix} (0.6,0.3,0.1) & (0.5,0.2,0.1) & (0.4,0.2,0.1) & (0.8,0.5,0.3) & (0.7,0.3,0.1) \\ (0.7,0.4,0.3) & (0.4,0.2,0.1) & (0.5,0.3,0.2) & (0.4,0.3,0.2) & (0.8,0.4,0.1) \\ (0.5,0.3,0.1) & (0.6,0.3,0.2) & (0.9,0.5,0.2) & (0.5,0.2,0.1) & (0.6,0.3,0.1) \\ (0.4,0.2,0.1) & 0.8,0.5,0.3 & (0.6,0.3,0.1) & (0.6,0.3,0.1) & (0.5,0.2,0.1) \\ (0.8,0.3,0.2) & (0.9,0.3,0.1) & (0.5,0.2,0.1) & (0.3,0.2,0.1) & (0.7,0.3,0.2) \end{bmatrix}$$

$$NR = \begin{bmatrix} (0.7,0.4,0.2) & (0.8,0.3,0.1) & (0.8,0.5,0.3) & (0.7,0.6,0.5) & (0.7,0.5,0.3) \\ (0.8,0.3,0.1) & (0.6,0.4,0.2) & (0.6,0.3,0.2) & (0.5,0.3,0.2) & (0.6,0.3,0.2) \\ (0.6,0.2,0.1) & (0.5,0.3,0.1) & (0.7,0.2,0.1) & (0.4,0.2,0.1) & (0.5,0.2,0.1) \\ (0.5,0.3,0.2) & 0.3,0.2,0.1 & (0.5,0.2,0.1) & (0.6,0.3,0.1) & (0.3,0.2,0.1) \\ (0.4,0.2,0.1) & (0.4,0.3,0.1) & (0.6,0.3,0.2) & (0.8,0.4,0.3) & (0.7,0.3,0.2) \end{bmatrix}$$

$$NT = \begin{bmatrix} (0.8,0.5,0.2) & (0.7,0.3,0.1) & (0.6,0.3,0.2) & (0.5,0.3,0.1) & (0.3,0.2,0.1) \\ (0.6,0.3,0.1) & (0.5,0.2,0.1) & (0.3,0.2,0.1) & (0.8,0.5,0.1) & (0.6,0.4,0.2) \\ (0.7,0.6,0.2) & (0.8,0.3,0.2) & (0.5,0.3,0.1) & (0.6,0.2,0.1) & (0.7,0.5,0.2) \\ (0.5,0.3,0.1) & 0.6,0.2,0.1 & (0.4,0.2,0.1) & (0.8,0.3,0.2) & (0.7,0.3,0.2) \\ (0.6,0.2,0.1) & (0.7,0.3,0.2) & (0.6,0.5,0.1) & (0.5,0.3,0.1) & (0.6,0.3,0.1) \end{bmatrix}$$

Step 4: Take the combined CHMX as in case 1.

Step 5: we will multiply normalized PNHSMs with combined

CHMXs $\left(NCP = N_{PNHSM}[p_{ij}] \times CM_{P_1}, NCQ = N_{PNHSM}[q_{ij}] \times CM_{P_2}, NCR = N_{PNHSM}[r_{ij}] \times CM_{P_3}, NCT = N_{PNHSM}[t_{ij}] \times CM_{P_4} \right)$.

$$NCP = \begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.6,0.2,0.1) & (0.6,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \end{bmatrix}$$

$$NCQ = \begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0.8,0.2,0.1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) & (0.7,0.2,0.1) \\ (0.5,0.3,0.1) & (0.9,0.2,0.1) & (0,1,1) & (0,1,1) & (0.9,0.2,0.1) \\ (0.4,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0.8,0.2,0.1) \\ (0.8,0.3,0.2) & (0.9,0.2,0.1) & (0,1,1) & (0,1,1) & (0.9,0.2,0.1) \end{bmatrix}$$

$$NCR = \begin{bmatrix} (0.8,0.3,0.1) & (0.8,0.3,0.1) & (0,1,1) & (0.8,0.3,0.1) & (0,1,1) \\ (0.8,0.3,0.1) & (0.8,0.3,0.1) & (0,1,1) & (0.8,0.3,0.1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0.7,0.2,0.1) & (0,1,1) \\ (0.5,0.2,0.1) & (0.5,0.2,0.1) & (0,1,1) & (0.5,0.2,0.1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0.7,0.2,0.1) & (0,1,1) \end{bmatrix}$$

$$NCT = \begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) \end{bmatrix}$$

Step 6: The sum of the product matrices obtained in step 5.

$$\begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0.6,0.2,0.1) & (0.6,0.2,0.1) & (0,1,1) & (0,1,1) & (0,1,1) \end{bmatrix} +$$

$$\begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0.8,0.2,0.1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) & (0.7,0.2,0.1) \\ (0.5,0.3,0.1) & (0.9,0.2,0.1) & (0,1,1) & (0,1,1) & (0.9,0.2,0.1) \\ (0.4,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) & (0.8,0.2,0.1) \\ (0.8,0.3,0.2) & (0.9,0.2,0.1) & (0,1,1) & (0,1,1) & (0.9,0.2,0.1) \end{bmatrix} +$$

$$\begin{bmatrix} (0.8,0.3,0.1) & (0.8,0.3,0.1) & (0,1,1) & (0.8,0.3,0.1) & (0,1,1) \\ (0.8,0.3,0.1) & (0.8,0.3,0.1) & (0,1,1) & (0.8,0.3,0.1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0.7,0.2,0.1) & (0,1,1) \\ (0.5,0.2,0.1) & (0.5,0.2,0.1) & (0,1,1) & (0.5,0.2,0.1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0.7,0.2,0.1) & (0,1,1) \end{bmatrix} +$$

$$\begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0,1,1) & (0,1,1) \\ (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0,1,1) & (0,1,1) \end{bmatrix}$$

$$= \begin{bmatrix} (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.3,0.1) & (0.8,0.2,0.1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.3,0.1) & (0.7,0.2,0.1) \\ (0.9,0.2,0.1) & (0.9,0.2,0.1) & (0.8,0.2,0.1) & (0.7,0.2,0.1) & (0.9,0.2,0.1) \\ (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.8,0.2,0.1) & (0.5,0.2,0.1) & (0.8,0.2,0.1) \\ (0.9,0.2,0.1) & (0.9,0.2,0.1) & (0.7,0.2,0.1) & (0.7,0.2,0.1) & (0.9,0.2,0.1) \end{bmatrix}$$

Step 7: Calculate the weights of these problems:

$W(d_1) = 4.0$, $W(d_2) = 3.9$, $W(d_3) = 4.2$, $W(d_4) = 3.7$, $W(d_5) = 4.1$

Step 8: In this case also, the Learning Disability problem d_2 has the highest MV, therefore it is the optimal observation by all of the Psychologists.

The same result has been achieved in both cases of the algorithm.

Conclusion:

In this paper, we developed a decision-making method related to PNHSSs. To conform the validity of our established approach, a comprehensive numerical example has been developed

and the results of both the cases are compared at the end. Both cases provide the same optimum decision.

References

[1] D. Molodtsov, "Soft set theory-First results," Computers and Mathematics with Applications, vol. 37, no. 4-5, pp. 19-31, 1999.

[2] F. Smarandache, "Neutrosophic set, a generalization of the Intuitionistic fuzzy sets," International Journal of Pure and Applied Mathematics, vol. 24, no. 3, pp. 287 – 297, 2004.

[3] F. Smarandache," Extension of soft set to Hypersoft set, and then to Plithogenic Hypersoft set," Neutrosophic sets and system, vol. 22, pp. 168 – 170, 2018.

[4] G. Ramya, A. Francina Shalini, "Aggregate Operators of Pythagorean Neutrosophic Hypersoft Set,"Neutrosophic sets and system (Communicated).

[5] G. Ramya, A. Francina Shalini, "Matrix theory for Pythagorean Neutrosophic Hypersoft Matrices,"International Journal of Neutrosophic Science (Communicated).

[6] I. Deli and S. Broumi, "Neutrosophic soft matrices and NSM-decision making," Journal of Intelligent and Fuzzy Systems, Vol. 28, no.5, pp.2233-2241,2015.

[7] K. T. Atanassov, "Intuitionistic fuzzy sets," Fuzzy sets and systems, vol. 20, no. 1, pp. 87-96, 1986.

[8] L. A. Zadeh, "Fuzzy sets," Information and control, vol. 8, no. 3, pp. 338-353, 1965.

[9] M. Saqlain, S. Moin, M.N. Jafar, M. Saeed, and F. Smarandache," Aggregate Operators of Neutrosophic Hypersoft Set", Neutrosophic Sets and Systems, vol. 32, pp. 294-306, 2020.

[10] P. K. Maji, A. R. Biswas, and A. R. Roy, "An application of soft sets in a decision-making problem," Computers and Mathematics with Applications, vol. 44, no. 8-9, pp. 1077 – 1083, 2002.

[11] P. K. Maji, R. Biswas, and A. R. Roy," Soft set theory," Computers and Mathematics with Applications, vol. 45, no. 4-5, pp. 555 – 562, 2003.